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ELEMENTS OF ELECTRICAL ENGINEERING

A TEXTBOOK OF

PRINCIPLES AND PRACTICE

BY

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PREFACE

This book deals with the fundamentals of Electrical Engineering and their application in practice. It is intended as a short course for electrical students and also for non-electrical students in colleges. The author has used the methods of presentation which he has found most satisfactory as the result of his own teaching experience. No attempt has been made to describe all types of electrical machinery. The subjects discussed are those which the author's engineering experience has shown are most commonly needed by the average engineer and those which best serve to illustrate the fundamental laws. The student is expected to have a knowledge of the principles of electricity and magnetism, such as would be given in a course in Physics; but, for convenience, these principles are summarized in Part I.

It is now a generally accepted fact that the student should learn the fundamental principles underlying Electrical Engineering practice, rather than merely accumulate a mass of information regarding specific applications. The student does not, however, really understand these fundamentals until he has had practice in identifying the application of these fundamental principles in their numerous applications. The method of presentation is intended to bring out the physical significance of a result, and not to depend merely upon the mathematical derivation of a formula. In fact, the use of formulas is reduced to a minimum and the student is encouraged to derive the result from the fundamental laws, such as the force action on a conductor, or the voltage produced by electromagnetic induction. The specific applications described and the problems used have been chosen to provide practice in identifying the fundamental laws upon which the operation of a machine is based. The data used in the problems have been carefully chosen to represent good practice and results which might occur in practice. Special attention is given to magnetic circuits as these are not so well understood by the average student as electric circuits. A large proportion of the book is devoted to alternating-current circuits and machinery, because students have greater difficulty in understanding these subjects, and also because a large proportion of the electrical apparatus found commonly in practice is of the alternating-current type.

Extensive use has been made of illustrations of electrical machinery which have been supplied by various manufacturers. Acknowledgment of these is given in every case. Special acknowledgment should be made of the assistance rendered by the Westinghouse Electric and Manufacturing Co., the General Electric Co., and The Electric Storage Battery Co., who have not only supplied a number of illustrations but have also furnished performance data on their apparatus.

A. L. C.

Pratt Institute,
Brooklyn, N. Y.,
August, 1924.

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ELEMENTS OF ELECTRICAL ENGINEERING

PART I

ELECTRIC AND MAGNETIC CIRCUITS

CHAPTER I

ELECTRIC CIRCUITS AND THE ELECTRIC CURRENT

1. The Electric Circuit.—Electricity is a medium which provides a convenient means of transferring energy from one place to another, and of changing the form in which this energy exists. This transfer and conversion is accompanied by a "flow of electricity" which is known as the electric current. Electricity is, however, not a *source* of energy.

The electric circuit is the path for the electric current. If a flow of electricity is to take place in a circuit, there must be a complete path for the current. Such a path is called a closed circuit. If the path is interrupted so that no electricity flows, even if there is a source of electricity present, the circuit is said to be *open.* Every electric circuit must have a source of electrical energy if electricity is to flow and energy is to be transferred from one part of the circuit to another.

2. Electric Currents are of three classes: (*a*) continuous, (*b*) alternating, and (*c*) pulsating. A continuous current flows continuously in one direction and at a steady value. Thus, the current which would flow in a circuit containing a battery, Fig. 1 *c*, would be a continuous current, and could be represented by a straight line as in Fig. 1 *a*. The term direct current is used to refer to ordinary continuous currents which are approximately steady, such as the current produced by an ordinary continuous current generator. The curve for current from such a machine is represented by Fig. 1 *b*. The abbreviation d.c. is used to refer to ordinary continuous currents such as are produced by batteries or by continuous current generators.

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The flow of direct current is similar to the flow of water in a pipe line supplied by a centrifugal pump (Fig. 1 *d*). An alternating current reverses or flows in alternate directions regularly. This is illustrated by Fig. 2 *a*, which indicates that the current for a certain length of time flows in a positive direction through the circuit and then reverses and flows in the opposite direction for the same length of time. The action is similar to the flow of water produced by a reciprocating pump without valves (Fig. 2 *b*). The abbreviation a.c. is used in referring to such a current. A pulsating current pulsates or changes regularly in magnitude. This term is usually applied to a current which flows in one direction, as indicated by the curve (Fig. 3 *a*). The flow of water, due to a reciprocating pump with valves, would correspond to this class of current. A pulsating current is sometimes produced by reversing, through mechanical or other means, the flow of an alternating current such as shown in Fig. 2 *a*.

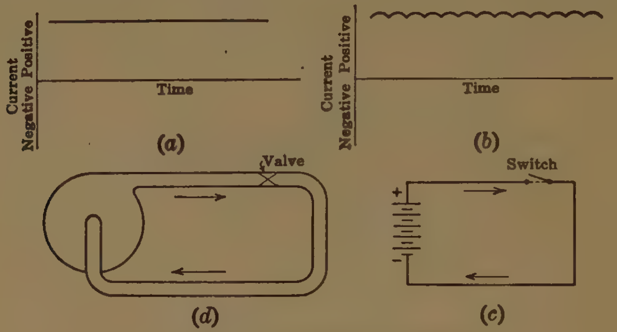

FIG. 1.—Continuous Current.

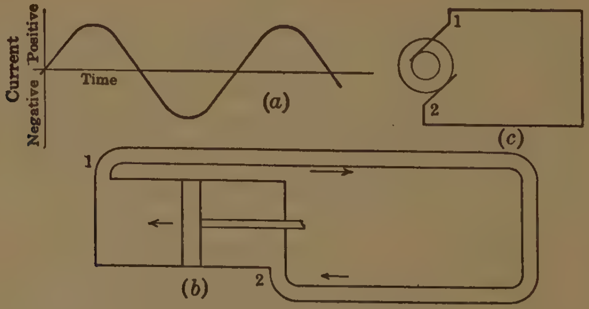

FIG. 2.—Alternating Current.

3. Effects Produced by an Electric Current.—Whenever an electric current flows in a circuit, it produces a heating effect and a magnetic effect and in some cases a chemical effect also. The heating effect occurs in the electric circuit itself and is shown by an increase in temperature of the circuit. It represents a transformation of the electrical form of energy into heat energy and always occurs to a greater or less degree whenever a current flows. The current which flows through the filament of an incandescent lamp heats the filament to a high temperature and nearly all the electrical energy transferred to the lamp is transformed into heat. The light which is produced is an indirect result of the heating effect. The heating effect represents a continual transformation of energy from the electrical to the heat form. The magnetic effect is shown by the ability of a circuit carrying a current to attract pieces of iron. This effect always exists whenever an electric current flows, although in many cases the effect is so weak that it is neglected in dealing with the circuit. As long as the current is steady the magnetic effect does not represent any continuous expenditure of energy, as does the heating effect. The chemical effect occurs when an electric current flows through a liquid. It may cause a metal to be transferred from one portion of the circuit to another, as in electroplating, or it may produce chemical changes in the portion of the circuit comprising the liquid and the two terminals of the circuit which are immersed in the liquid, as is done when a storage battery is charged.

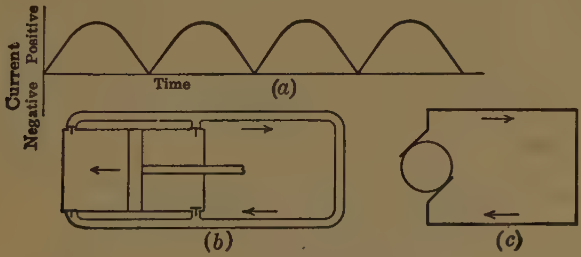

Fig. 3.—Pulsating Current.

4. Electromotive Force.—The force which causes an electric current to flow and thereby energy to be transferred, is called an electromotive force, (abbreviated, e.m.f.). A battery or other source of an electromotive force is not a *generator* of electricity but is simply a device which possesses a force capable of causing an electric current to flow.

5. Electrical Units.—The units commonly used when dealing with an electric circuit are:

(*a*) The unit of resistance or opposition to the flow of a current is the ohm, symbol *R*. The standard is the " international ohm " which is defined as the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass of a constant cross-sectional area and a length of 106.3 centimeters. A round copper wire, 1000 feet long and 0.1 inch in diameter has a resistance of approximately one ohm. For very high resistances, the term megohm is used. One megohm equals one million ohms. For very low resistance the microhm is used. One microhm is one-millionth of an ohm.

(*b*) The unit of current is the ampere, symbol I. This is a measure of the rate at which electricity is being transferred in the circuit and corresponds to gallons per second in a hydraulic system. It is defined as the practical equivalent of the unvarying current which, when passed through a standard solution of nitrate of silver in water, deposits silver at the rate of 0.001118 gram per second. For small currents the milliampere is used. One milliampere = 0.001 ampere. In some cases the microampere is used. One microampere equals one-millionth of an ampere.

(*c*) The unit of electromotive force (e.m.f.) is the volt, symbol E. An electromotive force tends to produce a current in a circuit, and corresponds to pressure in a hydraulic system. It is defined as the electrical pressure which, when steadily applied to a conductor having a resistance of 1 ohm, will produce a current of 1 ampere. An ordinary dry cell has an e.m.f. of 1.5 volts and an ordinary lead storage cell 2 volts. For small voltages the millivolt is used; 1 millivolt = 0.001 volt. For very high voltages the kilovolt is used; 1 kilovolt = 1000 volts.

(*d*) The quantity of electricity, symbol Q, is expressed by the product of current and time and corresponds to gallons or cubic feet in a hydraulic system. The common unit is the ampere-hour which is the quantity of electricity transferred by 1 ampere in one hour. The quantity transferred in one second is the ampere-second or coulomb. Hence 1 ampere-hour $= 60 \times 60 = 3600$ coulombs. The ampere-hours for a circuit can be found by multiplying the strength of the current in amperes by the time in hours during which the current flows.

(*e*) The unit of electric power is the watt, symbol P, and is defined as the power developed as heat in a resistance of 1 ohm by a direct current of 1 ampere. Electric power is frequently expressed in kilowatts (abbreviated kw.). One kilowatt equals 1000 watts. One horsepower equals 746 watts. One kilowatt is approximately $1\frac{1}{3}$ horsepower.

(*f*) The energy or work done in a circuit is expressed by the product of power and time, symbol W. The common unit is the kilowatt-hour (kw.hr.) which is the energy expended in a circuit during one hour when the power is being developed at a constant rate of 1000 watts. The watt-hour would be the energy corresponding to a power of 1 watt for one hour. The watt-second or energy developed by a power of 1 watt for one second is called the joule. The energy in kilowatt-hours is found by multiplying the power in kilowatts by the time in hours during which this power is developed. The energy equivalent of 1 kilowatt-hour is 3415 B.T.U., 2.655×10^6 foot-pounds, or 1.341 horsepower hours.

6. Conductors and Insulators.—Substances, which, when connected to a source of electromotive force, allow an appreciable current to flow, are called conductors. The most common conductors are metals, carbon, salts in solution, or melted salts. Substances which do not permit an appreciable current to flow are called insulators, as for example, air, glass, rubber, cotton, oils. It should be noted that no substance is a perfect insulator, but the amount of current which will flow through so-called insulators is inappreciable as compared with the current which would flow through conductors. Temperature has a decided effect upon the amount of current which some substances will conduct. Thus, glass, which is an insulator at ordinary temperatures, is a good conductor when at a red heat.

7. Properties of an Electric Circuit.—Electric circuits possess certain properties which influence the flow of electricity in the circuit. They are:

(*a*) **Resistance,** or the opposition to the flow of current.[1] It is manifested whether the current is steady (direct-current), pulsating or alternating, and causes a transformation of electrical energy into heat energy. In its effect, resistance can be compared to the friction of water flowing in a pipe.

(*b*) **Inductance,** or the opposition produced when a current in a circuit varies in strength. Its effect disappears when the current is steady and hence need commonly be considered only when the current is pulsating or alternating.[2] In its effect, inductance is similar to the inertia of water flowing in a pipe or to the inertia of a rotating mass such as a flywheel.

(*c*) **Electrostatic Capacity,** or the ability of an electric circuit to store up electricity. This effect is of importance only when the voltage of the system is pulsating or alternating. In its effect, electrostatic capacity is similar to the action of a spring which is compressed and released.[3]

PROBLEMS ON CHAPTER I

1–1. A storage battery discharging through a resistance has a terminal voltage of 6.15 volts at the beginning of discharge. After two hours, the voltage is 6.09 volts. Assuming that the terminal voltage decreases uniformly during the discharge period, plot a discharge curve between terminal voltage and time.

2–1. Plot a curve showing the reading of the voltmeter (Fig. 2–1) when the rotating commutator is in different positions.

3–1. A certain device is supplied from a source of alternating e.m.f. like that shown in the curve, Fig. 2*a*. The current can flow in a positive direction only. Plot a curve showing the current in the circuit during the time when terminal 1 (Fig. 2*c*) is positive and also when it is negative.

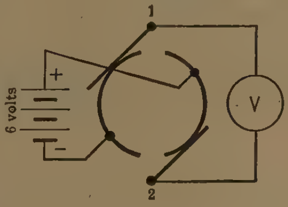

FIG. 2–1.

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[1] See Article 11.

[2] A more complete discussion of inductance is given in Articles 59 and 222.

[3] A more complete discussion of capacity is given in Articles 67 and 233.

4–1. Two sources of alternating e.m.f., *A* and *B*, similar to that shown in Fig. 2 are connected in series. Each of them produce the same shape of curve and the same maximum value of e.m.f. resembling the curve shown in Fig. 2*a*. The e.m.f. of *B* is a maximum when the e.m.f. of *A* is zero.

(*a*) Plot two curves like Fig. 2*a* to the same coordinates, showing the variation of e.m.f. of sources *A* and *B*.

(*b*) Plot a third curve showing the combined e.m.f. of *A* and *B*.

5–1. Name three ways by which the presence of a current of electricity in an electric circuit may be detected.

6–1. How is the magnetic effect of an electric current made manifest?

7–1. A certain lifting magnet requires the continual expenditure of 7260 watts in order to produce the magnetic field. It is capable of holding a load of 35,000 pounds of steel slabs. What becomes of the energy represented by this continuous input of power?

8–1. The insulation of a certain machine was found to have a resistance of 3,125,000 ohms. Express this in megohms.

9–1. The resistance of a certain block of copper is 0.00000135 ohm. Express this in microhms.

10–1. A certain voltmeter requires a current of 0.0094 ampere for full scale deflection. What is the current in milliamperes for full scale deflection?

11–1. A galvanometer gives full scale deflection with 0.0000126 ampere flowing. Express this in microamperes.

12–1. A d.c. millivoltmeter used with a shunt requires 0.049 volt for full scale deflection. Express this in millivolts.

13–1. A transmission line operates at a potential difference of 220 kilovolts between wires. Express this in volts.

14–1. A storage battery is charged for eight hours at an average rate of 45 amperes. What is the total quantity of electricity which is passed through the battery?

15–1. An electric condenser of a certain size is capable of storing 0.01 coulomb of electricity.

(*a*) If the condenser were discharged in 0.001 second, what would be the average value of current which flows on discharge?

(*b*) If the condenser were discharged in 0.0004 second, what would be the average value of the discharge current?

CHAPTER II

POWER AND WORK

8. Power in an Electric Circuit.—Power is the rate of doing work or the rate at which energy is transferred in the circuit. It is customary to speak of the power input or power output of a piece of apparatus when we mean the rate at which energy is supplied to or delivered by the apparatus. It has been proved experimentally that when a steady current of I amperes flows in a circuit having a resistance of R ohms, the power expended in the circuit is

$$P = I^2R \text{ watts.} \quad \ldots \ldots \ldots \quad (1)$$

Also, since this current will require a voltage across the terminals of this resistance of $E = IR$ volts,[1] the power is

$$P = EI \text{ watts.} \quad \ldots \ldots \ldots \quad (2)$$

One horsepower = 550 foot-pounds per second = 746 watts. The horsepower is, therefore, approximately equal to $\frac{3}{4}$ kilowatt and 1 kilowatt to $1\frac{1}{3}$ horsepower.

A size of an incandescent lamp commonly used requires the continual expenditure of 50 watts to maintain a current of 0.455 ampere, with a pressure of 110 volts. An electric motor delivering one horsepower requires a continual input of electric power of about 1000 watts or 4.35 amperes at 230 volts.

9. Work.—The work done in an electric circuit is the total amount of electrical energy transferred in the circuit. If the energy is being transferred continuously for one hour, at the steady rate of 1 kilowatt, the total energy transferred in the hour, or the work done, is 1 kilowatt-hour (kw.-hr.). It has been determined experimentally that 1kw.-hr. equals 3415 British thermal units.

An incandescent lamp which receives 50 watts continuously for 10 hours receives $50 \times 10 = 500$ watt-hours of electrical energy. This is equivalent to 0.68 horsepower-hour or 1708 B.T.U.

10. Efficiency and Losses.—Whenever a device is used to transform energy from one form to another or to transfer it from one place to another, a certain amount of energy is "lost," that is, it is changed into a form which is not usable. Usually this loss appears as heat. The difference between the energy input and energy output of the device is the energy loss, and the efficiency is the ratio of output to input. If the rate of energy input is constant, i.e., if the power is constant, the efficiency is also the ratio of power output to power input.

---

[1] See Article 23.

PROBLEMS ON CHAPTER II

1–2. An incandescent lamp requires 0.833 ampere at 120 volts. What is the power required to operate this lamp?

2–2. (*a*) How much current is required for a 500-watt incandescent lamp operating at 115 volts?

(*b*) If eight of these lamps are supplied from a single branch circuit, what should be the minimum size of fuse to be used in protecting the circuit?

3–2. A room is lighted by two 50-watt lamps and four 25-watt lamps. If the cost of energy is 8 cents per kw.-hr., calculate the monthly bill for lighting the room, assuming an average use of $3\frac{1}{2}$ hours per day and 30 days in the month.

4–2. An electric hoist has a mechanical efficiency of 75 per cent and a motor efficiency of 89 per cent.

(*a*) Find the kilowatt input to the motor when hoisting a load of 2500 pounds at a uniform velocity of 550 feet per minute.

(*b*) What would be the current required to drive the motor if it were supplied from a 550-volt d.c. system?

5–2. At 7 cents per kilowatt-hour, what is the cost per week of 60 hours to run a motor having an average load of 4 horsepower and efficiency of 80 per cent?

6–2. A certain motor load totals 800 amperes at 110 volts.

(*a*) What is the horsepower?

(*b*) If the voltage were 220, what would the current be for the same horsepower?

(*c*) What current, if the voltage were 600?

7–2. A coil of iron wire having a resistance of 75 ohms is used to carry a current of 15 amperes. The coil is immersed in water for cooling purposes. The water enters the receptacle at 20° C. and leaves at 80° C.

(*a*) What is the required weight of water per hour in pounds?

(*b*) What is the quantity in gallons per hour?

Assume weight of water 8 pounds per gallon. One B.T.U. is the heat required to raise the temperature of one pound of water one degree Fahrenheit. Neglect radiation from receptacle.

8–2. An electric furnace requires 45 amperes at 220 volts. How many kilowatts are required for operation?

9–2. A factory has 1500, 100-watt lamps, 500, 200-watt lamps, and 100, 500-watt lamps. Assume a generator efficiency of 91 per cent and a loss in the wiring of 12 per cent of the power at the lamps. Calculate the horsepower required to drive the generator supplying these lamps.

10–2. A generator is rated at 150 kw. and the manufacturer states that the efficiency at rated load is 91.5 per cent. What would be the horsepower required to drive this generator when carrying full load?

11–2. A 5-horsepower motor has an efficiency of 85 per cent and operates on a 230-volt system. Calculate the current input to the motor at full load.

12–2. Referring to Prob. 8–2, if energy costs 2 cents per kw.-hr., calculate the yearly cost for operating this furnace 10 hours per day, 300 days per year.

13–2. If the motor in Prob. 11–2 is operated at an average load of 75 per cent of its rating and runs 8 hours per day for 300 days per year, calculate the yearly cost for energy at a rate of 4 cents per kw.-hr. Motor efficiency 85 per cent.

14–2. A motor, when delivering 50 horsepower, has an input of 182 amperes at 230 volts. What is the efficiency of the motor?

CHAPTER III

RESISTANCE

11. Calculation of Resistance.—We have seen in Article 5 that the resistance of an electric circuit opposes the flow of an electric current in a circuit. The resistance of an electric circuit is a property of the conductor forming the circuit and depends upon the length, cross-sectional area, and temperature; and upon the material, whether copper, iron, brass, etc. It is expressed by the equation

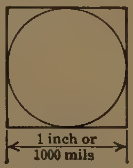
$$R=K\frac{L}{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where R is the resistance in ohms, L is the length, A the cross-sectional area, and K is a constant called the " specific resistance " or " resistivity." It follows from Equation (3) that the resistivity of a given material is numerically equal to the resistance of a wire or bar of the material which has unit length and unit cross-sectional area. The commonly used unit of resistivity is the circular mil-foot which is equal to the resistance of a wire 1 mil in diameter and 1 foot long. (See Article 14.) It will be seen from Equation (3) that the resistance of a conductor varies directly as its length, and inversely as its cross-section.

12. The Circular Mil.—The area A in Equation (3) may be expressed in square inches, square centimeters or other units, but the commonly used unit is the circular mil. This is defined as the area of a circle one mil or one-thousandth of an inch in diameter. The area of any solid conductor of circular cross-section can be found by squaring the diameter expressed in mils, thus:

Example 1.—A conductor has a diameter of 0.460 inch or 460 mils. Its area, expressed in circular mils, is therefore $A=460^2=211{,}600$ cir. mils.

A round conductor 1 inch or 1000 mils in diameter would have an area of $1000^2=1{,}000{,}000$ cir. mils or 0.7854 square inch (Fig. 4). Hence, 1 square inch is equivalent to $1{,}000{,}000\div0.7854=1{,}273{,}000$ cir. mils. The equivalent circular mils in a rectangular or square cross-section of conductor can be found by determining the area in square inches and using the above relation, or the area can be expressed in square mils and then reduced to circular mils. If the dimension of the square shown in Fig. 4 is expressed in mils, the area in square mils is $1000 \times 1000 = 1{,}000{,}000$ sq. mils. Since this square is equivalent to 1,273,000 cir. mils, 1 sq. mil = 1.273 cir. mils or 1 cir. mil = $1{,}000{,}000 \div 1{,}273{,}000 = 0.7854$ sq. mil. The square mil is a convenient unit to use when dealing with conductors having a square or rectangular cross-section. In using Equation (3), however, the area A should, in general, be expressed in circular mils; otherwise a different value of K must be used for circular and other cross-sections.



Circular Mils.
FIG. 4.

13. Wire Gages.—The cross-sections of standard sizes of electrical wires as used in the United States are not specified in square inches, but in circular mils or by numbers referred to a certain standard, called a wire gage. This standard is the American Wire Gage, formerly known as the Brown and Sharpe (B & S) wire gage.[1] The numbers extend from 0000 to 40, with a definite variation of section for each successive size. The cross-section doubles with every increase of three numbers. A No. 10 copper wire has a diameter of approximately 0.1 inch and a resistance of 1 ohm per 1000 feet. Wires larger than No. 0000, which is 0.46 inch in diameter, are designated by their circular mil equivalent.[2] The number of circular mils for a stranded conductor is found by adding the circular mils for each strand. It is customary to make up standard stranded conductors having the same copper cross-section as the larger sizes of solid wires. (See Table 2.) These stranded conductors are slightly larger in outside diameter than the corresponding size of solid wire.

14. Circular Mil-foot.—The value of resistivity (constant K in Equation (3)), as commonly used in practice, is the resistance of a wire 1 mil in diameter and 1 foot long. The value is therefore expressed in ohms per circular mil-foot. For ordinary " soft drawn " copper of 100 per cent conductivity, at 25° C. the resistivity $K = 10.6$ ohms per circular mil-foot. For "harddrawn" copper, which is about 97 per cent conductivity, $K = 10.9$ ohms per circular mil-foot. The values of K at any other temperature can readily be found by means of Equation (4) as explained in the next article. By the use of the proper constant K and Equation (3) the resistance of any conductor can be determined.

---

[1] See Tables 1 and 2, in Appendix for standard sizes.

[2] See Table 2.

Example 1.—Find the resistance at 25° C. of a copper bar 10 feet long with a section 3 inches by $\frac{1}{4}$ inch.

The area $A = 3 \times \frac{1}{4} = \frac{3}{4}$ square inch,

or

$$0.75 \times 1{,}273{,}000 = 955{,}000 \text{ circular mils.}$$

Hence the resistance is

$$R = 10.6 \frac{10}{955{,}000} = 0.000111 \text{ ohm at } 25° \text{ C.}$$

15. Change of Resistance with Temperature.—The resistance of most metals increases considerably with an increase of temperature. Some alloys, however, such as manganin, show practically no such change, while some conductors, such as carbon and electrolytes, decrease their resistance with an increase of temperature. The relation is

$$R_t = R_o(1 + at), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where R_t is the resistance at a temperature t° C. and R_o is the resistance at 0° C. The constant a, called the temperature coefficient of resistance is 0.00427 for copper and about the same for other non-alloyed metals with the exception of iron and steel.

Example 1.—The field of a generator has a resistance of 60 ohms at 0° C. What would the resistance be if the temperature were 80° C.?

$$\begin{aligned} R_{80} &= 60\ (1 + 0.00427 \times 80) \\ &= 80.5 \text{ ohms.} \end{aligned}$$

The coefficient a varies with different initial temperatures. A few values are given below:

For R_o, $a = 0.00427$;
For R_{20}, $a = 0.00393$;
For R_{40}, $a = 0.00364$.

Equation (4) can, therefore, be used for any initial temperature, provided the correct coefficient is used. Equation (4) then takes the form:

$$R_{t_2} = R_{t_1}\{1 + a_1(t_2 - t_1)\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where t_1 is the initial temperature and t_2 the final temperature in degrees Centigrade. The value of a_1 for any initial temperature t_1 can be found by the expression:

$$a_1 = \frac{1}{234.5 + t_1}.$$

Example 2.—The temperature of an armature when "hot" is 75° C. and the resistance, 0.0102 ohm. What would be the "cold" resistance at 25° C.?

$$a_{75}=\frac{1}{234.5+75}=0.00323;$$

$$R_{25}=0.0102\{1+0.00323(25^\circ-75^\circ)\}$$

$$=0.0102\times0.8385=0.00855\text{ ohm.}$$

It is frequently necessary to determine the temperature rise when the " cold " and " hot " resistances are given. This can be computed by the aid of Equation (5), but a more convenient form is:

$$\text{Temperature rise}=t_1-t=\frac{R_{t_1}-R_t}{R_t}(234.5+t), \quad . \quad . \quad (6)$$

where R_t = " cold " resistance;
t = " cold " temperature, in degrees Centigrade;
R_{t_1} = " hot " resistance;
t_1 = " hot " temperature, in degrees Centigrade.

Example 3.—The initial or cold resistance of a transformer winding is 0.435 ohm at a temperature of 25° C. When hot, the resistance measures 0.505 ohm. What is the temperature rise of the winding?

$$\text{Temperature rise}=\frac{0.505-0.435}{0.435}(234.5+25)$$

$$=41.8^\circ\text{ C.}$$

It will be seen that for copper the approximate value of a used in Equations (4) and (5) is 0.004; hence, we may state the following rule for the approximate change in resistance of a copper conductor: *The change in resistance of a copper conductor is approximately four-tenths of one per cent per degree Centigrade.*

Example 4.—Referring to Example 3, the change in resistance was $0.505-0.435=0.070$ ohm or 16.1 per cent of the cold resistance. Therefore, the temperature rise is $\frac{16.1}{0.4}=40^\circ$ C.

16. Resistance of Insulating Materials.—It was stated in Article 6 that certain materials, such as glass, paper, rubber, etc., are called insulators because they allow only a very small current to flow. Hence, their resistance is very high. For example, a cube of hard rubber one centimeter on a side may have a resistance of 1.5×10^{11} ohms between opposite sides, while a cube of copper of the same size would have a resistance of 1.724×10^{-6} ohms, or the rubber has about 10^{17} times the resistance of the copper. When a conductor is covered or " insulated " by rubber, paper, or other similar material, the amount of current which leaks through this insulation is extremely small if the insulation is in good condition. The resistance which is offered to a flow of current through this insulation is called the " insulation resistance " and is determined in practice by measuring the small leakage current which flows through the insulation by means of a sensitive galvanometer.

If an insulated conductor is placed in a tank of water and a voltage applied between the water and the conductor, a small leakage current will flow. This current will increase as the voltage is increased until finally the insulation will be broken down or punctured and a hole burned in it, giving a path of low resistance between conductor and water. The property of an insulating material which enables it to resist puncture in this way is called its dielectric strength. This is measured in volts for a specified thickness of insulation. Thus, a certain treated cloth tape 10 mils thick showed an average puncture voltage of 5000 volts. Then the maximum dielectric strength could be said to be 500 volts per mil.[3]

PROBLEMS ON CHAPTER III

1-3. A No. 0000 stranded copper conductor has an area of 212,000 cir. mils and a resistance of 0.0500 ohm per 1000 feet. A No. 0 wire is one-half the cross sectional area of a No. 0000 wire. What would the resistance be of 1000 feet of No. 0 conductor?

2-3. What is the resistance, at 25° C., of one mile of copper wire 300 mils in diameter?

3-3. What is the resistance at 25° C. of a copper bar, 3 inches wide, $\frac{1}{8}$ inch thick, and 15 feet long?

4-3. (*a*) Determine the area in circular mils of a 2 inch $\times$2 inch $\times\frac{3}{16}$-inch angle.

(*b*) If steel has eleven times the resistance of copper, calculate the resistance of 100 feet of this steel angle at 25° C.

5-3. What is the total resistance of 75 feet of a conductor composed of three copper straps, each $\frac{3}{4}$-inch wide and 0.010-inch thick?

6-3. A trolley road is built with rails weighing sixty pounds per yard.

(*a*) What is the resistance per mile for each rail?

(*b*) If the rails are cross-bonded, what is the total track resistance per mile?

Assume resistance of steel to be eleven times that of copper and a temperature of 25° C. Weight of steel, 0.28 pound per cubic inch. Neglect resistance of joints.

7-3. A dynamo has a shunt field resistance of 11.5 ohms and an armature resistance, not including brushes, of 0.00138 ohm at a temperature of 25° C. After running at full load for several hours, the resistances are found to be 13.3 ohms for the field and 0.00158 ohm for the armature. What is the final temperature of (*a*) the field; (*b*) the armature?

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[3] See also Article 66, Chapter IX.

8–3. A direct current feeder consists of a 000 A. W. G. stranded wire, 6500 feet long.

(*a*) What would be the total resistance at 0° F.?

(*b*) What would be the total resistance at 100° F.?

(*c*) What would be the percentage increase in resistance when the temperature rises from 0° F. to 100° F.?

9–3. A transformer has two windings. The resistance of the first is 0.51 ohm and of the other 0.0048 ohm after the transformer has been standing for some time in a room at a temperature of 16° C.

(*a*) What would the resistance be if the room had a temperature of 25° C.?

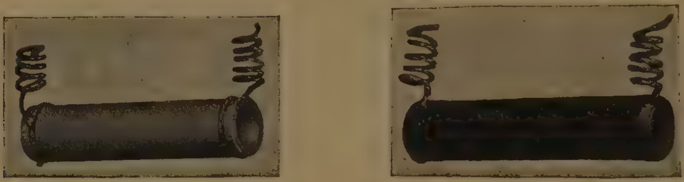
(*b*) After the transformer had been running for several hours at full load, the resistance of the windings was measured and found to be 0.61 ohm and 0.00575 ohm respectively. What is the average temperature rise above 25° C. of each winding?

10–3. The shunt field of a dynamo has a resistance of 85 ohms when the machine is not running and the room temperature is 25° C. Assuming that the average temperature rise of the field when the machine is running is 45° C., how many feet of No. 14 A. W. G. German silver wire (150 ohms per cir. mil foot) must be inserted in series with the field when starting cold to keep the field current the same as when hot?

CHAPTER IV

RHEOSTATS AND RESISTORS

17. Rheostats.—We have seen in Chapter III that all conductors possess the property of resistance to a greater or less extent. Because of this resistance, when current flows in an electric circuit, there is a loss of voltage, and a portion of the electricity is converted into heat. As a rule it is desirable to keep the loss due to resistance to a minimum, but in some cases additional resistance is introduced intentionally to control the flow of current or for other purposes. Thus, where electrical energy is used for heating, the conversion to heat energy is accomplished by introducing the necessary resistance in the circuit. A conductor, which is used primarily because it possesses resistance, is called a resistor. Copper or aluminum conductors are not suitable for resistors because they have a low specific resistance so that the resistor would be too bulky. Iron and steel have from six to twelve times the resistance of copper and, therefore, are more suitable for resistors. Alloys are extensively used as resistors because they can be made to have a very high specific resistance. German silver is a well-known alloy of this kind, but for most heating devices, nickel-chromium alloys are used because they will withstand high temperatures without oxidation. Resistors which are intended to have a high resistance and to carry a comparatively small current are commonly built of round wire or flat ribbon wound on an insulating tube. In some cases, after the wire is wound on the tube, it is covered by an



(*a*) (*b*)

FIG. 5.—Vitrohm Resistor. (*a*) Unit before enameling; (*b*) Unit after enameling. *The Ward-Leonard Electric Co.*

insulating enamel which is baked on and which protects the conductor from oxidation and assists in dissipating the heat produced (Fig. 5). When a resistor of low-resistance and large-current capacity is required, wire is not suitable and cast-iron grid resistors are used (Fig. 6).

A rheostat consists of a group of resistors arranged so that the amount of resistance in the circuit can be varied. One type of rheostat is shown diagrammatically in Fig. 7. The resistors are in series and connected to contacts over which a contact arm may be moved. In the position shown in Fig. 7, the current must pass through all the resistors. By moving the arm towards the right, some of the resistors are cut out of circuit and the amount of resistance in the circuit is thereby reduced. Rheostats are very commonly used for adjusting the field current of dynamos, for limiting the current when starting a motor, and for regulating the speed of motors while running.[1]

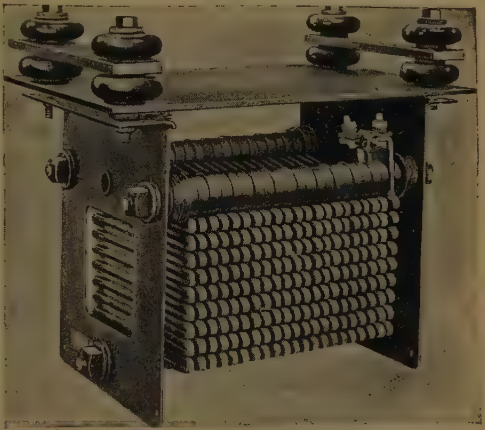


Fig. 6.—Grid Resistor. *The General Electric Co.*

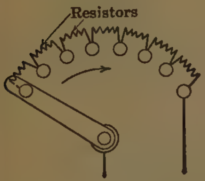


Fig. 7.—Diagram of a Rheostat.

Figure 8 shows two types of rheostats. Since heat is produced in any resistor, it must be designed to dissipate this heat. In many cases, the rheostat is designed to be cooled by a free circulation of air; sometimes forced ventilation, by means of a fan, is used. In all cases, however, the rheostat has a definite maximum current-carrying capacity, which determines its rating. If this rated current is exceeded, the resistors become overheated and the conductor may burn out. A rheostat will carry a greater load for a short time than it will continuously so that care must be taken not to use it for purposes for which it is not designed.

18. Current-adjusting Rheostats.—Besides the machine rheostats described in the preceding article and shown in Fig. 8, rheostats are extensively used for controlling the current consumed by a circuit. **Theater-dimmers** are rheostats connected in series with the lamps used on the stage and in the auditorium of a theater so that the voltage across the lamps may be reduced and the illumination decreased as desired. These dimmers usually take the form of a face-plate type of rheostat similar to the field rheostat shown in Fig. 8. They are usually assembled in groups with separate rheostats for different circuits and with control devices arranged so that the resistance of the different circuits can be controlled independently or together. **Battery-charging rheostats** are used to regulate the current taken when charging a storage battery from constant-potential mains. As the battery is charged, the voltage at its terminals increases, so that the charging voltage must be increased if a constant

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[1] Further description of motor starters and controllers is given in Chapter XXI.

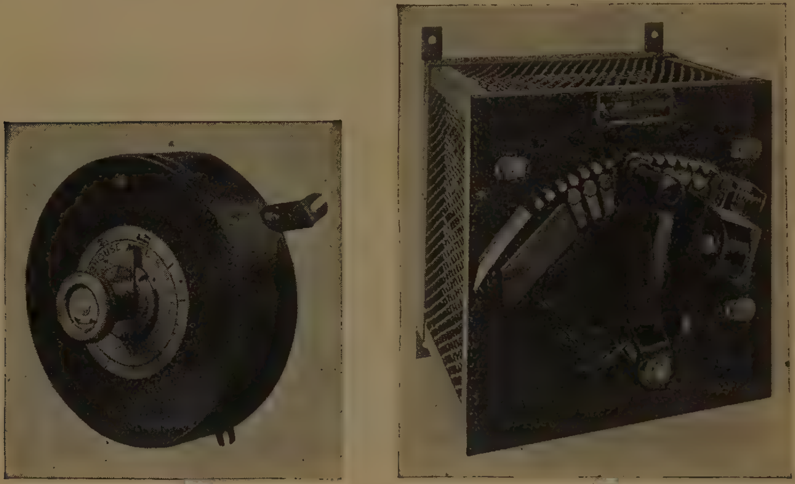

(a) (b)

Fig. 8.—Examples of Rheostats. (a) Field rheostat. *West. Elec. & Mfg. Co.*; (b) Armature rheostat. *Ward-Leonard Elec. Co.*

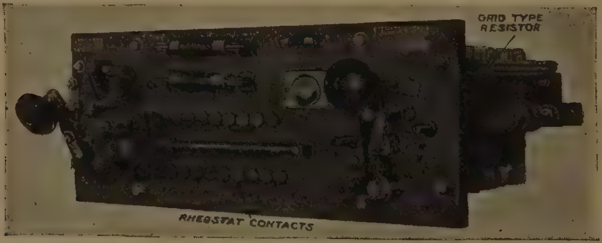


Fig. 9.—Battery Charging Rheostat. 90 amperes capacity. *The Cutler-Hammer Mfg. Co.*

current is to be maintained. This is done by cutting out some of the resistance in series with the battery, by moving the rheostat arm to a different contact point. Figure 9 shows a charging rheostat and panel. In recent years, the use of electric **heaters and furnaces** has become very common. The principal advantage over the use of coal or gas is the cleanliness of the electric heat and the more effective manner in which the heat can be employed. With existing prices for electrical energy, it is seldom that electricity can be used for heating buildings, but it can be used in special instances, as, for example, for the heating of railway cars. It is also used for

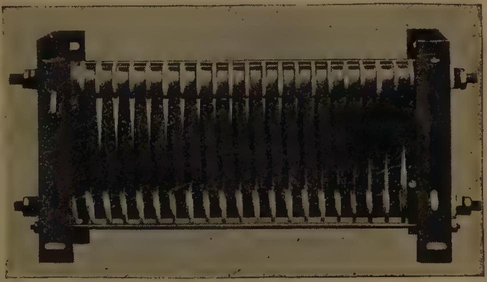


FIG. 10.—Oven Heater. *The Westinghouse Elec. & Mfg. Co.*

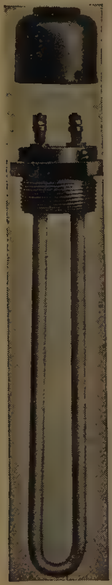


FIG. 11.

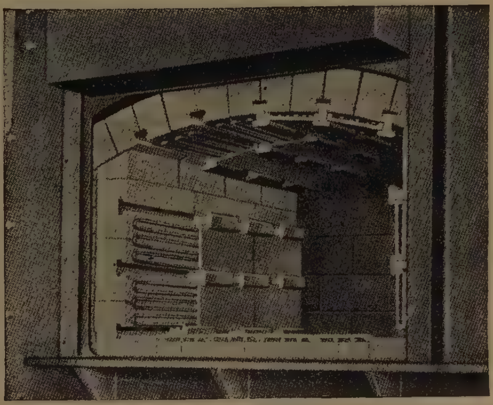


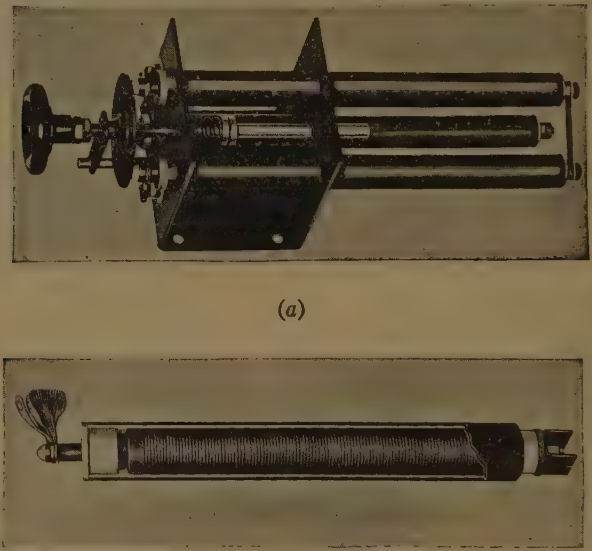
FIG. 12.

FIG. 11.—Immersion Heater. Hair-pin type. Single heat up to 2 kw. capacity, 21 inches long. *The General Electric Co.*

FIG. 12.—Resistance Furnace. Sides and top cut away to show resistors. *The Westinghouse Elec. & Mfg. Co.*

baking and drying ovens. One form of oven heater is shown in Fig. 10. Resistors used as heaters are run at a moderate temperature and are arranged for a free circulation of air so that the heat can be distributed as uniformly as possible in the space to be heated. Electricity is also extensively used for heating liquids the resistor being commonly placed inside a metal shield and the whole immersed in the liquid. Figure 11 shows one type of immersion heater.

Electric furnaces of the resistance type are now used extensively for japanning, enameling, and heat treating of steel. The resistor is usually a high resistance conductor built into the walls of the furnace. A furnace of this sort is shown in Fig. 12. Resistance furnaces are suitable for temperatures up to about 2000° F.; above that the heat is produced by an electric arc. Arc furnaces of many different designs are used very extensively for the production of tool and alloy steels, and for the making of castings. One type of steel furnace is shown in Fig. 254.[2] Electric furnaces for steel making up to a capacity of 60 tons are now in use.


(a)

(b)

Fig. 13.—Carbon Pile Rheostat. (a) Complete rheostat; (b) Section of resistor showing carbon disks. *The Allen-Bradley Co.*

Either a direct or an alternating current may be used in heaters or furnaces, but alternating current is usually employed, because it is simpler to transform the voltage to the value suitable for the furnace and also because, in most cases, the power supply is alternating current.

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[2] See also Article 267.

19. The Carbon Pile Rheostat consists of one or more columns of carbon disks contained in an iron tube with an insulating lining (Fig. 13). The current passes from one end of the column to the other and the resistance is varied by changing the pressure between the ends, an increase in pressure decreasing the contact resistance between disks. This type of rheostat is used for battery charging and for starting and controlling motors.

20. Liquid Rheostats make use of the resistance of a column of liquid, usually water, to which sodium carbonate or salt has been added. The principle of action is shown in the barrel rheostat of Fig. 14. The two terminals of the circuit are connected to two V-shaped iron plates which are insulated from each other by the block of wood to which they are attached. By raising or lowering these plates a greater or less surface is immersed in the solution and the resistance is varied accordingly. The amount of salt required is determined by the voltage and the current to be carried. For high voltages there are usually enough impurities in the water so

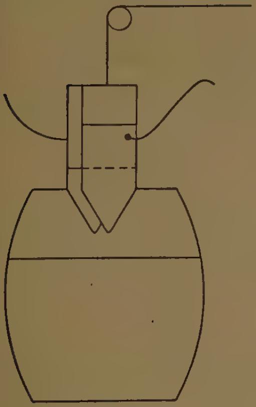

FIG. 14.

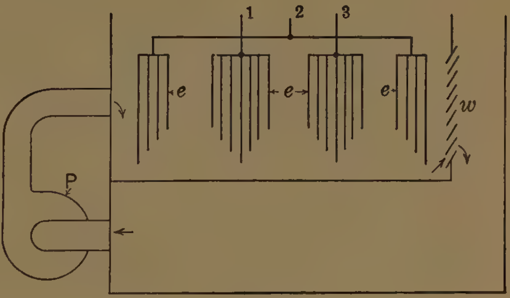

FIG. 15.

FIG. 14.—Barrel Rheostat.

FIG. 15.—Liquid Rheostat for a Three-phase Motor. The three leads from the motor are connected to terminals 1, 2, and 3. The pump *P* circulates liquid through the upper section of the tank, the liquid leaving through adjustable weir *W*. By closing more or less of the shutters at *W*, the level of the liquid can be changed, thus changing the depth that the electrodes are immersed in the liquid. This device is used for controlling large induction motors.

that very little salt need be used. Liquid rheostats are useful for temporarily loading a generator and are also employed in the rotor circuit of induction motors, where a uniform variation of resistance is necessary. Figure 15 shows a rheostat designed for this purpose. In this case the resistance is varied by changing the height of the liquid instead of moving the plates.

21. The Size of a Rheostat is determined by the amount of power it is intended to absorb, whether it is intended for continuous or for intermittent service, and the method of cooling. An ordinary wooden barrel rheostat like Fig. 14 will carry 15 kw. continuously without boiling violently, and, if arrangement is made to circulate cold water through the barrel, 150 kw. or more can be carried. The capacity of a rheostat is not determined by the current alone, but by the product of this current and the voltage across the rheostat terminals; in other words, by the power consumed in the rheostat.

CHAPTER V

THE DIRECT-CURRENT ELECTRIC CIRCUIT

The flow of electricity through an electric circuit can be compared to the flow of water in a hydraulic system. A direct current of I amperes (Fig. 16 *a*) is caused to flow through an electric circuit having a resistance of R ohms by the electrical pressure of E volts produced by the generator G. A flow of water (gallons per minute) is caused to flow through the pipe line (Fig. 16 *b*) against the frictional resistance of the pipe, by a pressure of P pounds per square inch produced by the centrifugal pump.

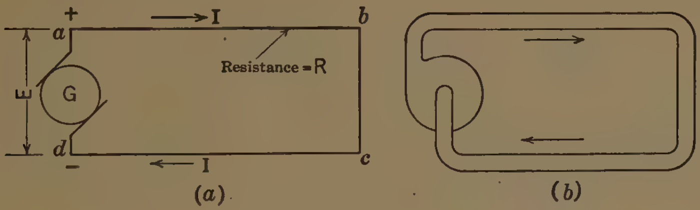

FIG. 16.—Electric and Hydraulic Circuit.

22. The Direction of Current in a conductor is indicated by arrows placed beside the conductor or by symbols placed in a circle representing a cross-section of the conductor. Thus, Fig. 17 *a* shows a side and end elevation of a conductor with current flowing from left to right. When looking at the end of the conductor we can imagine we see the tail of the arrow or that the current is flowing away from the observer. As the conductor (Fig. 17 *b*), has current flowing in the opposite direction we see the point of the arrow represented by a dot.

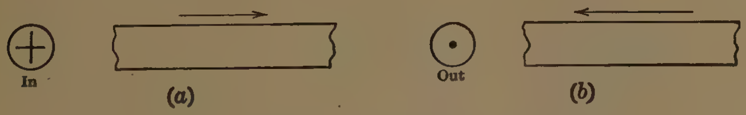

FIG. 17.—Direction of Current in a Conductor.

23. Ohm's Law.—It has been determined experimentally that the voltage required to force a current through a circuit of constant resistance R is proportional to the current. This is known as Ohm's Law. The practical electrical units are so chosen that Ohm's Law is expressed by the equation:

$$E = IR, \quad \ldots \ldots \ldots \quad (7)$$

where E is the electrical pressure in volts, I is the current in amperes, and R the resistance in ohms.

Example 1.—An electric heater supplied at 110 volts takes 4.5 amperes. The resistance is then

$$R = \frac{110}{4.5} = 24.5 \text{ ohms.}$$

Ohm's Law must be applied with care when a voltage E is acting upon a circuit having a source of electromotive force as well as resistance, as, for example, when charging a storage battery or operating a motor. In this case, if E is the voltage applied to the battery or motor terminals, which causes a current of I amperes to flow, and R is the resistance of the battery or motor, E will not equal RI, but will be considerably greater. In fact, most of the impressed voltage E is used to balance the counter voltage of the motor or battery, while the small remainder forces the current through the circuit according to Ohm's Law.[1]

24. Fall of Potential in an Electric Circuit.—Referring to Fig. 16 *b*, the pressure at the discharge end of the pump is greater than at the intake, and if the friction is uniform throughout the pipe, the pressure uniformly decreases as we go from the discharge to the intake. In the electric circuit, the terminal of the source or generator from which the current leaves is called the positive terminal and the other the negative terminal (Fig. 16 *a*). The entire voltage E is consumed in overcoming the resistance R, and if this resistance is uniform, there is a uniform fall of potential from a to d. Thus, point b is at a lower potential than a. This drop of potential between a and b (E_{ab}) is also called the potential difference between a and b. It should be noted also that a is positive with respect to b, and b is positive with respect to c, etc.

In an electric circuit, we use the term electromotive force (e.m.f.) to mean the voltage generated by the source, such as a generator or a battery. We use the terms difference of potential or voltage drop to mean a portion of the e.m.f which is consumed in a particular part of the circuit. Thus, in Fig. 16 *a*, if the generator has a negligible resistance, *E* is the e.m.f. of the generator and this is consumed by the combined drop or fall of potential in parts, *ab*, *bc*, *cd* of the circuit. This can be illustrated by a diagram (Fig. 18). The lines 1, 2 and 3 can represent to scale the portions of the impressed e.m.f., *E* which are required to overcome the resistances between *ab*, *bc*, and *cd* respectively and to force the current through the circuit (Fig. 16 *a*).

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[1] This is more completely explained when dealing with motors. See Article 105, Chapter XIII.

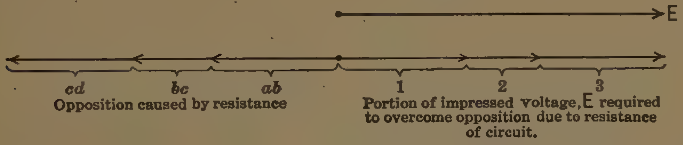
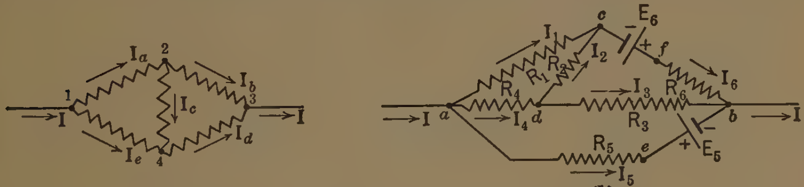

E
cd
bc
ab
Opposition caused by resistance
1
2
3
Portion of impressed voltage, E required to overcome opposition due to resistance of circuit.

FIG. 18.—Action and Reaction in an Electric Circuit.

The sum of parts 1, 2 and 3 must equal *E* since they balance the entire opposition to the flow of current.

In a *source* of electrical energy, such as a generator, the direction of current in the circuit is the *same* as the direction of the e.m.f. produced by the source. Thus, in Fig. 16 *a*, the direction of e.m.f. in the generator is from *a* to *d* through the external circuit *abcd* and this is also the direction of the current produced by this e.m.f. In a *receiver* of electrical energy, such as a motor or a storage battery, the direction of current is *opposite* to the direction of the e.m.f. produced by the receiver.



(*a*) (*b*)

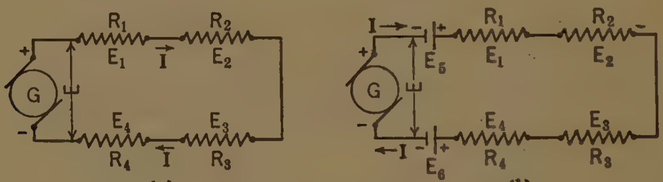
FIG. 19.—Currents in a Divided Circuit.

25. Fundamental Laws of the Electric Circuit.—Besides Ohm's Law, mentioned in Article 24, there are two other fundamental laws which are of importance when dealing with electric circuits. They were first formulated by Kirchoff, and as applied to direct-current circuits may be stated as follows:

(1) *The total current flowing towards a junction in the circuit must equal the total current flowing away from this junction.* Or, if we consider a current leaving a junction as negative and flowing towards a junction as positive, then the algebraic sum of the currents at any

junction point is zero. Thus, with currents flowing in Fig. 19 *a* as shown; $I=I_a+I_e$, $I_a=I_b+I_c$, $I_e=I_d-I_c$, etc. The truth of this law is apparent when it is realized that the quantity of electricity which flows to a certain point in a given time must equal the quantity of electricity which flows away from the point in the same time; otherwise there would be a storage of electricity at that point, which is obviously impossible in the kind of circuit considered.

(2) *In any complex network of conductors, the difference of potential between any two points has a fixed and definite value and is equal to the algebraic sum of the potential drops by any path from one point to the other.* Thus, in Fig. 19 *b*, if the currents and electromotive forces produced by the batteries are as shown in the diagram, the potential with respect to point *a* will *decrease* by an amount I_1R_1 through part *ac*, will *increase* E_6 volts due to the voltage of the battery in branch *cb*, and will *decrease* by an amount I_6R_6 in branch *cb*. Hence,



(*a*) (*b*)

FIG. 20.—Series Circuits.

$$E_{ab}=I_1R_1-E_6+I_6R_6.$$

Similarly, bv path *acdb*, we have:

$$E_{ab}=I_1R_1-I_2R_2+I_3R_3.$$

It is important to note, therefore, that the difference of potential between two points has, at a particular instant, one and only one value, regardless of the number of paths and the resistances and electromotive forces in the different paths.

26. Series Circuits.—A series circuit has only one path or branch for the current (Fig. 20 *a*). Applying the laws stated in the previous article we can say:

(1) The current I is the same in each part of the circuit.

(2) The algebraic sum of the difference of potential across the various portions of the circuit equals the total voltage impressed on the circuit. Thus, for Fig 20 *a*,

$$E=E_1+E_2+E_3+E_4.$$

Also, since

$$E_1 = IR_1, \text{ etc.}$$

$$\begin{aligned} E &= IR_1 + IR_2 + IR_3 + IR_4 \\ &= I(R_1 + R_2 + R_3 + R_4). \end{aligned}$$

The total resistance R is according to Ohm's Law:

$$R = \frac{E}{I}.$$

Hence

$$R = R_1 + R_2 + R_3 + R_4. \quad \ldots\ldots \quad (8)$$

Or the total resistance R for a series circuit is the sum of the resistances in the different parts.

When there is a source of e.m.f., such as a battery in the circuit, the polarity of this battery must be considered. If the polarity is in the same direction as that of the main source, i.e., the generator G in Fig. 20 *b*, then the voltage of the battery should be added to E, since it is an auxiliary source of electricity. If, however, the polarity is such that it opposes the main source, as does E_6, then its voltage should be subtracted from E. Hence, for the circuit shown in Fig. 20 *b*,

$$\begin{aligned} E + E_5 - E_6 &= E_1 + E_2 + E_3 + E_4 \\ &= I(R_1 + R_2 + R_3 + R_4) \\ &= IR. \end{aligned}$$

Therefore

$$I = \frac{E + E_5 - E_6}{R}.$$

The portions of this circuit in which the current flows from negative to positive, as in E and E_5 are sources of electricity, while the portions in which the current flows from positive to negative, as in E_1, E_2, E_3, E_4, and E_6, are receivers or consumers of electricity. The power delivered by the several sources must equal the power consumed by the various receivers and this is shown to be true if we multiply the equation above by the current I:

$$EI + E_5I = E_1I + E_2I + E_3I + E_4I + E_6I.$$

The terms on the left-hand side represent the entire power supplied to the circuit and those on the left, the entire power consumed by the circuit. Series circuits are commonly used for street lighting service.

Example 1.—A series circuit like Fig. 20 *a* has resistances as follows: $R_1 = 5$ ohms, $R_2 = 10$ ohms, $R_3 = 15$ ohms, $R_4 = 20$ ohms. What voltage E must be furnished by the generator in order to force 5 amperes through the circuit? The total resistance is $R=5+10+15+20=50$ ohms. Hence the total voltage is $E=IR=5\times 50=250$ volts. As a check we may calculate the voltage required for each part. Thus, $E_1=5\times 5=25$ volts, $E_2=5\times 10=50$ volts, $E_3=5\times 15=75$ volts, $E_4=5\times 20=100$ volts. Hence $E=25+50+75+100=250$ volts as before.

Example 2.—Assume E_5 (Fig. 20 *b*) is a storage battery giving 6 volts and E_6 another battery giving 32 volts. Assume the resistances the same as for Example 1. What voltage E is required to force 5 amperes through this circuit? It will be seen that E_5 is connected so as to assist the voltage E; hence the total voltage which is effective in forcing current through the circuit is $E+E_5$. The voltage E_6 constitutes an opposition the same as the resistances. The drop across each resistance is the same as for Example 1, because the current is the same. Hence the total voltage which must be overcome can be equated to the total available voltage thus,

$$E+6=32+25+50+75+100,$$
$$E=276 \text{ volts.}$$

As a check, this can be solved as follows:

The voltage drop across each resistance is the same as before. Therefore, the total drop due to the four resistances is $25+50+75+100=250$ volts. The total voltage to be supplied is then $250+E_6=250+32=282$ volts. But E_5 supplies 6 volts, hence the generator must supply

$$E=282-6=276 \text{ volts.}$$

27. Parallel Circuits have two or more branches or paths for the current (Fig. 21). Applying Kirchoff's Laws (Article 25), we can say that for a parallel or multiple circuit:

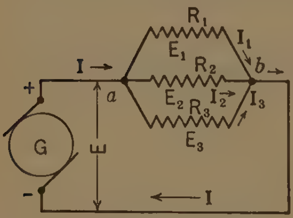


FIG. 21.—Parallel Circuit.

(1) The fall of potential from *a* to *b* must be the same for each branch. That is, $E_1=E_2=E_3=E$.

(2) The total current I entering the junction point *a* (or leaving *b*) must be the algebraic sum of the currents in the several branches. Thus, $I=I_1+I_2+I_3$. Where there is no source of e.m.f. in any of the branches as in Fig. 21 the total or combined resistance R can be determined easily, since

$$I=\frac{E}{R}, \quad I_1=\frac{E}{R_1}, \text{ etc.}$$

Substituting these values, we have:

$$\frac{E}{R}=E\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right),$$

$$R=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}. \quad \dots\dots\dots \quad (9)$$

The current in a particular branch can be found by dividing the terminal voltage by the resistance of the branch; thus, $I_1 = E \div R_1$. It should be noted that in any parallel circuit *the combined resistance R must always be less than the resistance of any single branch.* Also, if the voltage across the terminals remains constant, a change in resistance of one branch does not change the current in the other branches. Where the total current remains constant, however, a change in resistance of one branch changes the current in all the branches.

Example 1.—A circuit of three branches (Fig. 21) has resistances of $R_1 = 5$ ohms, $R_2 = 10$ ohms, $R_3 = 15$ ohms. If 110 volts is impressed on the circuit, calculate the total current and the current in each branch.

$$R = \frac{1}{\frac{1}{5}+\frac{1}{10}+\frac{1}{15}} = 2.73 \text{ ohms.}$$

The total current is

$$I = \frac{110}{2.73} = 40.3 \text{ amperes.}$$

The current in the different branches is

$$I_1 = \frac{110}{5} = 22 \text{ amperes;}$$

$$I_2 = \frac{110}{10} = 11 \text{ amperes;}$$

$$I_3 = \frac{110}{15} = 7.3 \text{ amperes.}$$

Total 40.3 amperes.

Example 2.—If the resistance R_1 were halved, what effect would this have on the currents, assuming that the voltage remains constant?

$$I_1 = \frac{110}{2.5} = 44 \text{ amperes.}$$

I_2 and I_3 will be the same as before because the terminal voltage has not changed. Hence the total current is

$$I = 44 + 11 + 7.3 = 62.3 \text{ amperes.}$$

28. Power in a Circuit.—We have seen (Article 8) that the power in a circuit is $P = EI$ watts, where E is the difference of potential at the terminals of the circuit and I is the current which flows, due to this voltage. This equation applies to any circuit whether it contains a source of counter e.m.f. or not. If there is no counter e.m.f. in the circuit, the power is all consumed by the resistance of the circuit, and is converted into heat. If there is a counter e.m.f., power is also consumed in forcing a current through the circuit in opposition to this counter e.m.f.

29. Voltage Drop in a Feeder.—As an example of the application of some of the principles explained in the last two paragraphs consider the following problem:

Example 1.—A feeder 500 feet long carries a load of motors, totaling 50 kw. and lamps totaling 5 kw. The voltage is 250 at the loads. Calculate the size of wire to limit the drop in the feeder to 6 per cent of the voltage at the load. Calculate the per cent energy loss and per cent drop for the size of wire finally chosen. The total current is

$$I=\frac{50{,}000+5000}{250}=220 \text{ amperes.}$$

The allowable drop is $250\times0.06=15$ volts. The required resistance per 1000 feet of conductor is

$$R=\frac{15}{220}=0.0682 \text{ ohm.}$$

According to Table 2, No. 000 wire has 0.0642 ohm resistance and is the nearest standard size.

The drop is, therefore,

$$E=0.0642\times220=14.1 \text{ volts.}$$

The per cent energy loss is

$$\frac{\overline{220}^2\times0.0642\times100}{55{,}000}=5.65\%$$

The per cent voltage loss is

$$\frac{14.1\times100}{250}=5.65\%$$

More detailed methods of feeder calculation are given in Chapter XXIII.

PROBLEMS ON CHAPTER V

1–5. What would be the fall of potential on one mile of 300,000 cir. mil stranded conductor when carrying 275 amperes? (See Table 2 in appendix.)

2–5. The shunt field winding of a dynamo is connected in series with an ammeter to a 110-volt supply, and the current is observed to be 5.8 amperes.

(*a*) What is the resistance of the winding?

(*b*) What current would flow if a resistance of 10 ohms were connected in series with the field?

(*c*) What would be the potential difference for the field in (*b*)?

3–5. It is desired to use a 50 millivoltmeter as a 100 ampere meter on a 500-volt circuit.

(*a*) What resistance must the shunt have?

(*b*) Same for a 1000 ampere meter?

(*c*) What would be the voltage on the instrument if the shunt became opened during use?

4–5. In measuring a certain current, a "shunt" having a resistance of 0.00025 ohm is connected in the circuit and a millivoltmeter attached to its terminals reads 0.05 volt.

(*a*) What is the value of the current?
(*b*) Why is the resistance of the shunt made so low?

5–5. An electric soldering iron requires 5 amperes at 115 volts.

(*a*) What is the resistance of the heating element?

(*b*) If the supply voltage dropped to 105 volts, what would be the approximate temperature of the iron as compared with the temperature at normal voltage?

6–5. It is desired to use a piece of copper bus-bar, 3 inches × $\frac{1}{4}$ inch in section as a shunt for a millivoltmeter in order to convert it to an ammeter. The millivoltmeter requires 53 millivolts for full scale deflection. It is desired to produce full scale deflection with a current of 750 amperes in the bus-bar. How far apart on the bus-bar must the leads of the millivoltmeter be connected in order to produce the desired deflection?

7–5. Referring to Fig. 19*a*, the currents in the branches are:

$$I_a = 15 \text{ amperes}, \; I_e = 35 \text{ amperes and } I_c = 8 \text{ amperes}.$$

Calculate values of I_b, I_d, and I.

8–5. In Fig. 19*a*, assume that the total current $I = 75$ amperes, and that $I_e = 45$ amperes, and $I_d = 38$ amperes.

(*a*) Calculate the currents I_b, I_a and I_c.
(*b*) What would be the direction of the current in the circuit between *2* and *4*?

9–5. Referring to Fig. 19*b*, assume currents as follows: $I = 55$ amperes, $I_1 = 25$ amperes, $I_5 = 15$ amperes, $I_3 = 20$ amperes. Find I_4, I_2, and I_6. What will be the direction of current in branch *cd*?

10–5. Referring to Prob. 9–5, let $R_4 = 2.5$ ohms, and $R_3 = 3.1$ ohms.

(*a*) What is the potential difference between *a* and *b*?
(*b*) If $E_5 = 6$ volts, calculate R_5.
(*c*) If $R_1 = 1.9$ ohms, calculate R_2.
(*d*) If $R_6 = 3.5$ ohms, calculate E_6.

11–5. A 150 voltmeter, having a resistance of 16,000 ohms is to be used with a multiplier as a 600-volt instrument. What resistance must the multiplier have?

12–5. A certain voltmeter has a resistance of 10,000 ohms and a 150 volt scale.

(*a*) How much resistance must be inserted in series with this meter to adjust it to give full scale deflection with 600 volts?

(*b*) Calculate the current required to give full scale deflection on 600 volts with the resistance determined in (*a*). How does this compare with the current required to give full scale deflection without the external resistance?

13–5. A generator giving a potential of 150 volts is used to charge a storage battery consisting of 54 cells, connected in series, at a rate of 100 amperes. Assume that each cell has a resistance of 0.0004 ohm and that the counter e.m.f. of each cell is 2.2 volts at the start and 2.5 volts after charging for 8 hours. Resistance of the leads 0.03 ohm.

(*a*) How much additional resistance would be required to limit the current to 100 amperes at the beginning and after 8 hours charging?

(*b*) What would be the power input to the battery in each case?

(*c*) What percentage of the total power is delivered to the battery?

14–5. A series lighting system has fifty 6.6 ampere incandescent lamps, each requiring 310 watts. The line resistance is 25 ohms. What is the total voltage required for the circuit?

15–5. A four-pole, 230-volt d.c. shunt motor is operated with a field rheostat in the shunt field circuit. The drop across the field rheostat is 19 volts. What would be the drop across each field spool?

16–5. Two voltmeters, each having a 150-volt scale, are connected in series across a 125-volt circuit. One voltmeter has a resistance of 150,000 ohms and the other a resistance of 15,000 ohms. What would be the reading of each instrument?

17–5. It is desired to measure a voltage of about 700 volts and no single voltmeter of sufficient range is available, but a 600-volt and a 150-volt instrument can be secured.

(*a*) How would they be connected to make the desired measurement?

(*b*) Assuming that both voltmeters require the same current for full scale deflection, determine the reading of each when the total voltage is 720.

18–5. A d.c. voltmeter, having a 150-volt scale, has a resistance of 150,000 ohms. It is used to measure the insulation resistance of a machine by connecting it in series with the unknown resistance. The deflection on the meter is 7.5 volts. When the voltmeter is connected directly to the supply, without the unknown resistance in series, it reads 117 volts. Calculate the insulation resistance of the machine.

19–5. Two resistances of 50 and 100 ohms respectively are connected in parallel between two points A and B between which the voltage is 100.

(*a*) What current would flow in each resistance and in the line?

(*b*) If the 100-ohm circuit were reduced to 0.5 ohm, what current would flow in each?

20–5. The field of a 15-horsepower shunt motor has a resistance of 25 ohms, and the armature has a resistance of 0.04 ohm. The voltage of the supply circuit is 110 volts.

(*a*) What current would flow if the field were connected to the supply?

(*b*) What current would flow in the armature if it were connected to the supply in parallel with the field with the motor at a standstill? What would be the current in the field?

(*c*) If a 5-ohm rheostat were included in the armature circuit when starting, what would be the total current taken by the motor?

21–5. Given resistances of 4 and 6 ohms in parallel and in series with these, a resistance of 7.6 ohms.

(*a*) What current would be sent through each of these resistances by 110 volts applied to the whole combination?

(*b*) If each resistance were halved, what would the currents be?

(*c*) What would the voltage be across the 6-ohm resistance?

22–5. Two resistances of 3 and 5 ohms respectively, are connected between points A and B. Two resistances of 1 and 2 ohms respectively are connected between *C* and *D*. Between *A* and *C* there is a resistance of 0.5 ohm and between *B* and *D* there is a resistance of 0.5 ohm. Between points *D* and *E* and also between points *C* and *F*, there is a resistance of 0.2 ohm. A voltage of 125 is applied between points *E* and *F*.

(*a*) Calculate the total resistance of the circuit from *E* to *F*.

(*b*) What is the current in each branch and in the main circuit?

23–5. It is desired to measure a current of about 325 amperes but the only available meters are rated at 100 and 300 amperes.

(*a*) How would they be connected to measure the current?

(*b*) If both meters have the same potential drop for full scale deflection, what would be the reading of each when the total current is 325 amperes?

24–5. The voltage drop across the series field of a compound generator (short shunt) is 8.2 volts when the machine is carrying 530 amperes.

(*a*) What is the resistance of the series field?

(*b*) A resistance or "shunt" is placed in parallel with the series field. It is desired that 0.8 of the total current be carried by the series field. Calculate the resistance of the shunt.

25–5. A trolley road is built with steel rails weighing 75 pounds per yard, and with a trolley wire 460 mils in diameter. What is the total resistance at 25° C. of 5 miles of the road, assuming rails welded at joints and cross-bonded?

26–5. The trolley wire in Prob. 25–5 is reinforced by a 500,000 cir. mil feeder run from the power house to a point 4 miles distant. What would be the total resistance of 5 miles of the road in this case?

27–5. A single track trolley line has rails with a resistance of 0.081 ohm per mile (each rail). The rails are cross-bonded. Trolley wire has a resistance of 0.34 ohm per mile. A 300,000 cir. mil feeder is run from the power house to a point three miles distant and tapped to the trolley wire. An electric car taking 100 amperes is at a point 4.3 miles from the power house. Voltage at power house 600. What is the voltage at the car?

28–5. Five 625-ohm, 20-candle power and four 392-ohm 32-candle power, 125-volt tungsten lamps are all in parallel. The wires connecting them to the generator have a total resistance of 2 ohms.

(*a*) What is the total resistance of the circuit?

(*b*) What is the current in each lamp and in the feeder?

(*c*) What is the voltage required at the generator?

29–5. A storage battery consisting of 55 cells connected in series is used to operate 100 lamps having a resistance of 220 ohms each, and 50 lamps having a resistance of 300 ohms each, all the lamps being connected in parallel. The lamps are 200 feet from the battery. Size of wire 325 mils diameter. At the beginning of the time, the voltage at the terminals of each battery cell is 2 volts, and at the end of 8 hours the voltage has dropped to 1.8 volts per cell.

(*a*) What is the voltage at the lamps at the beginning and end of the run?

(*b*) What is the power output of the battery in each case?

(*c*) What is the total energy output from the battery during the 8 hours?

30–5. A generator is rated at 500 kw. at 250 volts. What is the full-load current output?

31–5. An electric car requires 3 kw. for heating. The trolley voltage is 550.

(*a*) What current would be required for the heater?

(*b*) If the heater were divided into four equal sections, connected in series, what would be the resistance of each section?

32–5. An electric flatiron is rated at 460 watts.

(*a*) What current would it require if designed for a 115-volt circuit? What would be the minimum size of fuse which should be used to protect this iron?

(*b*) What current would be required for an iron of this rating designed for a 32-volt circuit? What size of fuse would be required?

33–5. A storage battery is being charged from a 115-volt source through a resistance. The potential difference at the terminals of the battery is 92 volts and the current is 55 amperes.

(*a*) What is the value of the resistance?

(*b*) What is the power input to the battery?

(*c*) What per cent of the power taken from the supply is lost in the resistance?

34–5. (*a*) What size of copper wire is required to deliver current at 110 volts to a 15-hp. motor having an efficiency of 89 per cent, the motor being 2000 feet from the generator and the voltage at the generator being 125 volts?

(*b*) What size would be required if the voltages were 220 and 250 volts respectively?

NOTE.—Base percentages on delivered values.

35–5. A motor, located 1000 feet from the generator, requires 20 amperes at 500 volts. The wire used for the feeder is No. 6 A.W.G.

(*a*) What is the percentage of volts lost?

(*b*) What is the power lost?

(*c*) What is the percentage of power lost?

36–5. A 50-hp., 230-volt motor is to be operated a distance of 575 feet from the switchboard. The efficiency is 91 per cent, and the voltage loss in the line is not to exceed 18 volts.

(*a*) What is the minimum size of rubber covered wire which could be used for this motor, neglecting voltage drop? (See Table 2, appendix).

(*b*) What size should be used to meet the specified requirements regarding voltage drop?

37–5. A building is to be lighted by twenty-five 200-watt lamps which are to be supplied from a generator 500 feet away. The voltage at the building is to be 115 volts when all lamps are lighted.

(*a*) What must the generator voltage be if a loss of 8 per cent is allowed for the line supplying these lamps?

(*b*) If the generator voltage were held constant, what would be the voltage at the building when only fifteen lamps were lighted?

(*c*) What must be the variation in the generator voltage in order to keep the voltage at the building constant at 115 volts when the number of lamps is decreased from twenty-five to zero?

CHAPTER VI

MAGNETS AND MAGNETISM

30. Magnetism.—A body which possesses the power to attract pieces of iron is called a magnet, and the state or condition of a body by means of which this attraction takes place is called magnetism. The power of attraction is made evident at certain points on the body called poles. Ordinary simple magnets have a north (N) and a south (S) pole. The north pole of a magnet is that pole which would point north if the magnet were suspended so that it were free to turn. Unlike poles (north and south) attract and like poles (north and north or south and south) repel. We shall see later that magnetism is always associated with a circuit in which a current of electricity is flowing; in fact, in practice, the magnetic effect is always produced by means of an electric current.

31. Magnetic Field.—The space surrounding a magnet is called its magnetic field. In practice, the term is applied to the space near the magnet where the force of attraction or repulsion is appreciable. The extent of this field depends upon the strength of the magnet. Experiment shows that the force of attraction or repulsion in a magnetic field varies in different parts of the field and, in general, decreases with an increased separation between the pole of the magnet and the body upon which the force acts. The magnetic field also exerts a force in a particular direction, as can be shown by placing a compass in a magnetic field. The direction in which the north pole of the compass points is called the direction of the magnetic force at that point. This means that the direction is from the N to the S pole of the magnet, through the field (Fig. 22).

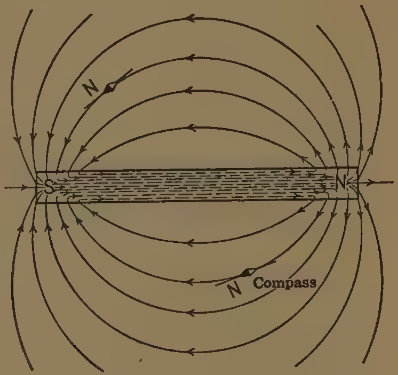


FIG. 22.—Field around a Bar Magnet.

32. Magnetic Lines of Force.—If a small compass is moved from the N to the S pole of a bar magnet (Fig. 22) the direction of motion always being that in which the north pole of the compass is pointing, it will trace a smooth curve between the N and S poles of the magnet, and this curve indicates the direction in which the magnetic force acts at the various points on the curve. This imaginary line is called a magnetic line of force. When a magnetic field is plotted by means of iron filings (Fig. 23), the direction of these lines is clearly indicated.

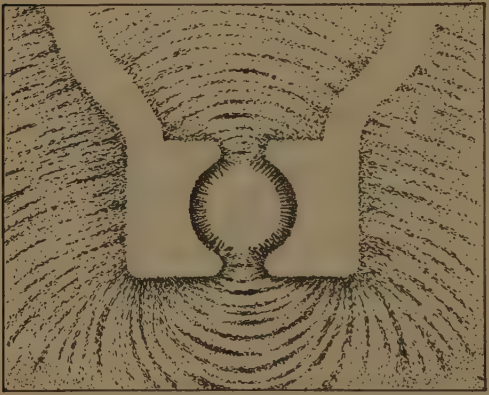


Fig. 23.—Field of a Weston D. C. Instrument.

The term " line of force " as used in the preceding paragraph indicates only the direction of the magnetic force at different points in the magnetic field and hence there could be an infinite number of these lines in any particular case. In practice, it is customary to use the term " line of force " or " line of magnetic induction " to define the amount of magnetic effect. The " line " thus defined is used as a unit of measurement and is known as the maxwell. When so used, the number of " lines " or maxwells per unit of cross-sectional area is taken proportional to the strength of the magnetic effect at that point. These imaginary lines are considered to exist inside the magnet as well as outside and to form closed loops or a complete circuit.

33. Magnetic Flux.—In an electric circuit, the number of amperes which flows through a particular cross-section of the electric circuit is a measure of the total current which flows through this section. In a magnetic circuit, the number of lines of force or maxwells which cross a particular section of the circuit is known as the magnetic flux. Thus, if a magnetic circuit has a cross-sectional area of 10 square inches, and if 50,000 lines of force or maxwells cross each square inch of this area, then the magnetic flux per square inch is 50,000 maxwells and the total flux for the entire section is $10 \times 50{,}000 = 500{,}000$ maxwells. The symbol Φ stands for flux in lines or maxwells.

34. Flux Density is the flux per unit area, i.e., the lines per square centimeter or per square inch. Flux density is represented by the letter B. If the lines are uniformly distributed over an area A, then the flux density $B = \Phi \div A$. The unit of flux density is one line per square inch or per square centimeter. One line per square centimeter is called a gauss. Thus, if a certain magnetic circuit has a cross-sectional area of 10 square centimeters and a total flux of 100,000 lines, the density $B = 100{,}000 \div 10 = 10{,}000$ gausses or lines per square centimeter. Flux density is sometimes expressed in kilolines per square centimeter or per square inch, a kiloline being 1000 lines.

35. Residual Magnetism.—The flux which remains in a piece of iron or steel after the magnetizing force has been removed is called the residual magnetism or residual flux. Residual magnetism plays an important part in the starting of self-excited generators. (See Article 96.) The residual magnetism of wrought iron, soft steel, or cast iron, is very small, while hard steel retains a much larger proportion of the flux after the magnetizing force is removed. Hardened steel is therefore always used in the manufacture of permanent magnets such as are used for electric meters of various kinds and for small electric generators called magnetos.

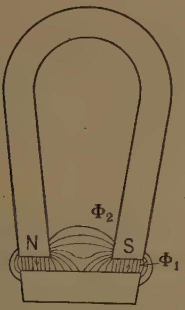


FIG. 24.—Leakage Flux.

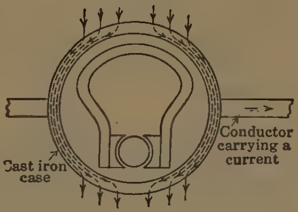


FIG. 25.—Magnetically Shielded Instrument.

36. Magnetic Leakage.—It is impossible to confine magnetic flux to as definite a magnetic path or circuit, as it is to confine an electric current to a definite electric circuit. Hence in many magnetic circuits a considerable portion of the total flux produced does not pass through the portion of the circuit where it can be used, and this part of the flux is called a leakage flux. For example, if the magnetic field around the permanent magnet shown in Fig. 24 were examined, it would be found that a considerable amount of the total flux Φ does not pass through the armature but " leaks " across between the poles and, therefore, is not effective in producing a pull on the armature. This flux Φ_2 is called the leakage flux while the flux Φ_1, which produces a pull on the armature, is called the useful flux. The total flux Φ is equal to the sum of the useful and leakage fluxes or $\Phi = \Phi_1 + \Phi_2$. It will be shown in a later article that the amount of the leakage flux depends upon the relative magnetic resistance (called reluctance) of the useful and the leakage paths.

Because it is impossible to "insulate" magnetic flux, it is sometimes difficult to eliminate the effect which stray magnetic fields have upon sensitive electric measuring instruments. Thus, a conductor carrying a large current produces a strong magnetic field which may affect the reading of meters close by. The meter is, therefore, shielded by placing it in a cast-iron case so that the stray lines of force find a path through the case and do not influence the meter. This is shown in Fig. 25.

PROBLEMS ON CHAPTER VI

1–6. A solenoid, in which there is a steady current flowing, has a core 10 centimeters in diameter. The current is sufficient to give a total flux of 75,000 lines.

(*a*) What is the flux density in the core?

(*b*) Assuming the total flux kept constant, what would be the density with a core 5 centimeters in diameter?

2–6. The pole face of a dynamo is 12 inches long × 9 inches wide. The flux density at the pole face is 49,000 lines per square inch.

(*a*) What is the total flux per pole?

(*b*) If the armature core has teeth with a width equal to the width of the slots, calculate the flux density in the teeth. Neglect fringing effect at the pole tips and assume all flux passes through the teeth.

3–6. A transformer requires a total flux of 1,100,000 lines for proper design. The iron to be used is of such a quality that a flux density of 30,000 lines per square inch is allowable.

(*a*) What is the required cross-section of the transformer core?

(*b*) If a density of 20,000 lines were used, what would be the core cross-section?

4–6. Referring to Prob. 2–6, assume that the leakage of flux between pole tips is 18 per cent of the useful flux at the pole face.

(*a*) What will be the total flux in the field cores and yoke of the machine?

(*b*) The field cores are 9 inches × 6.5 inches in cross-section. Calculate the flux density in the cores.

CHAPTER VII

MAGNETIC EFFECT OF THE ELECTRIC CURRENT

37. Magnetic Field around a Conductor.—Whenever an electric current flows in a circuit, a magnetic field is produced by the current. The direction of this field bears a definite relation to the direction of the current. For a conductor of circular cross-section carrying a current, the magnetic field can be represented by concentric circles, as shown in Fig. 26, when the return current is a considerable distance from the conductor. The relation between the direction of the field and the direction of the current can be remembered by the use of the **thread rule** (Fig. 26), which is, that if a screw with a right-hand thread is turned in the direction of the magnetic field the screw will move axially in the same direction as the current in the conductor. The field of two conductors with current in opposite directions is shown in Fig. 27.

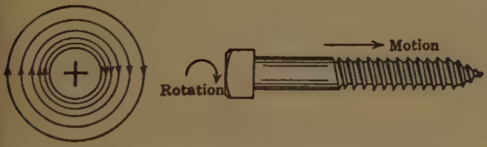


FIG. 26.—Magnetic Field around a Conductor.

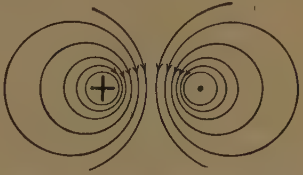


FIG. 27.—Magnetic Field for Two Conductors.

38. Electromagnets.—When current is passed through a coil of wire (Fig. 28), magnetic poles are produced at each end of the coil and it acts like a magnet. It is, therefore, called an electromagnet. These magnetic poles are produced even if the coil is wound on a non-magnetic " core " or support, but the poles are stronger if this core is made of magnetic material. The polarity of an electromagnet depends upon the direction of current in the coil. A rule defining this direction is: If a right-hand screw placed in the axis of the coil is turned in the direction of the current flow, the screw will move axially in the direction of the lines of force, i.e., from S to N, in Fig. 28.

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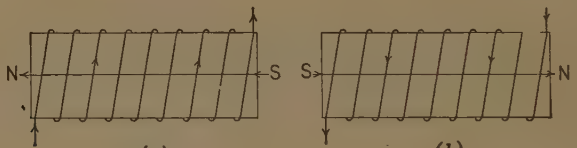
39. Magnetomotive Force.—If N turns of insulated wire were wound upon a wooden ring or "core" (Fig. 29), and a current were passed through this conductor, a magnetic flux would be produced in the core. The amount of this flux would depend directly upon the number of turns N and current I, and inversely upon the "magnetic resistance" or reluctance $\mathcal{R}$ of the core. This is expressed by the equation:

$$\Phi=\frac{0.4\pi NI}{\mathcal{R}}. \quad \quad (10)$$



(a) (b)

FIG. 28.—Field of a Solenoid.

It will be seen that this equation is similar in form to Ohm's Law for the electric circuit. The flux Φ corresponds to the current in an electric circuit, and the quantity $\mathcal{R}$, which is the reluctance of the path in which the flux is produced, corresponds to the resistance of the electric circuit. The expression $0.4\pi NI$ corresponds to the electromotive force in an electric circuit and is therefore called the magnetomotive force (m.m.f.) of the coil. This magnetomotive force exists in the closed magnetic circuit consisting of the core (Fig. 29) and produces the flux Φ. The magnetomotive force is represented by the symbol F, therefore,

$$F=0.4\pi NI=1.257NI. \quad \quad (11)$$

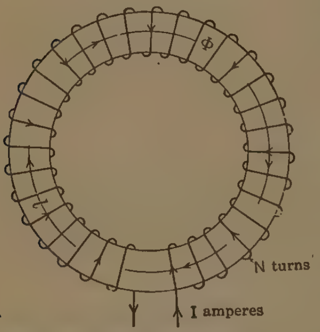


FIG. 29.—Magnetomotive force of a Solenoid.

This gives the magnetomotive force of any coil of N turns in which a current of I amperes is flowing. The factor 0.4π results from the manner in which the magnetic units are defined. Expressed in the c.g.s. system, the unit of magnetomotive force is the gilbert as determined from Equation (11). The product NI is called the ampere-turns of a coil and since the magnetomotive force F, expressed in gilberts, is proportional to the ampere-turns, Equation (11), the term ampere-turn is frequently used as a measure of the magnetomotive force. One ampere-turn = 1.257 gilberts.

40. The Magnetic Reluctance.—$\mathcal{R}$ in Equation (10) is proportional directly to the length (l) and inversely to the area (A) of the magnetic circuit and hence is similar to the resistance of an electric circuit. In the example shown in Fig. 29 the area of the magnetic circuit A can be taken as the area of the wooden core, and the length l as the average circumference of this core, in which case the reluctance is

$$\mathcal{R}=\rho\frac{l}{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where ρ is a factor the value of which depends upon the kind of material composing the magnetic circuit. For non-magnetic circuits $\rho=1$. For magnetic materials ρ depends upon the flux density as well as upon the kind of material, and furthermore the area A and length l is not always so easy to determine as in the case of Fig. 29. Hence, it is not always convenient to use Equation (10) for calculating the flux in a magnetic circuit. A different method is given in Article 42.

41. Flux Density.—Referring again to the magnetic circuit of Fig. 29, the area A is uniform and the total flux Φ is the same at all points in the ring; therefore, the average flux density at any point in this magnetic circuit is $B=\Phi\div A$. Combining this with Equations (10) and (12) we have

$$\begin{aligned} B&=\frac{0.4\pi NI}{\mathcal{R}A} \\ &=\frac{0.4\pi NI}{l}=1.257\frac{NI}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13) \end{aligned}$$

This equation can be used to determine the ampere-turns required to produce a specified flux density in non-magnetic materials. If l is given in centimeters, and B in lines per square centimeters, then

$$NI=0.796Bl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For lengths in inches and the density in lines per square inch we have

$$NI=0.3132B'L. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Example 1.—Suppose the ring, Fig. 29, has an area of 3 square inches and a length of 25 inches. Calculate the ampere-turns required to produce a total flux of 3500 lines in the core.

The density is $B'=3500\div3=1165$ lines per square inch. The required ampere-turns is

$$NI=0.3132\times1165\times25=9120 \text{ ampere-turns.}$$

If an iron core were substituted for the wooden core of Fig. 29, the flux produced by a given number of ampere-turns would be very much greater. Equation (13) then becomes

$$B_m = \mu \frac{0.4\pi NI}{l}, \quad \dots\dots\dots \quad (16)$$

where μ is a factor called the permeability of the magnetic circuit. The magnetic units are so chosen that the permeability of air is unity. Many other substances such as wood, glass, copper, aluminum, and almost all metals, also have a permeability of unity because when they are substituted for air in the magnetic circuit the flux remains unchanged. They are, therefore, called "non-magnetic." On the other hand, a few metals, of which iron and steel are the most important examples, have a permeability much higher than air; that is, for a given magnetomotive force, a much greater flux is produced. They are known as "magnetic" substances. The permeability of magnetic materials such as iron or steel is not a constant, but varies with the flux density, being highest at low densities and decreasing with higher densities. The permeability of the sheet steel employed for electrical machinery is about 2500 for the densities ordinarily used. The permeability of cast iron or cast steel is much lower.

It should be noted that the density B for a simple magnetic circuit, as illustrated in Fig. 29, depends upon the magnetomotive force per unit length (centimeter or inch) and the permeability of the material and is independent of the area of the magnetic circuit. The total flux produced, however, depends upon the area of the magnetic circuit and is given by the equation $\Phi = BA$, where A is the area of magnetic circuit.

42. Magnetization Curves.—It is apparent that, in order to use Equation (16) to calculate the flux produced in a circuit composed of magnetic material, it is necessary to know the permeability of the material, and, since this varies with the density, it is necessary to employ a curve derived from tests of the material. A simpler method in many cases is, however, to employ a magnetization curve which shows the relation between flux density and the magnetomotive force per unit length of magnetic circuit. These curves are plotted with lines per square centimeter as ordinates, and gilberts per centimeter as abscissae, or with lines per square inch and ampere-turns per inch. In either case the shape of the curve would be the same. The magnetomotive force per unit length of path, i.e., gilberts per centimeter or ampere-turns per inch is sometimes called the magnetizing force and is represented by the symbol H. The magnetization curves are, therefore, sometimes called "$B-H$" curves. Typical magnetization curves are shown in Fig. 30.

The meaning of these curves is shown by the following:

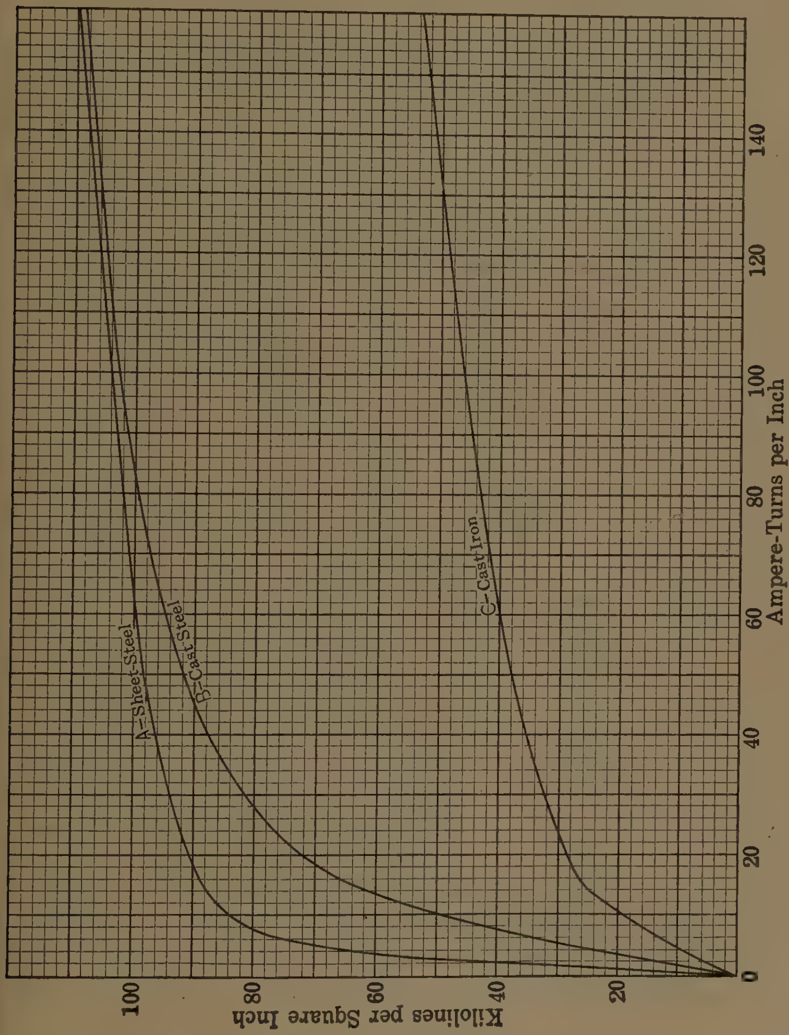


FIG. 30.—Magnetization Curves. *Curve A from American Sheet and Tin Plate Co. Curves B and C from Westinghouse Electric & Mfg. Co.*

Example 1.—A ring of cast steel which has a section 2 inches by 1 inch carries winding of 100 turns through which a current of 10 amperes is flowing. The average length of path is 20 inches. What is the flux density produced?

The magnetomotive force acting on the circuit 20 inches in length is $100 \times 10 = 1000$ ampere-turns. The magnetizing force per inch is $1000 \div 20 = 50$ ampere-turns per inch of length of circuit. From the curve of Fig. 30, it is found that

50 ampere-turns per inch will produce a flux density of 92 kilo-lines per square inch in cast steel.

The total flux Φ would be $2\times1\times92{,}000=184{,}000$ lines.

Example 2.—Given a ring of cast iron having a section 0.5 inch by 0.75 inch, and an average length of path of 15 inches, how many ampere-turns are required to produce a total flux of 15,000 lines?

The flux density is

$$B=\frac{15{,}000}{0.5\times0.75}=40{,}000 \text{ lines per square inch.}$$

From the curve, Fig. 30, it is found that 60 ampere-turns per inch are required to produce this flux density. Hence the magnetomotive force is $60\times15=900$ ampere-turns. This m.m.f. could be produced by a coil of 900 turns carrying 1 ampere or by a coil of 90 turns carrying 10 amperes, etc.

43. Magnetic Saturation.—It will be seen by referring to the magnetization curves (Fig. 30), that for low densities the flux density B is proportional to the magnetizing force, but, as the density increases, B increases more slowly until finally a distinct bend occurs (at about 85,000 lines per square inch, in the case of sheet steel). Beyond the bend, the curve rises very slowly, indicating that the flux increases only slightly with a very large increase in the magnetizing force. This phenomenon is called magnetic saturation. When such a high flux density is produced in any material that a large increase in H increases B only slightly, the material is said to be nearly saturated. Magnetic materials exhibit this characteristic, but air and non-magnetic materials do not; so their magnetization curve is a straight line.

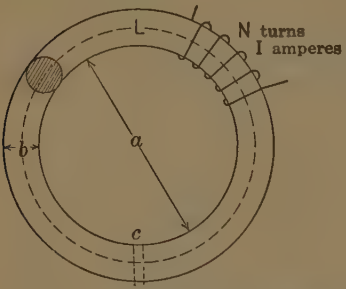


Fig. 31.—Magnetization of a Ring.

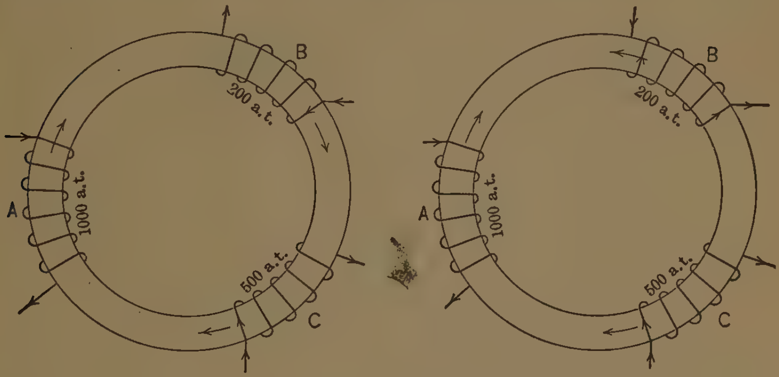
44. The method of calculating magnetic circuits, most commonly employed, requires the use of magnetization curves like those in Fig. 30. For a magnetic circuit like the steel ring (Fig. 31), the leakage of flux is so small as to be negligible, and hence, the total flux Φ is the same at all points in the magnetic circuit. The ampere-turns can then be easily determined.

Example 1.—Assume that the ring shown in Fig. 31, is composed of cast steel with dimension $a=15$ inches, and $b=1.5$ inches. Determine the ampere-turns required for the coil wound around this ring, to produce a flux $\Phi=100{,}000$ lines in the ring.

The area, $A=1.5^2\times0.7854=1.77$ square inches, and the average length of path, $L=\pi(15+1.5)=51.9$ inches. The flux density $B=100{,}000\div1.77=56{,}500$ lines per square inch. According to the curve, Fig. 30, a magnetizing force of 12.5 ampere-turns per inch is required to produce this flux density. The total number of ampere-turns (m.m.f.) is $12.5 \times 51.9 = 649$ ampere-turns for the coil.

Example 2.—Suppose that the ring shown in Fig. 31 is so constructed that half the length L or 26 inches is cast iron and the other half cast steel. Neglect the effect of the joints between the steel and cast iron. Determine the total ampere-turns required to produce a total flux of 75,000 lines in the magnetic circuit.

The flux density $B = 75{,}000 \div 1.77 = 42{,}400$ lines per square inch. Referring to the curves, Fig. 30, it is found that to produce this density in the cast iron, 74 ampere-turns per inch are required so that the total ampere-turns for the cast iron part of the circuit is $74 \times 26 = 1924$ ampere-turns. Similarly, for the cast-steel portion of the circuit the total ampere-turns would be $8 \times 26 = 208$. Since these two parts in series constitute the complete magnetic path, the total ampere-turns which must act on this circuit would be the sum of the ampere-turns for each part or $1924 + 208 = 2132$ ampere-turns for the entire ring. The coil must, therefore, be constructed to produce a m.m.f. of 2132 ampere-turns.



(*a*) (*b*)

FIG. 32.—Magnetizing Effect of Several Coils.

In connection with these two problems it should be noted that the magnetizing coil can be placed anywhere on the magnetic circuit and need not cover the entire circuit as did the winding on the ring shown in Fig. 29. Particularly, in Example 2, it is not necessary that one coil having 1924 ampere-turns be placed around the cast-iron portion of the circuit and another coil having 208 ampere-turns be placed around the cast-steel portion of the circuit, but a single coil of 2132 ampere-turns placed around the ring at *any* point will produce the required flux. The only essential is that the magnetic circuit shall pass *through the coil* or, in other words, shall be "linked with" the coil. It should be noted, however, that if several coils are placed on the same magnetic circuit, as shown in Fig. 32, then the total magnetomotive force effective in producing flux in the magnetic circuit is the algebraic sum of the ampere-turns of the coils. Thus in

Fig. 32 *a* each of the three coils, *A*, *B*, and *C*, are so arranged that they tend to produce a flux in the same direction as indicated. Hence the total m.m.f. is 1000+500+200=1700 ampere-turns. If the direction of current in *B* is reversed as shown in Fig. 32 *b* it would tend to establish a flux in a direction opposed to that of *A* and *C*, hence the resulting m.m.f. is 1000+500−200=1300 ampere-turns.

Example 3.—Assume that the cast-steel ring shown in Fig. 31 has a section cut out leaving an air gap 0.25 inch in length as shown at *C*. It is required to determine the m.m f. for the entire circuit to produce a flux of 75,000 lines across the air gap *C*. Because this gap is short, it can be assumed that the leakage of flux is negligible so that there will be 75,000 lines in the steel. The fringing or spreading of the lines at the air gap will be neglected.

The flux density at all parts of the circuit is $B=75{,}000 \div 1.77=42{,}400$ lines per square inch. The length of the steel path is $51.9-0.25=51.65$ inches. Using the curve, the ampere-turns for the steel portion of the path is $18.5\times 51.65=955$ ampere-turns. For the air path, Equation (15) can be used.

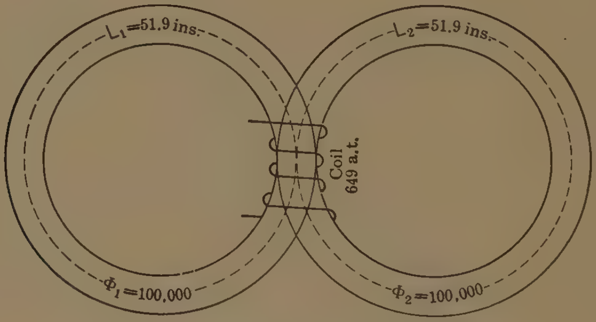


FIG. 33.—Magnetic Circuits in Parallel.

$$NI=0.3132B'L=0.3132\times 42{,}400\times 0.25=3320 \text{ ampere-turns.}$$

The ampere-turns required to produce the specified flux would be $955+3320=4275$ ampere-turns.

In Example 1, at the beginning of this article, it was shown that, when a coil giving 649 ampere-turns acted upon a cast-steel ring of the dimensions specified, a flux density of 56,500 lines per square inch is produced, and the total flux through the ring is 100,000 lines. Suppose this coil is also surrounded by another ring of exactly the same length, cross-section and material (Fig. 33). The m.m.f. acting on ring 1 is 649 ampere-turns and the m.m.f. acting on ring 2 is also 649 ampere-turns, since the two rings constitute separate and distinct closed magnetic paths which are linked with the coil. They are, in fact, parallel circuits so that each of them is acted upon by the entire m.m.f. The magnetizing force is, therefore, the same for each ring and is 12.5 ampere turns per inch. Hence, the density in each ring is 56,500 lines per square inch, the flux Φ in each ring is 100,000 lines and the *total* flux produced by the coil is 200,000 lines. In general, it may be said that when there are several distinct magnetic circuits constituting closed paths, linked with a coil, each of these paths is acted upon by the entire magnetomotive force of the coil.

45. Force on a Conductor in a Magnetic Field.—When a conductor carrying a current is in a magnetic field, a force is exerted upon the conductor, tending to move it at right angles to the direction of the current and of the magnetic lines. This is shown in Fig. 34, where a conductor carrying current away from the observer tends to move down, due to the combined influence of the field produced by the magnet *NS* and the field produced by the current in the conductor. The direction of the force depends upon the polarity of the magnet and the direction of the current. If the polarity of the magnet were reversed the force would be upwards; similarly, if the direction of the current were reversed, the direction of the force would be reversed. A simple method of determining the direction of the force is to plot the resultant field produced by the combination of flux due to the current and that due to the magnet as shown in Fig. 34. The direction of the fields is the same on the upper side of the conductor, while below it, the field due to the current tends to neutralize that of the magnet. Hence, there is a strong field above the conductor and a weak field below. The lines representing this field can be imagined to act like stretched rubber bands and hence the conductor would be forced downward towards the weaker side of the field.

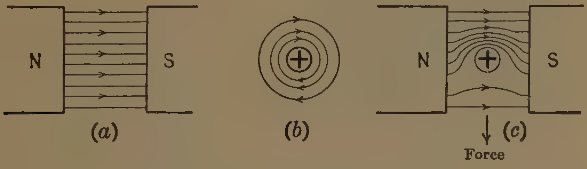


FIG. 34.—Force on a Conductor.

The force F, tending to move a conductor in a magnetic field, has been proved experimentally to be proportional to the flux density, the length of the conductor in the field, and the current in the conductor. This is given by the equation

$$F=\frac{BLI}{10}, \qquad (17)$$

where F is expressed in dynes, B in lines per square centimeter, I in amperes, and L in centimeters. Expressed in pounds, lines per square inch, and length in inches, the equation is

$$F' = \frac{B'L'I}{11{,}300{,}000} \text{ pounds. . . . . } \quad (18)$$

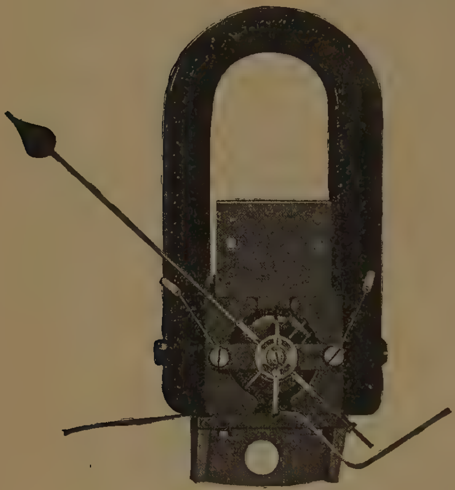


FIG. 35.—Moving System of a Weston D. C. Instrument.

This force relation has many important applications, of which the following examples are the most common.

(1) An electric motor consists of a steel frame having a magnetic flux so arranged as to act upon a number of conductors carrying current. The force which results, according to the principle stated in the preceding paragraph, causes motion of either the magnetic field or the conductor, and the machine can be used to drive a mechanical load.

(2) A coil consisting of a number of turns of very small wire is suspended in an intense magnetic field produced by a permanent magnet, as shown in Fig. 35. When current flows in the coil, a force is exerted on the coil and it turns on its pivots until the force exerted by the current is balanced by the tension of the controlling springs. A pointer attached to the coil gives an indication on a scale of the strength of the current in the coil. This is the principle of the so-called D'Arsonval type of ammeters and voltmeters, a commercial type of which is the Weston d.c. ammeter and voltmeter.

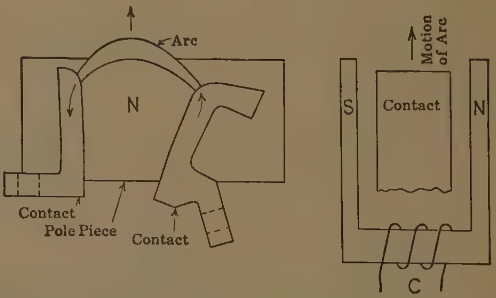


FIG. 36.—Magnetic Blowout as Applied to Switch Contacts. A magnetic field is produced between the contacts by the electromagnet NS which is excited by the coil C. This coil is in series with the contacts and carries the main current.

(3) If a magnet is brought near an electric arc, it tends to move this arc in a definite direction depending upon the polarity of the magnet (Fig. 36). If the magnet is strong enough, the arc will be greatly elongated and finally broken. This constitutes the "magnetic blowout" which is so generally used on direct-current circuit breakers, controllers, and various types of switches. An example of the magnetic blowout as applied to an electrically operated switch is shown in Fig. 36. (See also Fig. 41).

The forces acting upon two parallel conductors carrying current are shown by Fig. 37. If the currents are in opposite directions, the conductors repel each other (Fig. 37 *a*), while if in the same direction they attract (Fig. 37 *b*). Under short-circuit conditions, this force is very large, and cases have occurred in practice where feeders have been torn from their supports due to this force. In electric transformers, the coils are very likely to be distorted or displaced unless they are very carefully braced, while the end connections of generators, especially turbo-generators, require very substantial bracing to prevent their displacement under the extremely large forces which occur during a short circuit. Where the conductors are in the vicinity of magnetic material, the force is greatly increased, due to the greater flux produced by a given current.

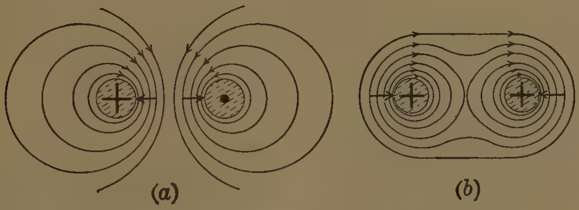


FIG. 37.—Force on Conductors.

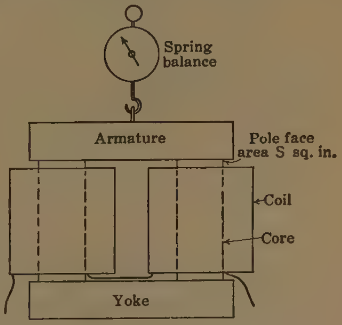


FIG. 38.—Pull of a Magnet.

46. Pull of a Magnet.—It is commonly known that an electromagnet, like that shown in Fig. 38, exerts a force on its armature which resists any attempt to remove it. It has been proved that the force tending to keep the armature against the poles depends upon the strength of the field and is given by the equation

$$F_m = \frac{B^2 S}{72{,}130{,}000} \text{ pounds}, \quad . \quad (19)$$

where F_m = pounds at the pole face;
B = flux density at the pole face in lines per square inch;
S = area of the pole face in square inches.

In the example shown in Fig. 38 the total pull on the armature is $2F_m$. If the armature is separated a slight distance from the poles, giving an air gap in the magnetic circuit, the pull for a fixed number of ampere-turns in the magnetizing coils will be very greatly reduced, since the introduction of the air gaps greatly increases the reluctance of the magnetic circuit, thus decreasing the total flux and hence the flux density. When the air gap so introduced is comparatively short, the fringing or spreading of the lines at the air gap can be neglected and the density B at the pole faces can be used in Equation (19).

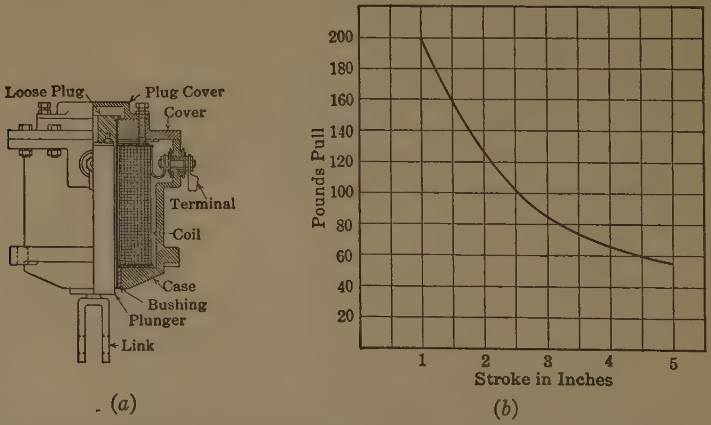
It should be noted that wherever a joint or an air gap occurs in a magnetic circuit there is a force tending to shorten this air gap or to bring the surfaces more closely into contact. The effect may be likened to that produced if the lines crossing the air gap were stretched elastic filaments.

Example 1.—Assuming that the maximum density for cast steel is limited to about 100 kilo-lines, because of commercial considerations, calculate the maximum pull, in pounds per square inch, which it is possible to produce when the armature of the magnet is in contact with the poles.

$$F=\frac{\overline{100,000}^2}{72,130,000}=138.8 \text{ pounds per square inch.}$$

APPLICATIONS OF ELECTROMAGNETS

47. Solenoids.—The term solenoid is commonly applied to an electromagnet consisting of a coil of insulated wire wound in one or more layers and having a movable iron plunger, although strictly any coil of wire which can be used to produce a magnetic field, if current is passed through the coil, is a solenoid. As used in practice, solenoids are ordinarily iron-clad; that is, the coil is surrounded by a frame of iron or steel and the plunger is arranged to move in or out of the



(a) (b)

FIG. 39.—Iron-clad Solenoid. *The Cutler-Hammer Mfg. Co.* (*a*) Sectional view; (*b*) Pull diagram, showing pull with different positions of the plunger.

center of the coil (Fig. 39). When the coil is energized, there is a tendency to pull the plunger into the core, thus decreasing the air gap. The pull of the solenoid increases rapidly as the air gap decreases, because the flux increases and the force is proportional to the square of the flux.

The size of a solenoid depends upon the force which it must exert and the distance of travel of the plunger. In general, it is best to design a solenoid to have a short travel and produce a large force, using levers to connect the plunger with the device to be operated, rather than to design it to give a smaller force with a longer travel. Figure 39 *b* shows the variation in pull of an iron-clad solenoid.

Solenoids are used for releasing the friction brake commonly employed on cranes (Fig. 40). When there is no power applied to the motor the solenoid is de-energized and the springs keep the brake closed. The solenoid coil is connected across the motor terminals so that when the power is applied the solenoid acts to overcome the pull of the springs and thus releases the brake and allows the motor to start. Any failure of power immediately allows the springs to apply the brake and prevents lowering of the load.

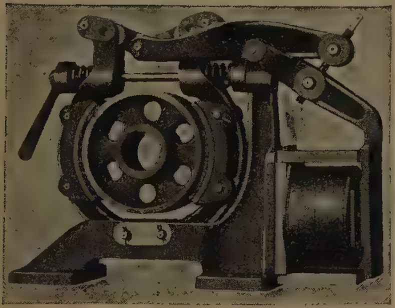


FIG. 40.—Crane Brake.
The Westinghouse Elec. & Mfg. Co.

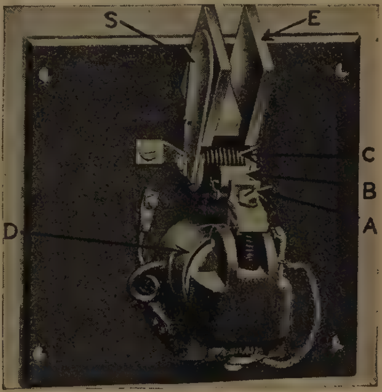
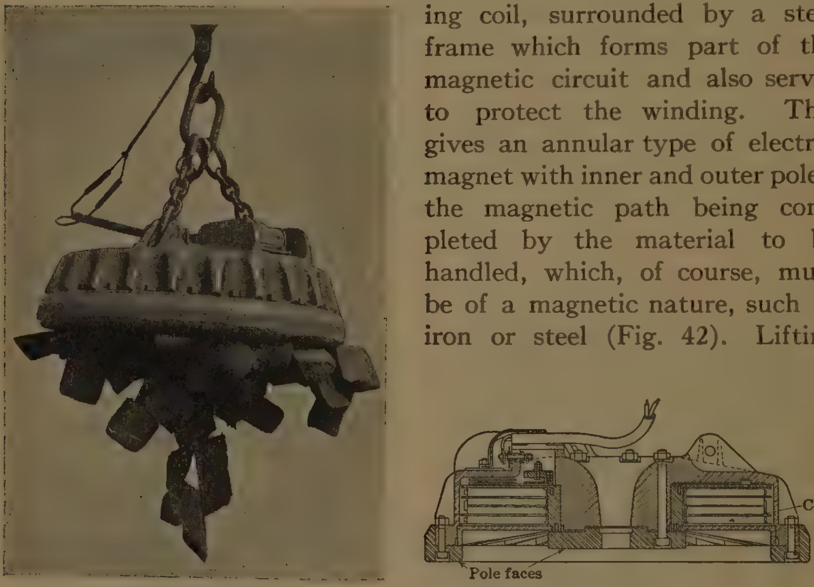


FIG. 41.—Magnetic Contactor.
The General Electric Co.

Electromagnets are used extensively for operating electric switches or contactors which are employed in automatic motor starters and controllers (Fig. 41). The essential parts are an operating coil *D*, surrounding an iron core and yoke, with a movable armature which carries a contact *A*. This is one terminal of the circuit. A stationery contact *B* forms the other terminal. When the coil is energized the armature forces *A* against *B*, thus closing the circuit. Coil *C* is also in the circuit and produces a magnetic field giving a magnetic blowout for the arc when the contents are opened. (See Article 45 and Fig. 36.)

48. Lifting magnets are used for handling iron or steel pieces in bulk, either as scrap or as finished material. As developed in practice, these magnets consist of an exciting coil, surrounded by a steel frame which forms part of the magnetic circuit and also serves to protect the winding. This gives an annular type of electromagnet with inner and outer poles, the magnetic path being completed by the material to be handled, which, of course, must be of a magnetic nature, such as iron or steel (Fig. 42). Lifting magnets are designed to use direct current. The amount of power required to handle a given amount of material depends upon whether it is scrap, is irregular in shape, or is in the form of bars or billets. For example, a certain 36-inch-diameter lifting magnet requiring 3.3 kw. for excitation can hold 15,000 pounds of billets or slabs, but only 200 to 600 pounds of scrap.



(*a*) (*b*)

FIG. 42.—Lifting Magnet. (*a*) Magnet lifting pig iron; (*b*) Section of magnet. *The Cutler-Hammer Mfg. Co.*

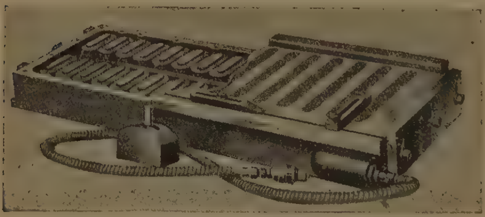


FIG. 43.—Magnetic Chuck. *The Heald Machine Co.*

49. Magnetic chucks are used with planers or grinders to hold small pieces of iron or steel that are to be machined, thus avoiding the necessity of employing the usual clamps. One type of magnetic chuck is shown in Fig. 43. It comprises exciting coils which are surrounded by a cast-steel housing having the bottom surface machined so that it can be placed on the planer or grinder bed. The upper face of the housing, which is also machined, is divided into a number of sections which are alternately of north and south polarity and form the magnet poles. The work is laid on these poles and thereby closes the magnetic circuit. The magnetic pull is sufficient to hold the work firmly in place. The coils are de-energized to remove the work. The amount of power used is small, a chuck having a face 10¾ inches by 34½ inches requiring only 300 watts.

50. Magnetic clutches are used where a load must be connected and disconnected frequently from the driving source. The exciting coil *C* is surrounded by an annular magnetic frame (Fig. 44) which is connected to the load. A movable disk armature *A* connected to the driving shaft through the spring plate *S* is placed close to the poles of this annular frame or housing. When the coil is excited by current supplied through slip rings *R*, the armature is pulled into contact with the magnet and the machine is driven by the friction between the disk and the friction surface *F*.

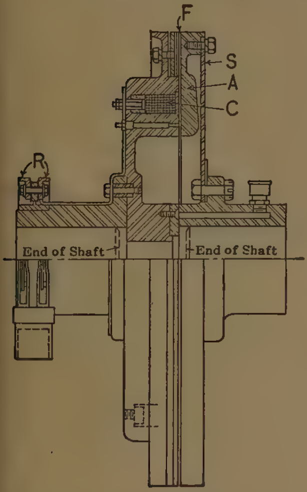


FIG. 44.—Magnetic Clutch. *The Cutler-Hammer Mfg. Co.*

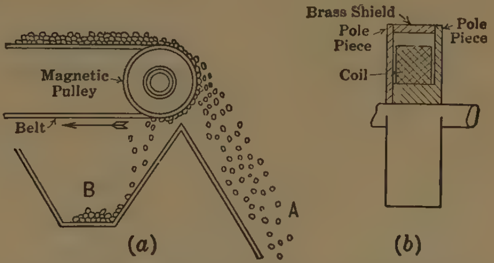


FIG. 45.—Magnetic Separator. (*a*) Principle of action; (*b*) Section of magnetic pulley showing coil and pole pieces. *The Cutler-Hammer Mfg. Co.*

51. Magnetic separators are used for concentrating certain magnetic ores of iron and for removing particles of iron or steel from pulverized material. The crushed ore is delivered onto a moving belt (Fig. 45) which passes over a magnetic pulley consisting of an iron shell excited by a coil. The magnetic ore sticks to the belt and is carried farther around than the non-magnetic material, such as rock or dirt, the latter dropping into a separate hopper *A*.

PROBLEMS ON CHAPTER VII

1–7. Draw a two-pole dynamo field frame and indicate required direction of current through the field coils in order to produce an N pole on the left side of the armature. Indicate also the direction of flux through poles and armature.

2–7. Draw an eight-pole dynamo field showing direction of current in the coils, polarity of field poles, and direction of flux.

3–7. Referring to the wooden ring, Fig. 29, assume that there are 500 turns with a current of 10 amperes in the winding. The average length l is 50 cms.

(*a*) Express the magnetomotive force of this winding in ampere-turns and in gilberts.

(*b*) If the ring has a diameter of 5 cms., calculate the flux density.

(*c*) Calculate the total flux produced by this winding.

4–7. (*a*) Assuming dimensions of the wooden core the same as in Prob. 3–7, calculate the ampere-turns required to produce a flux density of 500 gausses in the core.

(*b*) If the density were doubled, what would be the required magnetomotive force?

(*c*) If the current available is limited to 25 amperes, calculate the number of turns required to produce the flux specified in (*a*).

5–7. (*a*) Referring to Fig. 29, calculate the reluctance $\mathcal{R}$ for a non-magnetic ring when $l = 100$ cms. and the diameter of the section is 6 cms.

(*b*) Calculate the reluctance when $l = 100$ cms. and the area is one-half that specified in (*a*).

(*c*) What would be the reluctance if the cross-section were the same as in (*a*), but $l = 200$ cms.?

(*d*) What would the reluctance become if both l and the area of the ring were doubled?

6–7. It is desired to produce a total flux of 1,000 lines in a brass ring, having a mean length of 15 inches and a circular cross-section of 0.75 inch diameter.

(*a*) Calculate the required ampere turns.

(*b*) How much current would be required if $N = 50$ turns?

7–7. (*a*) Calculate the flux density produced if the winding specified in Prob. 3–7 were placed on a ring of magnetic material having a permeability of 150 and dimensions as specified in Prob. 3–7.

(*b*) Calculate the total flux produced.

8–7. A winding consisting of 850 turns carrying 2.5 amperes, is placed on a ring having a mean length of 15 inches and an area of 1.5 square inches. The total flux produced in the ring is 110,000 lines. Calculate the permeability of the ring.

9–7. Referring to Fig. 30, determine:

(*a*) The flux density produced by a magnetizing force of 48 ampere-turns per inch, when acting on a cast-iron and a cast-steel circuit.

(*b*) Which material is the better for use as a field frame for a dynamo? Why?

10–7. It is desired to produce a total flux of 1,200,000 lines in a magnetic circuit having an area of 25 square inches.

(*a*) Determine the magnetizing force if cast iron were used.

(*b*) Determine the magnetizing force for cast steel.

11–7. (*a*) Would it be feasible to use cast iron for a magnetic circuit having an area of 18 square inches if the total flux desired were 1,620,000 lines?

(*b*) What materials could be used?

12–7. Compare the effect on the magnetizing force of doubling the flux density in cast steel at low densities of approximately 20,000 lines per square inch, with the magnetizing force for higher densities such as 50,000 lines per square inch.

13–7. Given a ring of cast iron with an average length of magnetic path of 50 cms.

(*a*) What is the magnetizing force required to produce a flux density of 6000 lines per square cm.?

(*b*) How many turns would be required if the current were 5 amperes?

(*c*) If 100 turns were used, calculate the current.

14–7. If a cut were made in a ring of Prob. 13–7 and the ring spread out to 0.5 cm. air gap, what would be the total number of turns for the same density and five amperes current?

15–7. Referring to Prob. 3–6*a*, calculate the ampere-turns required to produce the specified flux if the average length of magnetic circuit were 32 inches and the material, sheet steel.

16–7. Referring to Fig. 31, $b=1.5$ inches and $a=18$ inches.

(*a*) Calculate the ampere-turns required to produce a total flux of 70,000 lines in cast iron.

(*b*) Calculate for the same density with cast steel.

17–7. Referring to Prob. 16–7, what would be the total flux produced if the material were cast steel and the magnetizing force determined in (*a*) were used?

18–7. A ring, having dimensions as in Prob. 16–7, has a piece cut out, leaving an air gap, $c=0.25$ inch (Fig. 31). Material is cast steel. (*a*) What would be the required ampere-turns to produce a total flux of 70,000 lines in the air gap?

(*b*) If the air gap were made 0.125 inch, what would be the required number of ampere-turns to produce the same total flux?

19–7. Referring to Prob. 16–7, assume that 15 inches length of the ring is of cast iron and the remainder of cast steel.

Calculate the total ampere-turns required to produce a flux of 70,000 lines in the ring.

20–7. A certain magnetic circuit has a winding consisting of 1550 turns in which 0.75 ampere is flowing. Linked with this magnetic circuit is a second electric circuit consisting of a single turn arranged to oppose the first winding. How much current must flow through the second circuit in order to make the flux in the magnetic circuit zero?

21–7. Referring to Fig. 34, what would be the direction of the force on the conductor if both the polarity of the field and the direction of the current were reversed?

22–7. Referring to Fig. 48*b*, assume direction of current in conductors *a* and *b* is as shown.

(*a*) Determine direction of force on the conductors.

(*b*) The flux density in the air gap is 4000 gausses, the active length of each conductor 50 cms., the current per conductor 50 amperes. Find the force in pounds acting on each of these conductors.

23–7. Referring to Fig. 38, the electromagnet has the following dimensions: armature 2 inches×4 inches, yoke 4 inches×3 inches, cores of circular section 3.5 inches diameter. The length of magnetic path is as follows: armature 16.1 inches, two cores 20 inches, yoke 16.7 inches. Total pull required 80 pounds. Material cast iron.

(*a*) Calculate the ampere-turns required for each coil, assuming the armature in contact with the poles. (Allow 0.005 inch air gap.)

(*b*) Calculate the ampere-turns required with a 0.5 inch separation between armature and pole faces.

CHAPTER VIII

INDUCED ELECTROMOTIVE FORCE

52. Methods of Producing an E.M.F.—There are only three methods employed in practice to produce an e.m.f. These are, in order of importance: (*a*) electromagnetic induction, (*b*) chemical action, (*c*) thermal action. The production of electricity upon the present enormous scale is dependent upon electromagnetic induction since it would be impossible to accomplish this result by the use of primary batteries (chemical action) or by thermal action. Electricity is produced by thermal action when the junction of two dissimilar metals is heated; and a device which produces an e.m.f. in this manner is called a thermocouple. The thermocouple is used in some types of measuring instruments, but is of no practical importance as a source of electricity for general power purposes. In this chapter, we are to consider the production of electricity only by electromagnetic induction.

53. Electromagnetic Induction.—It was discovered experimentally by Faraday that, whenever there is a change in the magnetic field surrounding a conductor, an electromotive force is induced in the conductor. This induced e.m.f. may be produced in several ways as follows:

(*a*) A conductor may move in a stationary magnetic field of constant strength.

(*b*) A stationary conductor may be exposed to a moving magnetic field of constant strength.

(*c*) The strength of the field surrounding the conductor may change without any motion of conductor or magnetic circuit.

The principle illustrated by (*a*) is involved in the operation of d.c. generators and motors, while (*b*) applies to nearly all a.c. generators and many a.c. motors; (*c*) is employed in spark coils, induction coils, and in transformers, which are so commonly used to convert electrical energy from one voltage to another.

54. The e.m.f. induced by motion of a conductor or a magnetic flux is the same whether the conductor rotates and the flux is stationary or the flux rotates and the conductor is stationary. This can be stated briefly by saying that the relative motion of the two determines the e.m.f. induced. Thus, if both conductor and flux rotate in the same direction at the same speed, no e.m.f. will be produced; if they rotate at the same speed, but in opposite directions, the e.m.f. induced would be *twice* that which would be induced if either were stationary. An e.m.f. is not induced when a conductor is moved parallel to the lines of force, but only when it moves at an angle with these lines. Thus, in Fig. 46, no voltage will be induced when the conductor is moved horizontally in the uniform field produced by the magnet *NS*. Any motion across the direction of the lines will, however, produce an e.m.f. in the conductor. For this reason, the conductor is said to " cut " the lines of force. The actual e.m.f. induced in the conductor depends upon the rate at which the flux is cut. That is, the e.m.f. is proportional to $\frac{\Phi}{t}$, where Φ is the flux cut in time t.

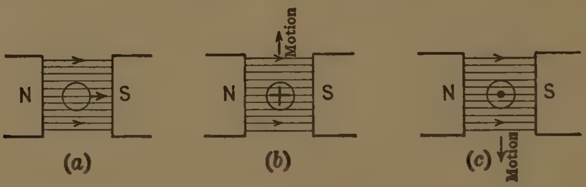


FIG. 46.—Induced E. M. F.

It should be noted that it is incorrect to say that a current is induced by cutting flux. An e.m.f. is always induced when flux is cut in the manner described, and if this e.m.f. acts on a closed circuit, current flows due to this e.m.f. If the circuit is open, however, the e.m.f. is still induced according to the above law, but obviously no current can flow since there is no closed circuit.

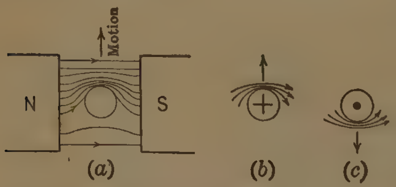


FIG. 47.—Direction of Induced E.M.F.

55. Direction of Induced E. M. F.—There is a definite relation between the direction of the flux, the motion of the conductor, and the e.m.f. induced in the conductor. Thus, in Fig. 46 *b*, an e.m.f. will be induced in a direction away from the observer when the conductor moves upward with the field polarity indicated. If the direction of motion reverses, as in Fig. 46 *c*, the direction of e.m.f reverses. A convenient rule for remembering these relations is shown in Fig. 47.

Imagine that the lines of force are elastic filaments which tend to crowd together and group themselves around the front side of the moving conductor (Fig. 47 *a*). This would give a field around the conductor as shown in Fig. 47 *b*. The direction of the induced e.m.f. is then the same as the direction of current required to produce a field around the conductor in the direction indicated; hence the induced e.m.f. will be, in this case, away from the observer. Reversal of the motion (Fig. 47 *c*) causes crowding of the lines on the bottom side of the conductor; hence it causes a reversal of the direction of the induced e.m.f.

If a conductor in which an e.m.f. is induced forms part of a closed electrical circuit, then current will flow, as was explained in the previous article. If the only source of e.m.f. in the circuit is that produced by the cutting of the flux, then the current will flow in the same direction as the induced e.m.f.; that is, the symbols in the circles (Fig. 46), would represent the direction of current as well as the direction of induced e.m.f. If, however, there is another source of e.m.f. in the circuit in addition to the induced e.m.f. then the current will flow in the *same direction* as the induced e.m.f. only so long as it is greater than any other e.m.f. which opposes it, so that there may be a condition where the current flows in the opposite direction to the induced e.m.f. Current flows in the same direction as the induced e.m.f. in a generator and in the opposite direction in a motor.

56. The Electric Generator.—If a loop of wire is rotated in a magnetic field (Fig. 48 *a*) an e.m.f. will be induced in each side of the loop and, according to the rule given in the previous article, will be away from the observer, for the side under the N pole and towards the observer for the side under the S pole, provided the rotation is clockwise as indicated. The two e.m.f.'s are, therefore, in such a direction that the voltage at the terminals of the loop *ab* is the sum of the e.m.f.'s in the two sides. If the two ends are connected to insulated rings (slip-rings) with which stationary brushes make contact (Fig. 48 *b*), the e.m.f. generated can be employed in forcing current through an external circuit. Such an arrangement constitutes an electrical generator of the simplest kind. A study of Fig. 48 will show that as long as one side *a* of the coil (called a conductor or " inductor ") is under the influence of the N pole, the e.m.f. will be in a direction away from the observer; when this side (*a*) is under the influence of the S pole, the direction of the e.m.f. in this side will be reversed. The e.m.f. between the brushes would, therefore, reverse also and could be represented by a curve (Fig. 48 *d*) which shows the magnitude of the e.m.f. produced during one complete revolution and the polarity of brush 1 with respect to brush 2. In other words, this coil would produce an alternating e.m.f. at the brushes and the machine is a simple a.c. generator or alternator.

A unidirectional e.m.f. at the brushes can be obtained if the connections to the brushes are reversed at the instant the e.m.f. reverses. Thus, if two insulated segments, *s*, *s* (Fig. 49) are connected to the ends of the coil *ab*, an e.m.f. at the terminals 1, 2 can be maintained constantly in a definite direction. This is illustrated by the curve (Fig. 49 *c*), which represents the polarity of the terminals and the magnitude of the e.m.f. generated in the coil during a complete revolution. The machine shown in Fig. 49 is an elementary form of a d.c. generator. In practice, a large number of loops or coils are used and frequently more than two poles are employed. Similarly, the number of segments *s s* which constitute the " commutator " of the d.c. generator is greatly increased. By this means, the pulsations of e.m.f. which are indicated in the curve (Fig. 49 *c*) are smoothed out,

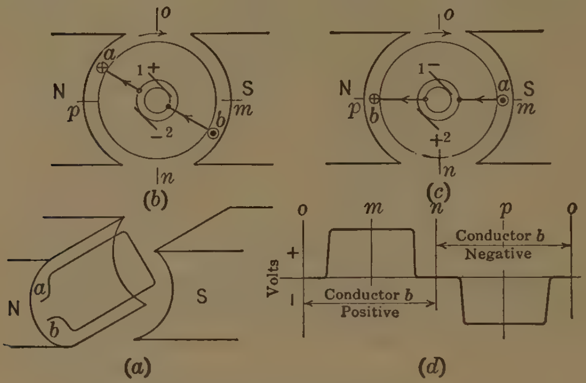


FIG. 48.—Elementary Generator, A. C.

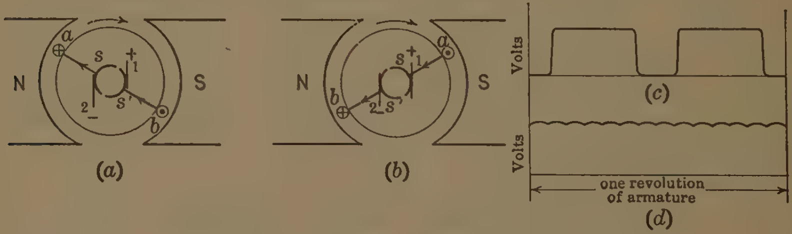


FIG. 49.—Elementary Generator, D. C.

thus producing an e.m.f. which is of nearly constant strength (Fig. 49 *d*).

57. Calculation of Induced E.M.F.—An e.m.f. of one volt is induced in a conductor when it cuts magnetic flux at the rate of one hundred million (10^8) lines per second. Hence, in any case, it is necessary to determine only the rate at which lines are being cut, to determine the e.m.f. in volts. The rate of cutting is always $\Phi \div t$, as was stated in Article 54. If the flux is uniformly distributed so that the conductor cuts a field of constant flux density for the time t there will be a steady e.m.f. induced during the entire time t with a value of

$$E=\frac{\Phi}{t\times 10^8}\text{ volts. . . . .} \quad (20)$$

If, however, the field is not uniform, the e.m.f. will vary during the time t, but the average e.m.f. induced will still be given by Equation (20). As long, then, as the flux density remains constant and the velocity of the conductor is constant, the rate of cutting is constant and a steady value of e.m.f. is induced.

Example 1.—A certain conductor cuts a uniform field of 500,000,000 lines in 2 seconds. What is the steady value of e.m.f. induced while cutting at this rate?

$$E=\frac{500{,}000{,}000}{2\times 10^8}=2.5\text{ volts.}$$

Example 2.—A conductor cuts 1000 lines in one thousandth of a second. What is the steady e.m.f. induced as long as it cuts at this rate?

$$E=\frac{1000\times 1000}{10^8}=0.01\text{ volt.}$$

Example 3.—A conductor cuts 100,000 lines in one-hundredth of a second, and, in the next two-hundredths of a second, it cuts 250,000 lines. What is the e.m.f. induced?

For the first period of 0.01 second the rate is $100{,}000 \div 0.01 = 10{,}000{,}000$ lines per second.

For the second period of 0.02 second the rate is $250{,}000 \div 0.02 = 12{,}500{,}000$ lines per second.

Hence, the steady voltage during the first period is $E_1 = 10{,}000{,}000 \div 10^8 = 0.1$ volt and for the second period it changes to $E_2 = 12{,}500{,}000 \div 10^8 = 0.125$ volt.

The total flux cut is $100{,}000 + 250{,}000 = 350{,}000$ lines. The total time for cutting is $0.01 + 0.02 = 0.03$ second. The average rate of cutting is $350{,}000 \div 0.03$, and the average e.m.f. for the entire time of 0.03 second is

$$E_{av.}=\frac{350{,}000}{0.03\times 10^8}=0.117\text{ volt.}$$

The average voltage also could be found as follows:

$$\begin{array}{lll} \text{First period} & 0.01\times0.1 & =0.001 \\ \text{Second period} & 0.02\times0.125 & =0.00250 \\ \hline & 0.03 & 0.00350 \end{array}$$

$$E_{av.}=\frac{0.0035}{0.03}=0.117 \text{ volt as before.}$$

The student should fix firmly in mind the idea of rate of cutting. A rate of 10^8 lines per second does not mean necessarily that the conductor actually cuts 10^8 lines, but simply that, if it continued to cut lines at this uniform rate for one second, it would then have actually cut 10^8 lines. In the same way, we speak of a train having a speed of 60 miles per hour. This does not mean necessarily that the train actually travels 60 miles, since it may have traveled only 30 miles during one-half an hour, or 15 miles during a quarter of an hour. The rate is, however, $30\div0.5$ or $15\div0.25$ or 60 miles per hour.

Example 4.—Referring to Fig. 49, assume that the loop revolves at a speed of 1800 r.p.m., and that there is a total flux of 10,000,000 lines passing between the N and S poles. Assume that one pole face covers 30 per cent of the entire circumference.

(*a*) Calculate the average e.m.f. induced in one side of the coil during a revolution.

(*b*) Calculate the e.m.f. induced in one side of the coil while it is under a pole, assuming the flux uniformly distributed.

Solution.—(*a*) It is apparent that twice during a revolution the conductor (*a*) has no voltage induced in it and that the total flux that it cuts during one revolution is $2\times10{,}000{,}000$ lines. The machine makes $\frac{1800}{60}=30$ revolutions per second, hence this flux is cut in $\frac{1}{30}$ second. The average rate of cutting is $\frac{2\times10{,}000{,}000}{\frac{1}{30}}$. And the average voltage is

$$E=\frac{2\times10{,}000{,}000\times30}{10^8}=6 \text{ volts.}$$

(*b*) While the conductor is under a pole there is a steady voltage induced because the field is uniform. The flux cut by the conductor in traveling across one pole face is 10,000,000 lines. The time taken to travel this distance is

$$\tfrac{1}{30}\times0.3=0.01 \text{ second.}$$

The rate of cutting is

$$10{,}000{,}000\div0.01,$$

and the e.m.f. is

$$\frac{10{,}000{,}000}{0.01\times10^8}=10 \text{ volts.}$$

Hence, actually during each revolution the e.m.f. in the conductor is 10 volts for 30 per cent of the time, then zero for 20 per cent, then 10 volts in a reversed direction for 30 per cent of the time and finally zero for the remaining 20 per cent of the revolution. The average voltage could, therefore, be found by averaging these values thus:

$$\begin{array}{rr} 0.30\times10=3 \\ 0.20\times\ \ 0=0 \\ 0.30\times10=3 \\ 0.20\times\ \ 0=0 \\ \hline 1.00\qquad\quad 6 \end{array}$$

$$E=\frac{6.0}{1.0}=6 \text{ volts as found in } (a).$$

58. Eddy Currents.—If a solid mass of iron is rotated in a strong magnetic field (Fig. 50 *a*), the iron will soon become heated, indicating a considerable loss of energy in the iron. This heating is due principally to the current which flows as a result of the e.m.f. induced in the iron as it rotates in the magnetic field. The section of iron under the N pole will have an e.m.f. induced in a direction away from the observer and under the S pole in the opposite direction, and, since there is a closed path of very low resistance between the two sides of the cylinder, a large current will flow through the cylinder in the direction indicated by the arrows. This current will cause the cylinder to heat.[1] If the iron cylinder is divided into a number of thin disks which are insulated from each other (Fig. 50 *b*) the e.m.f. induced across a single disk is very much smaller, and the resistance to the flow of current is much greater. Consequently the eddy currents in these disks are much less than before and an armature built in this manner can be run in a magnetic field without excessive heating. It is for this reason that the steel core which supports the armature windings of a dynamo is always built from thin disks or laminations. Thick laminations are cheaper than thinner ones but give greater eddy current losses. In practice, a thickness of

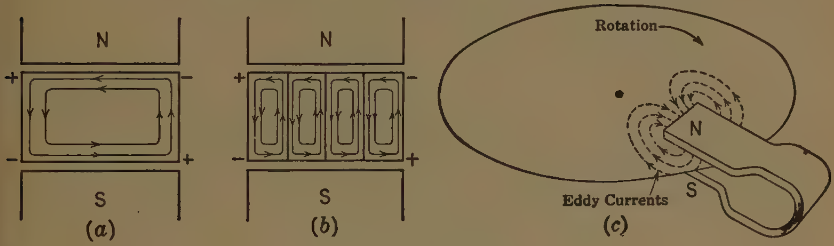


FIG. 50.—Eddy Currents.

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[1] There is also some heating due to "hysteresis." This is discussed in Article 146.

from 0.014 inch to 0.025 inch has been found to satisfy usual requirements.

The principle just described, is made use of in the "damping disk" of watt-hour meters. A copper or an aluminum disk rotates between the poles of powerful permanent magnets (Fig. 50 *c*) and thereby eddy currents are produced in the disk. These currents react upon the magnetic field in such a manner that there is a "drag" on the disk which tends to stop it. As soon as it stops, there are no eddy currents and hence no drag, while the faster the disk turns, the greater is the force required to turn the disk. This force is produced by the metering element of the instrument in such a manner as to give an indication of the particular quantity to be measured.

59. Self Induction.—We have seen that if a conductor forming an electric circuit is linked with[2] a magnetic flux which is changing in strength, an e.m.f. will be induced in the conductor. It has been proved experimentally that if this flux acts on a closed circuit, the induced e.m.f. will cause a current to flow in such a direction as to tend to produce a flux in opposition to the inducing flux, thus opposing the change in flux. This is known as Lenz's Law. It was stated in Article 37 that, whenever an electric current flows in a circuit, a magnetic field is produced, the strength of which is proportional to the strength of the current. Now this magnetic field is linked with the conductor carrying the current, and hence according to Lenz's Law just stated, an e.m.f. will be set up in the conductor, which will oppose the flow of current and tend to send current in the opposite direction. This is called a "self-induced" e.m.f. since it is produced in the same conductor which carries the current producing the magnetic field. Obviously, while the current is changing in strength, the field around the conductor will change and this self-induced e.m.f. will be generated. If the current and magnetic field are increasing, the induced e.m.f. opposes the voltage sending the current through the circuit and, hence, is a counter e.m.f. tending to send current in the opposite direction. That is, it opposes the increase in current. If the current in the circuit is decreasing, then the flux surrounding the conductor is decreasing. An e.m.f. is now induced in the opposite direction which tends to aid the impressed voltage and tries to prevent the decrease of current which is occurring.

60. Growth of Current in an Inductive Circuit.—The effect of self-induction in a d.c. circuit is illustrated in Fig. 51. If a d.c. potential of E volts is applied to a coil having a resistance of R ohms, the current, according to Ohm's Law, should be $I=E\div R$. The

[2] This expression is defined in Article 44.

current will not, however, reach a value of I amperes immediately upon the closing of switch S, but will reach this steady value only after an appreciable time has elapsed as shown in Fig. 51 *b*. During the period of time between 0 and 1, the impressed voltage E performs two functions. It overcomes the fall of potential due to the resistance R and it also balances the e.m.f. of self-induction. Hence, at any instant, such as point x, the total voltage is

$$E = e_R + e_L,$$

where e_R is the fall of potential due to resistance ($e_R = iR$) and e_L is the voltage required to overcome the self-induced e.m.f. The value

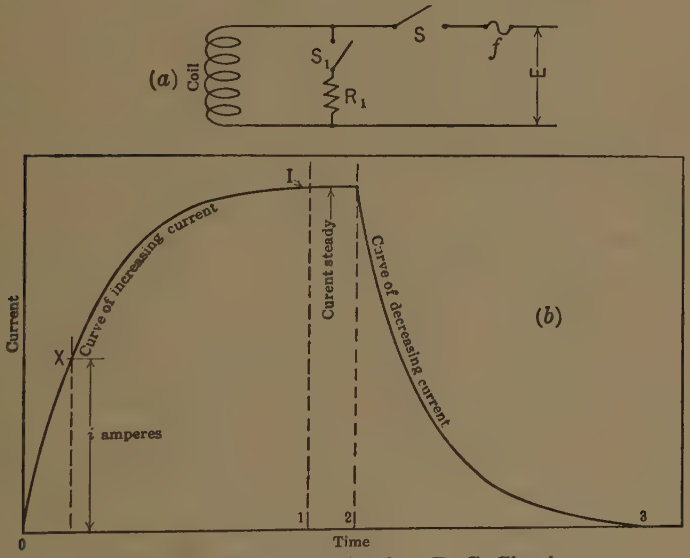

FIG. 51.—Self Induction in a D. C. Circuit.

of e_R is zero at the instant when the switch is closed, since i is zero and e_R increases until it is equal to E at the point 1, where the current reaches its steady value of I amperes. At the point 0, since e_R is zero, the entire impressed voltage E is used to increase the current and at this instant the current is increasing at the highest rate. Hence, the self-induced e.m.f. e_L is highest at the instant the switch is closed and becomes zero at point 1. An electric circuit, in which an appreciable e.m.f. is induced while the current is changing, is called an inductive circuit and the circuit is said to have self-induction.

61. Mechanical Analogy for Increasing Current in an Inductive Circuit.—The action taking place during the growth of a current in a d.c. circuit may be compared to starting a railway train and bringing it to a steady speed. In this process there are two reactions to be overcome by the tractive force of the locomotive. The first is the force necessary to overcome the train friction, which is small at low speed and increases with the speed; the second is the force required to accelerate the train. This force is large when the speed is increasing rapidly and is small when it is increasing slowly. It becomes zero when there is no change in speed. The total force exerted by the locomotive is analogous to the impressed voltage E of the electric circuit. The friction of the train corresponds to the resistance, and the force required to accelerate the train corresponds to the voltage e_L required to overcome the self-induced voltage of the coil. The speed of the train is represented by the strength of the current in the electric circuit. Obviously a train cannot be brought to a speed of 60 miles per hour instantaneously without exerting an infinite force upon it; similarly, a current of I amperes cannot be produced instantaneously in a coil without applying an infinite voltage to the circuit. At low speed, the retarding force due to friction is small and nearly the entire impressed force is used in accelerating the train, so that it rapidly increases in speed. When the electric circuit is first closed, the resistance drop is small, so that nearly the entire impressed voltage is used in increasing the current and it increases in strength very rapidly. As the speed of the train in increased, more and more of the total impressed force (which is assumed to be held constant) is consumed by the friction of the train, leaving a smaller amount available for acceleration, until finally a speed is reached where the friction load equals the impressed force, equilibrium is established, there is no acceleration, and the train runs at a constant speed. In the electric circuit, as the current becomes greater, a greater proportion of the impressed voltage E is consumed by the resistance, leaving a smaller amount for increasing the current until a point is reached where the current has attained such a value I, that it consumes the entire impressed voltage and there is nothing left for accelerating or increasing the current; hence, it stays at a steady value.

62. Stopping a Direct Current in an Inductive Circuit.—Referring again to Fig. 51, consider what will happen if the supply voltage E is removed from the coil after the current has reached its steady value I. If the switch S_1 is closed, the fuse f will blow, thus removing the voltage E from the coil and forming a circuit for the coil through the resistance R_1. The energy stored up in the magnetic field of the coil will then discharge through the resistance R_1 and the current will gradually decrease to zero. The strength of the current will be represented by the portion of the curve in Fig. 51 *b* between points 2 and 3. The current will not drop instantaneously to zero, because the e.m.f. of self-induction is now in such a direction as to *oppose* the decrease of current. That is, when the current is *increasing* between points 0 and 1, the e.m.f. of self-induction *opposes* the impressed voltage or tends to send a current in a direction opposite to the impressed voltage E; when the current is *decreasing*, the induced e.m.f. reverses and is in the *same* direction as E, thus tending to maintain the current in the circuit. The e.m.f. of self-induction does work on the circuit by forcing a current through the circuit against the reaction due to the resistance in the circuit which in this case is equal to $R+R_1$. If the resistance of the circuit is high the current will decrease rapidly, but a high induced e.m.f. will result.

63. Mechanical Analogy for Decreasing Current in an Inductive Circuit.—If a train running at a constant speed is brought to a standstill, the force necessary to accomplish this depends upon the time taken in the process. If the steam is shut off from the locomotive, the friction of the train will eventually cause it to stop. This friction corresponds to the resistance R of the coil and the time required corresponds to the time required to bring the current to zero when the coil is short-circuited. By applying the brakes, the train can be brought to a stop more quickly, but a greater force must be exerted to do this. This corresponds to the case in the electric circuit where an additional resistance R_1 is introduced, thus decreasing the current more rapidly. If the train is brought to a very sudden stop by colliding with another train, the effect is similar to that which would occur if an electric circuit were opened suddenly.

64. Discharge Resistance for Alternator Field.—The exciting winding of an alternator has a high inductance and, therefore, if the field current were interrupted suddenly by opening the field switch, a high voltage would be induced in a field winding and the insulation of the winding would probably be damaged. When it is necessary to open the field circuit of a dynamo, the field switch is provided with auxiliary contacts which are connected to a field discharge resistance. When the field switch is opened, the switch blades touch these auxiliary contacts before the field circuit is interrupted so that the discharge resistance is placed in parallel with the dynamo field. Further motion of the switch opens the supply circuit and the energy stored in the dynamo field discharges through the resistance, thus avoiding an excessive induced e.m.f.

65. Mutual Induction.—Whenever a magnetic field produced in a conductor, such as coil A (Fig. 52), links with another coil, B, any change in strength of this field will cause a change in the field of the second coil and, accordingly, will induce an e.m.f. in coil *B*. If circuit *B* were closed, a current would flow, due to this induced e.m.f. The direction of this current would be such as to tend to set up a field in opposition to the field produced by coil *A*.

Example 1.—An induction coil (Fig. 53) consists of a primary winding *P* through which a regularly interrupted direct current flows, and a secondary winding *S* of many more turns than *P*. The variations in flux due to the rapid changes in the primary current induce a high e.m.f. in the secondary winding which can be used for ignition for gasoline engines and for other purposes.

Example 2.—The static transformer so commonly used in a.c. power systems has a primary winding carrying a constantly varying current which produces a varying flux linking with a second winding in which a varying voltage is induced. By properly proportioning the number of turns of the windings, a particular a.c. voltage can be transformed to another value either higher or lower as desired.[3]

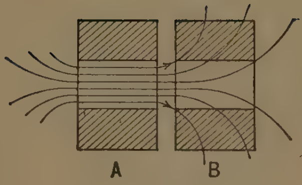

FIG. 52.

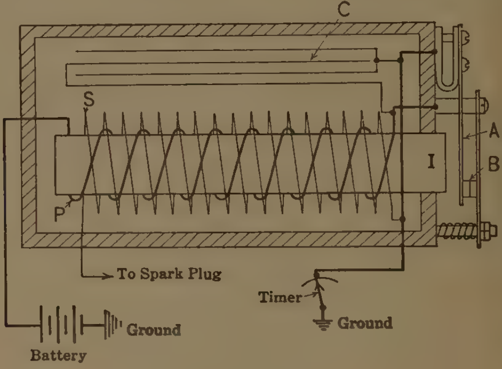

FIG. 53.

FIG. 52.—Mutual Induction.

FIG. 53.—Induction Coil. As applied to automobile ignition system. Primary winding *P* and secondary *S* are wound on iron core *I*. The primary circuit from battery is through vibrator contacts *B* and the timer. When circuit is closed by timer on engine, *I* attracts steel spring armature *A* breaking primary circuit at *B* and inducing a high voltage in the secondary which jumps the gap in spark plug and fires the charge in engine cylinder. Condenser *C*, bridged across *B*, reduces the burning and assists in interrupting circuit suddenly.

PROBLEMS ON CHAPTER VIII

1–8. A conductor is cutting a magnetic field at a velocity of 1500 cms. per second. The flux density of the field is 8500 gausses. The length of conductor in the field is 25 cms. Calculate the induced voltage.

2–8. The pole face of a dynamo is 25 cms. long, parallel to the axis of the armature and the flux density in the air gap is 8000 gausses. The conductors on the armature are 15 centimeters from the axis of the armature and the speed is 1200 r.p.m. What is the e.m.f. in volts induced in each armature conductor (25 cms. long) as it passes a pole face?

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[3] See Chapter XXIX.

3–8. The conductors of a generator are located 18 inches from the axis of the armature which is turning at 1200 r.p.m. There are four poles on the machine and the total flux per pole is 1,500,000 lines. A conductor cuts the flux during 65 per cent of the time for one revolution and the flux density is uniform.

(*a*) What is the e.m.f. induced in each conductor while it is cutting flux?

(*b*) What is the average e.m.f. per conductor during one complete revolution of the armature?

4–8. A potential of 110 volts is applied to an inductive circuit (Fig. 51). The resistance of the circuit is 3 ohms.

(*a*) What will be the current when it reaches its maximum steady value?

(*b*) After the potential has been applied for 0.01 second, the current is 19 amperes. What is the e.m.f. of self induction at this instant?

(*c*) After 0.02 second, the current becomes 28 amperes. What is the e.m.f. of self induction?

(*d*) What is the value of the e.m.f. of self induction at the instant the switch is closed?

5–8. If the self induction of the circuit in Prob. 4–8 were increased, what effect would this have upon the time required for the current to reach a steady value?

6–8. If a potential of 110 volts were applied to a circuit having a resistance of 3 ohms and negligible self induction:

(*a*) What would be the maximum value of the current?

(*b*) Plot a curve between time and current showing how the current changes, from zero when switch is closed, to the maximum value found in (*a*).

7–8. A circuit consisting of a coil with negligible resistance and a resistor having negligible self induction are connected in series to a 250-volt d.c. supply. The resistance of the resistor is 25 ohms and the self induction of the coil is 0.15 henry.[1]

(*a*) What will be the current after it has reached its maximum value?

(*b*) What will be the potential across the coil and the resistor at the instant the switch is closed?

(*c*) What will be the potential across each part when the current becomes steady at its maximum value?

8–8. The flux per pole in a d.c. dynamo is 13,500,000 lines and there are eight poles with all field coils connected in series and 370 turns per coil.

(*a*) What would be the average e.m.f. in the entire field circuit when the circuit is interrupted, if the flux were decreased to zero in 0.5 second?

(*b*) If the flux were decreased to zero in 2 seconds, what would be the average induced e.m.f.?

9–8. A potential of 10 volts is applied to a certain coil with 500 turns, having a resistance of 20 ohms. The current is 0.12 ampere after the circuit has been closed for 0.01 second. The current and flux are approximately proportional to the elapsed time. Calculate the flux linking with the coil when the current is 0.12 ampere.

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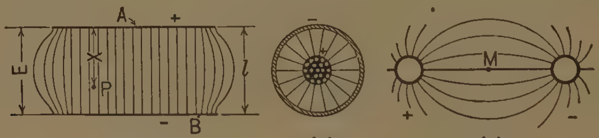
[1] See Article 222 for definition of this unit.

CHAPTER IX

ELECTROSTATICS AND CONDENSERS

66. The Electrostatic Field.—If a potential of E volts is applied to two metal plates, A and B (Fig. 54), which are insulated from each other, an electrostatic field is produced in the space between the plates. For the arrangement shown in Fig. 54 *a*, this field is uniform, so that the difference of potential between one of the plates, A, for example, and any point P in this field is proportional to the distance x measured from A. This electrostatic field has the property of imparting a certain amount of electricity to, or " charging " a body which is placed in the field. The distribution of the field is represented by lines drawn between the two bodies, as in Fig. 54. The number of these lines per unit area is made proportional to the intensity of the field. An electrostatic field always exists in any insulation or dielectric which separates conductors having a difference of potential, but it is, in general, of importance only where high potentials exist. The intensity or strength of an electrostatic field increases as the potential difference increases, and, when such a field exists, the dielectric is subjected to a strain which tends to " break down " or puncture the dielectric and render it conducting. Air will puncture more easily than solid dielectrics such as glass, rubber, porcelain, and liquid dielectrics such as mineral and vegetable oils. The term " dielectric strength " refers to the ability of a material to resist puncture and is expressed in terms of the difference of potential which a unit length will withstand without failure. The dielectric strength of air at normal pressure is about 100 volts per mil (0.001 inch) and of rubber from 300 to 500 volts per mil. Referring again to Fig. 54 *a*



(*a*) (*b*) (*c*)

FIG. 54.—Electrostatic Fields.

which has a uniform strength of electric field at all points between the plates, the fall of potential per inch of dielectric is $E_g = E \div l$ when l is the distance between the plates. This value E_g is called the potential gradient of the electrostatic field. In Fig. 54 *a* the gradient is constant because the field is uniform, but, with arrangements like Fig. 54 *b* or *c*, the electrostatic field is not uniform since there are more electrostatic lines per unit area near the conductors. Hence there is a greater dielectric strain near the conductors, which is expressed by saying that the voltage gradient is higher near the conductors than it is near the outside sheath in Fig. 54 *b* or the middle point M in Fig. 54 *c*. If the gradient at any point exceeds the dielectric strength, the material will be broken down at that point, but this does not mean necessarily that the entire dielectric will be broken down, as the portion where the gradient is lower may still be able to withstand the voltage. A partial breakdown of the dielectric in the region near the conductors occurs with very high voltage overhead lines, producing what is called corona around the wires. When viewed in the dark, this corona appears as a blue glow surrounding the conductors. The air in which the corona is formed is said to be ionized.

When a difference of potential exists between two conductors separated by a dielectric, there is not only a stress in the dielectric, but also a force tending to bring the two conductors together. Two bodies charged to the same potential, either positive or negative, with respect to a third body, such as the ground, tend to repel each other. This force action which exists with charged bodies is made use of in the **electrostatic voltmeter** which has a movable and stationary system of plates which are charged by the voltage to be measured. The force produced by the electrostatic field is small for moderate voltages so that, in general, electrostatic voltmeters are most suitable for measuring the higher voltages. This type of voltmeter can be made for

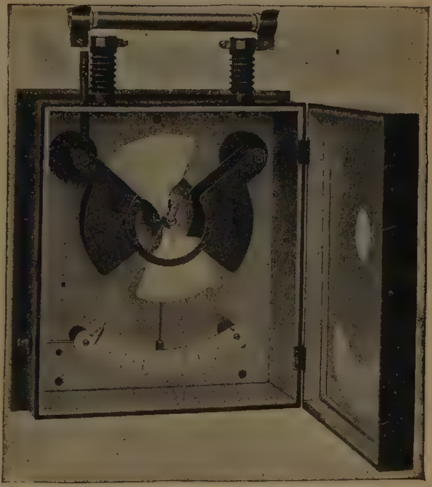


Fig. 55.—Electrostatic Voltmeter. Vane type, range up to 10 kilovolts. *The General Electric Co.*

any potential which it is possible to produce. One type of electrostatic voltmeter is shown in Fig. 55. Unlike ordinary voltmeters, these instruments consume no energy.

67. Charging a Condenser.—The combination of two conductors, separated by a dielectric, has the property of storing up a definite amount of electricity when a difference of potential exists between the two conductors. Since all conductors are insulated, either by air or other dielectric, this property is universal in electrical circuits. In many cases the amount of electricity thus stored is negligible, but in others it is an important factor. When two insulated conductors are designed primarily for the purpose of storing up electricity in this manner, rather than for carrying a current of electricity, the combination is called a condenser. The conductors of a condenser may consist of sheets of tin foil or other metal, or metallic coatings, on the outside and inside of a glass jar.[1] Whatever form they take, they are called the "plates" of the condenser, and the insulating material separating the plates is called the dielectric. If such a condenser were connected to a source of d.c. potential with a galvanometer or other current-measuring device connected in circuit (Fig. 56 *a*) and the switch *S* were closed, the potential would be applied to the plates *a* and *b* and the galvanometer would indicate a current. After a very short time, however, the galvanometer would return to zero, showing that the current had ceased. Plate *a* is now positive with respect to *b* and the difference of potential between them is the same as that of the source, viz., 110 volts. If the condenser were disconnected, plate *a* would remain at a positive potential with respect to *b*, provided the insulation between the plates were perfect. In practice, this difference of potential would cause a leakage between the plates so that the potential would not remain constant but would gradually fall to zero. A condenser which has been connected to a source of potential, as described, is said to be charged and the quantity of electricity which flows to the condenser during the process is called the charge of the condenser. If the condenser, which has been charged as shown in Fig. 56 *a*, were disconnected from the supply, and a circuit between the plates closed through the galvanometer (Fig. 56 *c*), there would be a momentary deflection of

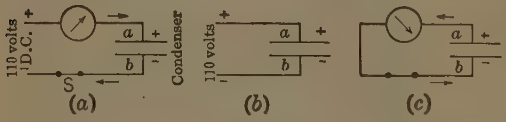


FIG. 56.—Charge and Discharge of a Condenser.

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[1] Overhead lines and underground cables also act like condensers, although they are designed primarily to transmit electricity and not to store it.

the galvanometer pointer in the opposite direction to that when the condenser was charged, thus indicating a current in a reverse direction or a discharge of the condenser. After a short time the potential between the plates would fall to zero and the current would cease. In a condenser with a perfect dielectric the quantity of electricity which flows on discharge would be exactly the same as for the charge. In practice, usually a small amount of the charge is lost by leakage. The quantity of electricity or the amount of charge which the condenser can receive depends upon the potential, the area of the plates, and the thickness and kind of dielectric.

68. Electrostatic Capacity.—The size of a condenser is specified by stating its electrostatic capacity, which is a measure of the quantity of electricity or charge which can be stored in the condenser. The electrostatic capacity is defined as the ratio of charge to the difference of potential between the plates or

$$\text{Electrostatic capacity } C=\frac{q}{e}. \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Instead of "electrostatic capacity," the shorter term "capacity" is generally used. Many writers use the term "capacitance" instead of "capacity." For a condenser of fixed dimensions, the charge is directly proportional to the difference of potential between the plates; hence, the ratio $\frac{q}{e}$ is constant and C, the capacity is constant, and is as much a fixed quantity as the resistance of a wire of particular dimensions. The charge of electricity q is equal to the average current in the circuit multiplied by the time required to accumulate the charge, that is, $q=I_{av}\times t$. If this charge is expressed in coulombs (ampere-seconds) and e in volts, the capacity C is expressed in a unit called the farad. A condenser which will hold a charge of 1 coulomb when a difference of potential of 1 volt is applied to the plates has a capacity of 1 farad. The farad is a very large unit so that, in practice, the microfarad, which is one-millionth of a farad, is used:

$$1 \text{ farad}=10^6 \text{ microfarads.}$$

The capacity of a condenser of definite dimensions depends upon the kind of dielectric between the plates. The capacity with air as a dielectric is taken as a standard for comparison with other dielectrics. Thus, if a certain air-insulated condenser had a capacity of C microfarads, a condenser of the same dimensions with another kind of dielectric would have a capacity of KC microfarads. The constant K by which the capacity of an air condenser is multiplied to obtain the capacity for another kind of dielectric is called the specific inductive capacity of that dielectric. In general, K is greater than one for the dielectrics commonly used. In the table below are given values of K for dielectrics commonly used; the reader is referred to electrical handbooks for more complete tables.

Values of Specific Inductive Capacity of Dielectrics

Material	Specific Inductive Capacity K
Air	1.0
Glass	5.5 to 10.0
Mica	2.5 to 6.0
Transformer oil	2.5
Paraffined paper	2.0 to 3.3

The capacity of simple forms of condensers can be calculated from the dimensions. For parallel plate condenser like Fig. 54, the capacity in microfarads is:

$$C=\frac{KA}{4.452\times d\times 10^6}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

where K = specific inductive capacity;

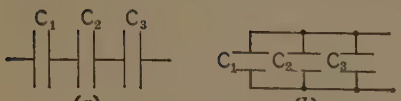
A = active area in square inches of one plate of the condenser (one side of plate A in Fig. 54);

d = thickness of dielectric in inches.

Example 1.—Find the size of plate required for a 1 micro-farad condenser, using paraffined paper 1 mil thick as a dielectric. Assume $K=2$.

$$A=\frac{4.452\times 0.001\times 10^6\times 1}{2}$$

$$=2{,}226 \text{ square inches} \quad \text{or} \quad 15.45 \text{ square feet.}$$



(a) (b)

Fig. 57.—Condensers in Series and Parallel.

69. Condensers in Series and in Parallel.—Where several condensers are connected in series (Fig. 57 *a*) the total capacity is

$$C_S=\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}. \quad . \quad . \quad . \qquad (23)$$

Example 1.—Three condensers, respectively 5, 10, and 15, micro-farads capacity are connected in series. Calculate the total capacity.

$$C_S=\frac{1}{\frac{1}{5}+\frac{1}{10}+\frac{1}{15}}=\frac{1}{0.367}=2.63 \text{ m.f.}$$

The total capacity of several condensers connected in parallel (Fig. 57 *b*) is

$$C_p = C_1 + C_2 + C_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

Example 2.—Three condensers, respectively 5, 10, and 15, micro-farads capacity are connected in parallel. Calculate the total capacity.

$$C_p = 5 + 10 + 15 = 30 \text{ m.f.}$$

70. Examples of Condensers Used in Practice.—Condensers made from tin foil, with thin paraffined paper as a dielectric, are very commonly used in practice, especially for telephone work. A 4 micro-farad condenser of this type occupies a space of $1\frac{3}{4}$ inches by $2\frac{3}{4}$ inches by $4\frac{5}{8}$ inches. Condensers for use on power systems for improving the power factor (see Article 322) are constructed of metal foil with a treated paper dielectric. They are built to operate at 2300 volts. Condensers connected across the circuit breakers of the high-tension magnetos used for automobile ignition are made with mica or paraffined paper as a dielectric, although mica is far superior to paper and is used for high-grade work. The capacity of these condensers varies with different kinds of magnetos, but a typical outfit uses a mica condenser of about 0.3 microfarad. Very small capacities of from 0.002 to 0.00025 microfarad are used for radio receiving apparatus. These condensers are usually air-insulated when the capacity is to be adjustable and mica- or paper-insulated when of fixed capacity.

71. The Cottrell precipitation process makes use of an electrostatic field to remove dust from the gases given off in various metallurgical processes. The dust-laden gases are passed through a flue in which are electrodes charged to a d.c. potential of about 100,000 volts, which is obtained by rectifying an alternating voltage by means of a commutator.[2] One electrode, usually the negative, consists of a wire, chain or strip, which facilitates the formation of the corona; the other electrode (positive) is a tube, plate or screen, which minimizes the formation of corona at its surface. The negative electrode is called the discharge electrode and the positive, the collecting electrode. When the corona discharge occurs, the gas around the discharge electrode becomes ionized and the resulting ions travel towards either the positive or the negative electrode, depending upon their polarity. The negative ions travel towards the positive or collecting electrode and charge negatively any solid or liquid particles which may be suspended in the gas. These negatively charged particles are, therefore, attracted to the collecting or positive electrode. The dust thus collected is removed at intervals by shaking the collecting

[2] See rectifiers, Chapter XXXIV.

electrode, with the electrical potential removed and the flow of gas stopped. The process is used in cement mills, smelters, and other chemical plants, to remove valuable particles, such as gold, silver, and potash, from the gases or to prevent the escape into the air, of substances injurious to surrounding vegetation.

PROBLEMS ON CHAPTER IX

1–9. A sheet of fiber 0.5 inch thick is tested for dielectric strength by placing it between two metal plates and applying a potential difference of 48,000 volts between the plates.

What is the potential gradient in the fiber?

2–9. The dielectric strength of oiled cloth, used for insulating electrical conductors is 750 volts per mil. Two layers of this cloth, each 10 mils thick, are to be used. Assuming a uniform potential gradient in this cloth, determine the maximum potential difference across this insulation if the potential gradient is not to exceed 10 per cent of the dielectric strength.

3–9. What is the capacity of a condenser which holds a charge of 0.05 coulomb when a difference of potential of 500 volts is applied?

4–9. A transmission line 100 miles long has a capacity of 0.0078 microfarad per mile. How much electricity could be stored in the line if 150,000 volts were applied?

5–9. Determine the area of each plate of a paraffined paper condenser which would have the same capacity as the transmission line of Prob. 4–9. Assume both sides of each plate active, a dielectric thickness of 2 mils and $K = 2.5$.

6–9. A condenser is to be built using interleaved aluminum plates, each 4 inches ×6 inches, with air as a dielectric. The separation between plates is 20 mils.

(*a*) Calculate the capacity if a total of 23 plates were used.

(*b*) Calculate the area of each plate which would be required to give the same capacity if the plates were separated by mica 5 mils thick instead of by air.

7–9. Find the combined capacity of four condensers, each of 5 m.f. capacity when connected in series.

8–9. Find the combined capacity of four condensers, each of 5 m.f. capacity when connected in parallel.

9–9. Four condensers, respectively 3, 5, 7.5, and 10 m.f. capacity, are connected in series. What is the combined capacity?

10–9. Two condensers, respectively 5 and 15 m.f. capacity, are connected in parallel and a condenser of 3 m.f. is connected in series with the two. What is the total capacity of the entire combination?

11–9. It is desired to obtain a capacity of 0.000625 m.f. The sizes of the condensers available are 0.0015, 0.001, 0.0005, and 0.00025 m.f. Determine the number, capacity of each, and method of connection to give this capacity.

PART II

DIRECT-CURRENT MACHINERY

CHAPTER X

PARTS OF A DYNAMO

72. A generator is a machine designed to convert mechanical to electrical energy; a motor is designed to convert electrical to mechanical energy. Both generators and motors consist essentially of a group of conductors moving in a magnetic field. These conductors carry current; hence a force action results (Article 45), and they also have a voltage induced in them because they cut the magnetic field (Article 54). The term dynamo is applied to either a generator or a motor. Alternating-current generators are commonly called alternators. Neither the generator nor the motor is a source of energy; they are simply converters of energy from one form to another.

73. Generator and Motor Action.—We have seen in Chapter VIII that an e.m.f. is induced in a conductor whenever it cuts a magnetic field. A generator is, therefore, simply a collection of conductors so arranged that they can be cut by a magnetic field. Usually, in d.c. generators, the field is stationary and the conductors on the armature revolve; in a.c. machinery the armature conductors are usually stationary and the field revolves. Electrically there is no difference in the voltage induced for the same relative motion, but there are certain mechanical advantages that determine which design is best in a particular type of machine.

In Chapter VII it was shown that a force is exerted upon a conductor whenever it carries current and is under the influence of a magnetic field. A motor, therefore, is essentially a collection of conductors, so arranged that they can be supplied with current and can move in a magnetic field. It is apparent that the essential parts of the generator and the motor are the same; viz., a group of conductors which can be moved relatively to a magnetic field. In fact d.c. generators and motors are essentially alike in their construction and such machines operate as generators if supplied with mechanical energy, or as motors if supplied with electrical energy. There is a greater difference in the construction of a.c. dynamos, but in general these also can be used as generators or as motors.

74. The rating of a dynamo is a statement of the power which the machine will deliver under specified operating conditions. With a few exceptions the rating which is stamped on the manufacturer's name-plate is the *output* of the machine. The rating of a generator is expressed in kilowatts and of a motor in horsepower.[1] The rating is not complete unless a statement of the normal operating voltage of the machine is also given. Thus, a generator rated at 500 kw. at 250 volts, will *deliver* 500 kw. or $500{,}000 \div 250 = 2000$ amperes when operating at a terminal voltage of 250 volts. Similarly, a motor rated at 50 hp. at 115 volts will deliver 50 hp. when operating on a 115-volt supply system. The current taken by the motor would not however be $\frac{50 \times 746}{115} = 324$ amperes, but would be greater than this because of the losses in the machine. The factors which affect the rating of a dynamo will be discussed in Chapter XVIII.

The name plate which is always found on an electrical machine also contains additional information, such as speed, frequency in case of an a.c. machine, duration of allowable load, and similar data as to the operating conditions under which the rated or normal output may be secured.

75. Fundamental Parts of a D.C. Dynamo.—If we examine a d.c. dynamo we find that it consists of a group of copper conductors mounted in a slotted cylindrical core and arranged to form a part of a magnetic circuit composed principally of a mass of steel or cast iron. The copper conductors, with their core, constitute the armature of the machine, while the mass of steel surrounding the armature is the field frame. Associated with the armature is the commutator to which the copper conductors are connected. The armature, being mounted on a shaft, can rotate, and the conductors, therefore, can move relative to the magnetic flux produced by the field frame. This flux is produced by means of coils of wire called field coils, mounted on the frame of the machine. Only in the case of small machines is it feasible to provide the required flux by means of permanent magnets, because only a low flux density is possible and there is also a loss of flux due to vibration and aging. The particular type of design used at present is the result of development and the attempt of the designers to employ the arrangement of parts which gives satisfactory results for the lowest cost.

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[1] The standardization rules of the American Institute of Electrical Engineers recommends that motor ratings be expressed in kilowatts, but this recommendation has not as yet been adopted generally by motor manufacturers.

76. Production of E.M.F. in a Generator.—It has been shown in Article 54 that an e.m.f. is induced in a conductor when it moves in a magnetic field in such a manner as to cut across the lines of force of that field. The voltage induced in a single conductor is, however, rather small, so that a large number of conductors is required if the generator is to produce a voltage suitable for the usual requirements. Suppose that a number of conductors are arranged around the surface of a hollow iron cylinder (Fig. 58). These conductors are located in a magnetic field produced by the iron pole pieces N S. The flux passes across the air gap and through the cylinder or core. If the conductors are rotated in a clockwise direction, there will be voltages induced in the conductors as indicated in Fig. 58. Since the field is of uniform strength, each conductor will have a steady voltage induced in it as long as it is under one of the pole pieces N or S, and the voltage will be zero when the conductor is moving between the poles. Furthermore, the direction of the induced voltage will be different under the two poles according to the rule stated in Article 55. If the voltages thus produced in these conductors are to be combined so as to obtain a higher voltage than that generated by a single conductor, they must be connected in such a way that the voltages add. One method of doing this results in an arrangement called a Gramme ring winding.

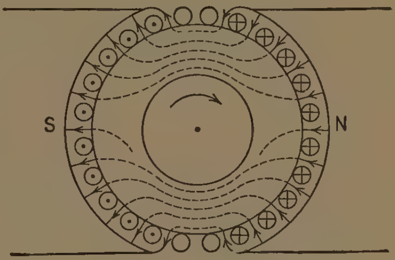


FIG. 58.—Elementary Dynamo.

77. The ring winding is formed by joining the conductors shown in Fig. 58 by wires passing through the center of the ring, thus forming an endless helix (Fig. 59); that is, the winding is closed upon itself. No e.m.f. is generated in the portion of the winding inside the iron core since the flux divides and passes through the iron core rather than through the central air space. The conductors under one pole are thus connected in such a manner that their e.m.f.'s add so that between points *a* and *b* there would be a difference of potential equal to the sum of the voltages induced in the conductors under either pole. The direction of the voltage on each side is such as to tend to send a current from *b* to *a*, but, since these two voltages are equal and opposed to each other, no current can flow through the windings. There will, however, be a difference of potential between points *a* and *b*, the value of this being equal to the sum of the voltages induced in half of the conductors, and *a* will be positive with respect to *b*. If stationary contacts placed at points *a* and *b* are connected to an external load (Fig. 59 *a*), a steady direct current will flow as long as the generator rotates at constant speed and the field strength is held constant. The current I which flows in the external circuit divides in the armature into two parts and flows through opposite sides of the ring. The winding is, therefore, said to have two paths.

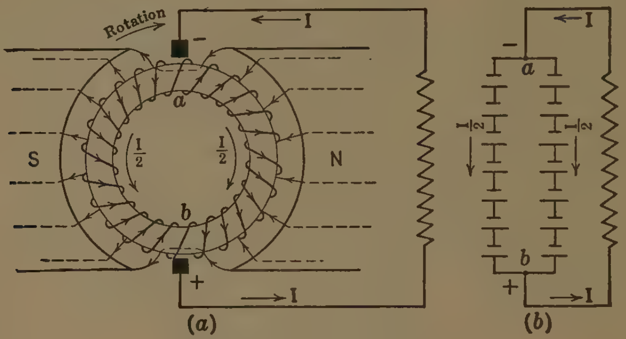


FIG. 59.—Ring Winding.

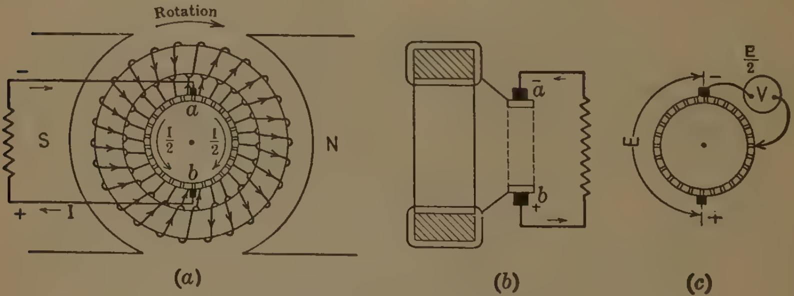


FIG. 60.—Bipolar Ring Winding.

While machines have been built in which contact was made directly with the conductors, as shown in Fig. 59, the common method is to connect the winding to insulated copper segments forming a commutator. Connection between the winding and the external circuit is then made through the medium of brushes bearing on this commutator (Fig. 60). If the distribution of voltage on the commutator between positive and negative brushes were measured by connecting one voltmeter terminal to the negative brush and moving the other terminal along the commutator towards the positive brush (Fig. 60 *c*) it would be found that with the movable terminal near the negative brush the reading is small, but as more and more conductors were included in the circuit by moving the contact nearer the positive brush, the voltage would increase until, finally, when the movable terminal reaches the positive brush, the full voltage of the machine would be observed. The effect of each conductor in contributing its voltage to the total voltage of the machine is similar to the effect when a number of cells of battery are connected as shown in Fig. 59 *b*.

78. Multipolar Ring Winding.—If the ring winding illustrated in Fig. 60 were placed in a multipolar field the e.m.f. induced in the conductors would be as indicated in Fig. 61 since the field poles have alternately north and south polarity. The voltages induced under the S poles are, therefore, opposite to the voltages under the N poles and, since the total voltage induced under each pole is the same, these voltages balance each other and no current can circulate through the winding. The effect is the same as when four sets of battery cells are connected in parallel (Fig. 61*b*). Between points *a* and *b* or *a* and *d* there is, however, a difference of potential represented by the sum of the voltages in the conductors under one pole. Similarly there are equal voltages between *c* and *d* or *c* and *b*. Points *a* and *c* are of the same polarity and can therefore be connected together; the same is true for points *b* and *d*. The voltage at the terminals of the machine is, therefore, the total voltage induced in the conductors under one pole. If the circuit is closed through an external load, the load current *I* divides into four parts in the armature and flows as indicated

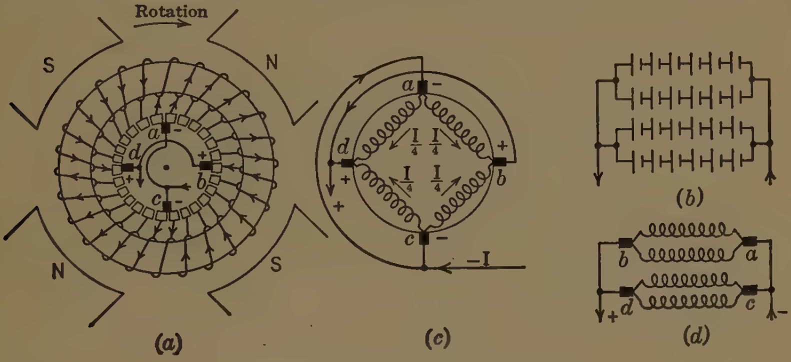
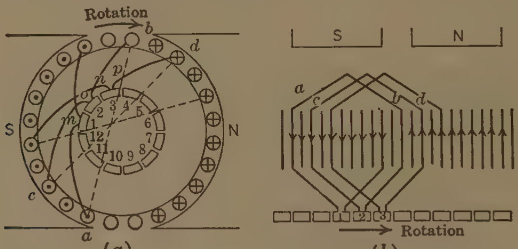


FIG. 61.—Multipolar Ring Winding.

in Fig. 61 *a* and *c*. The winding evidently has four paths for the current. If I is the current at the terminals and E the potential difference between terminals, then the output of the machine is $P=EI$ watts.

79. Drum Windings.—The ring winding has the disadvantage that it is difficult to manufacture and repair because each turn has to be threaded through the central opening in the core. It also requires an excessive amount of wire because only one side of each turn cuts flux. The ring winding has, therefore, been superseded by another type of winding in which all the wire is on the outside of the armature and none passes through the center of the core. This arrangement is called a drum winding. In Fig. 62 is shown a bipolar machine with a drum-wound armature. The direction of induced e.m.f. in the conductors under the two poles will be opposite as has already been explained. If two conductors *a* and *b*, for example, which are on opposite sides of the core are connected together, the voltages induced in the two sides will be added as shown in Fig. 62 *b*. Similarly, conductors *c* and *d* are connected together, and so on until all of the conductors have been assembled into what are called coils, each having two conductors. The coil ends *m*, *n*, *o*, *p*, etc., can then be connected to commutator segments in such a way that the coil voltages are added (Fig. 62). If this illustration is examined carefully it will be seen that the voltage between the brushes is equal to the sum of the voltages induced in one-half the total number of conductors. There are, therefore, two paths for the current which flows through the armature just as there were in the bipolar ring winding. There is this difference, however, that whereas in the ring winding each path was made up of conductors under a single pole, in the drum winding each path is composed of half the conductors under a south pole and half those under a north pole. The result, as far as total voltage is concerned, will, however, be the same if the total number of con-



(*a*) (*b*)

FIG. 62.—Bipolar Drum Winding.

ductors is the same in each case. For this reason it is convenient to represent a drum winding diagrammatically as shown in Fig. 63, where the commutator is omitted and the brushes are placed above the conductors. The curved arrows in Fig. 63 *a* indicate the direction of induced voltage and of current in the armature. The paths through the armature are illustrated in Fig. 63 *b*.

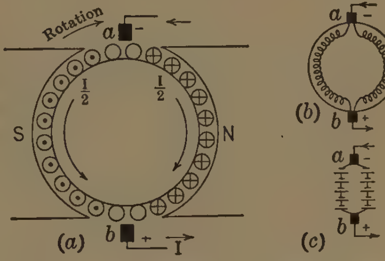


FIG. 63.—Circuits in a Bipolar Drum Winding.

80. Multipolar Drum Winding.—It is not possible to take a drum winding intended for a bipolar field as in Fig. 62 and place it in a four-pole field without change in its connections, as could be done with a ring winding. (See Article 78.) If this were done, the conductors which were connected together to form a coil would be under poles of like polarity (Fig. 64 *a*), the induced voltages would be in the same direction, and the voltage at the terminals of the coil would be zero. In any drum winding, therefore, the two conductors forming the coil must lie under poles of opposite polarity (Fig. 64 *b*); then the voltage of the coil will be the sum of the voltages of the conductors in the coil. This means that the coil must span a distance about equal to the pole pitch, i.e., the distance between centers of adjacent poles. Coils thus arranged may then be connected to a commutator to produce voltages similar to those of the multipolar ring winding (Fig. 61). This will give *four* paths through the armature as shown diagrammatically in Fig. 64 *c*. The conductors act like four groups of battery cells in parallel (Fig. 64 *d*). The actual arrangement of coils and connections is more complicated than is shown in Fig. 64. The connections for a four-pole drum winding are given in Fig. 65. If the

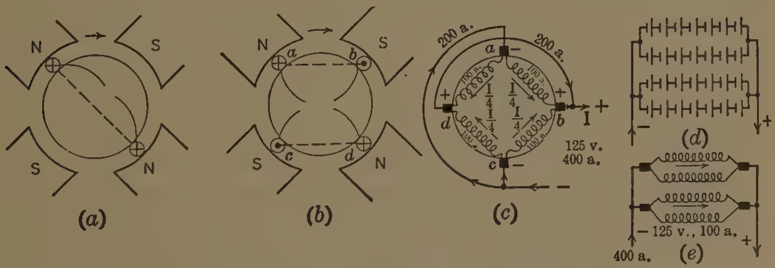


FIG. 64.—Circuits for a Lap Winding.

student will trace the paths through this armature from negative to positive terminal, it will be apparent that there are four paths through the armature, and one-fourth of the conductors are in each path. A drum winding which has as many paths as there are poles is called a **lap winding.** It is possible to take the same coils as in Fig. 64 and reconnect them to the commutator in such a way that there are only two paths in the armature (Fig. 66). Such a winding is called a **wave winding.** In this arrangement there are *two paths* regardless of the number of poles. Since there are two paths in the wave winding, the voltage for each path is the sum of the voltages induced in one-half the total number of conductors, instead of one-fourth as in the lap-wound machine of Fig. 64. Accordingly, if the four-pole, lap-wound machine gives 125 volts at its terminals, it would give 250 volts if the coils were reconnected to form a wave winding. In the lap-wound machine of Fig. 64 if the ampere capacity of the conductors is 100 amperes, then the total capacity of the machine is 4×100=400 amperes and the power output is $P=400\times125=50{,}000$ watts. In the wave-wound machine of Fig. 66, the ampere capacity of the

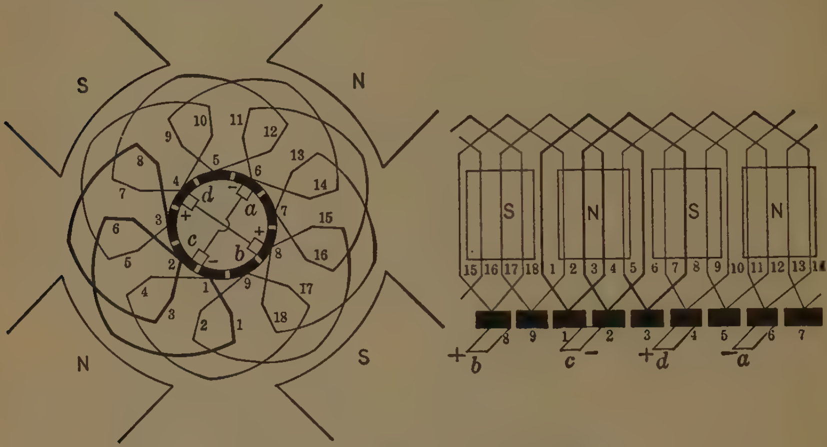


FIG. 65.—Example of a Lap Winding.

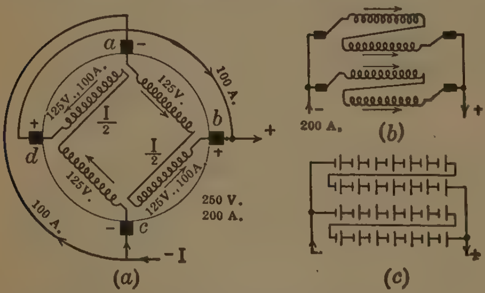


FIG. 66.—Circuits for a Wave Winding.

individual conductors would be the same and the machine could carry 100 amperes per path as before. The current capacity of the machine is then $2 \times 100 = 200$ amperes, the voltage at the terminals is 250 volts and the power output $P = 200 \times 250 = 50{,}000$ watts as before. In other words, the power output of a machine is not changed by changing the armature connections from lap to wave. The actual connections for a four-pole wave winding are shown in Fig. 67. Another peculiarity of the wave-wound machine is that it may be designed to use only two brushes, whereas the lap-wound machine must have as many brushes as there are poles. Thus, in Fig. 67 brushes *c* and *d* could be omitted. If only two brushes were used, they would be located a pole pitch apart. Wave windings have an advantage in small high-voltage machines because a larger size of conductor can be used. The fact that they can be designed for only two brushes is also a decided advantage in some applications, such as railway motors, where it is difficult to provide access to both top and bottom sides of the armature. For large machines, the lap winding must be used, however, since there is a limit to the amount of current which can successfully be carried in one path in a machine.

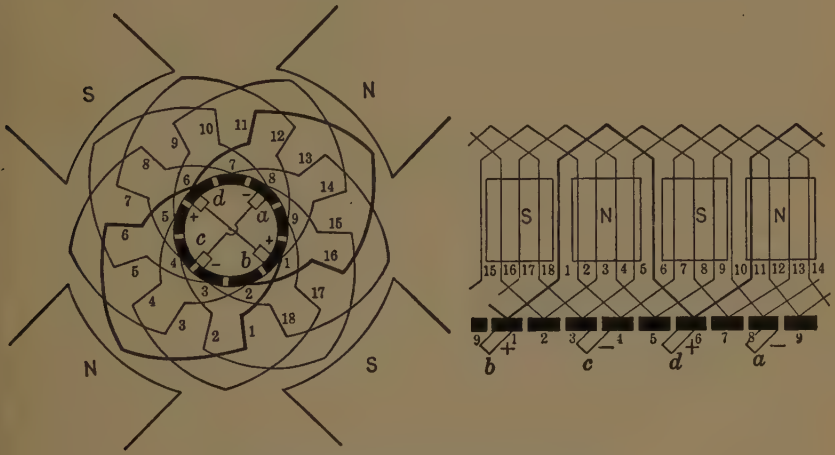


FIG. 67.—Example of a Wave Winding.

Although the illustrations show coils with only one turn, it should be noted that frequently the designer uses coils having a number of turns. The following definitions apply to drum windings:

(1) A coil is the portion of the armature winding connected between two consecutive commutator bars. Thus, in Fig. 65, the portion of the winding from bar 1 to bar 2 (i.e., conductors 1 and 6) constitutes a coil.

In Fig. 67, however, the coil is connected between bars 1 and 6, although it is made up of conductors 1 and 6 as before.

(2) A coil for a drum winding will have at least two conductors but may have four, six, eight, etc. A coil with four conductors is a two-turn coil. (See Fig. 68 *c*.) Sometimes two or more pieces of wire are placed in parallel as in Fig. 68 *d*. Since this does not add anything to the e.m.f. of the coil but simply increases the ampere capacity, this would still be a one-turn, two-conductor coil.

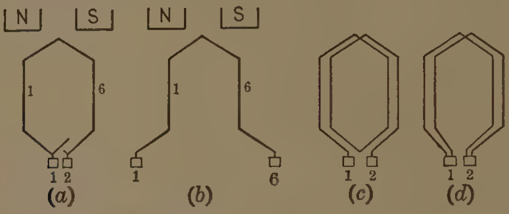


FIG. 68.—Coils for Drum Windings. (*a*) One-turn coil, lap winding; (*b*) One-turn coil, wave winding; (*c*) Two-turn coil, lap winding; (*d*) One-turn coil, lap winding.

PROBLEMS ON CHAPTER X

1–10. A compound-wound generator is rated at 500 kw., 125 volts.

(*a*) What is the full-load output in amperes?

(*b*) If this generator were rated at 500 kw., 600 volts, what would be the full-load current output?

2–10. The generator in Prob. 1–10 has an efficiency of 93 per cent at full load. What horsepower would be required to drive this machine at full load in (*a*) and (*b*), Prob. 1–10?

3–10. A direct current motor is rated at 50 hp., 115 volts, and has an efficiency at full load of 88 per cent.

(*a*) What is the full-load current input?

(*b*) If the motor were built for 550 volts instead of 115 volts, what would be the full-load current?

4–10. The generator in Prob. 2–8 is a two-pole machine with a total of 250 conductors on the armature. Each pole piece spans 35 per cent of the armature circumference. The conductors are arranged to form a ring winding like that shown in Fig. 60. What would be the e.m.f. between the brushes *a* and *b*?

5–10. A generator with a ring-wound armature like that shown in Fig. 60 has a potential of 250 volts at its terminals. There are 500 conductors on the armature and 65 per cent of the armature is covered by the field poles.

Calculate the e.m.f. generated in each conductor, assuming uniform flux density in the air gap.

6–10. The generator in Prob. 5–10 delivers 100 amperes to an external circuit. What is the current in each armature conductor?

7–10. The machine illustrated in Fig. 60 and described in Probs. 5–10 and 6–10 delivers direct current to the external circuit.

(*a*) Is the current which flows in the armature conductors direct or alternating?
(*b*) If your answer is "alternating," how do you reconcile this with Kirchoff's laws?

8–10. Referring to the machine of Prob. 5–10:

(*a*) What would be the voltage between adjacent commutator bars connected to conductors which were under a pole?
(*b*) What would be the voltage when these conductors were between poles?

9–10. The armature winding, referred to in Prob. 5–10, is placed in a four-pole field (Fig. 61). Assume that the voltage induced in one conductor is the same as in Prob. 5–10 and that the same percentage of the armature is covered by the poles.

(*a*) What would be the voltage between brushes *a* and *b*?
(*b*) How much current could be delivered to the external circuit without exceeding the ampere capacity of the conductors as determined in Prob. 5–10?
(*c*) What is the kw. output of this machine and how does it compare with the output of the armature when used as a bipolar machine (Prob. 5–10)?

10–10. A four-pole, 250-volt, 10-kw. generator is lap-wound. Calculate the current in each armature conductor when the machine is carrying full load.

11–10. The armature of the machine in Prob. 10–10 is reconnected to form a wave winding.

(*a*) Calculate the terminal voltage and current, assuming the same voltage and current per conductor as in Prob. 10–10.
(*b*) What would be the kw. output?

12–10. A four-pole generator rated at 7.5 kw., 125 volts, has an armature wound with 47 coils, each coil having two turns of No. 7 wire. There are two paths in the armature between terminals of opposite polarity. The length of one turn in the coil is 34 inches. What is the resistance of the armature between positive and negative terminals, neglecting brush contact resistance?

13–10. A four-pole motor rated at 2 hp., 115 volts, has an armature wound with 65 coils, each coil having 5 turns, and each turn consisting of two No. 18 wires. There are two paths in the armature between brush sets of opposite polarity. The length of one turn is 16.9 inches. What is the resistance of the armature between terminals, neglecting brush contact resistance?

14–10. Calculate the circular mils per ampere in the armature conductors of the generator in Prob. 12–10, when carrying full load.

15–10. A 125-volt generator has an armature wound with 87 coils, each coil consisting of 4 turns with two No. 13 wires per turn. There are four paths in the armature. On the basis of 600 circular mils per ampere, what would be the kw. rating of this machine? Neglect field current.

CHAPTER XI

CONSTRUCTION OF D.C. DYNAMOS

81. The Frame.—In Fig. 69 is shown an elevation and sectional view of a d.c. generator. The frame serves as a portion of the magnetic circuit so that the necessary flux may be provided at the air gap, and it also forms a housing or protection for the armature and a support for the bearings in the case of small machines. On the ring-shaped portion or yoke are inwardly projecting cores, which carry the exciting windings and have pole pieces, curved to fit the armature. The air gap is the space between the face of the pole pieces (or the pole face) and the surface of the armature. In addition to the main poles, which contain the exciting winding and serve as a path for the flux cut by the conductors, there are also small poles between the main poles (Fig. 124 *a*). These are called " interpoles " or more correctly commutating poles and, as their name implies, serve to assist in the commutation of the current in the armature. (See Article 140.) With the exception of very small machines, most modern d.c. dynamos are built with commutating poles. These poles have an exciting winding which is connected in series with the armature of the machine.

The frames of modern machines are usually made of cast steel, or rolled steel, although sometimes cast iron is used. It is of importance to make the magnetic circuit of as low reluctance as possible to reduce the size and cost of the exciting or field winding. This can be accomplished with steel because of its higher permeability. A machine with a cast-steel frame will, therefore, be much smaller, of lighter weight and will cost less than a machine with a cast-iron frame. The field poles are frequently built from laminated sheet steel. The frames of larger machines are usually divided to permit easy access to the armature for purpose of repair, and to facilitate shipment and handling during erection. The windings on the poles are so connected as to give alternately a north and south polarity in order to obtain the correct direction of e.m.f. in the armature conductors. (See Article 79.) Machines may be bipolar (i.e., two main poles) or multipolar. The machine in Fig. 69 is a multipolar, four-pole machine.

In most cases, the number of commutating poles is the same as the number of main poles, but this is not always necessary and, in small machines particularly, there may be only half as many commutating poles as there are main poles.

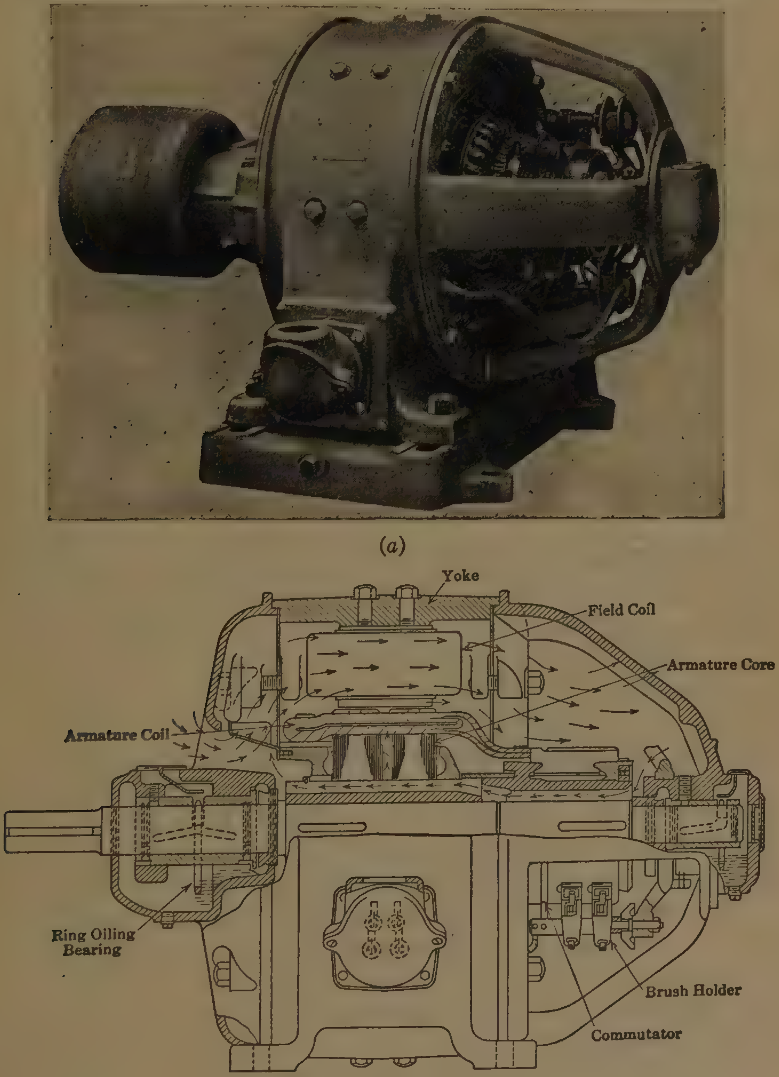


FIG. 69.—Construction of a D. C. Dynamo. (a) Side view of 30 hp., 230 v. motor. (b) Sectional view. *The General Electric Co.*

82. Bearings.—The bearings of all electrical machinery, except very large high-speed machines, are designed to be self-lubricating. The most common method of doing this is by means of oil rings, which hang on the shaft and are turned by it (Fig. 70). The lower side of these rings dip in a bath of oil, which is thereby carried to the shaft. Ball bearings are now quite common, particularly for motors. These bearings are packed with grease and have the advantage that they require less frequent attention than the ring oiling type and can also be made dust-proof more easily. A typical ball bearing for a motor is shown in Fig. 71. A waste-packed bearing is frequently used for railway motors and for industrial motors subject to severe operating conditions, such as are found in steel mills and in cotton mills. Turbine-driven alternators have the oil supplied to the bearings under pressure by means of a pump.

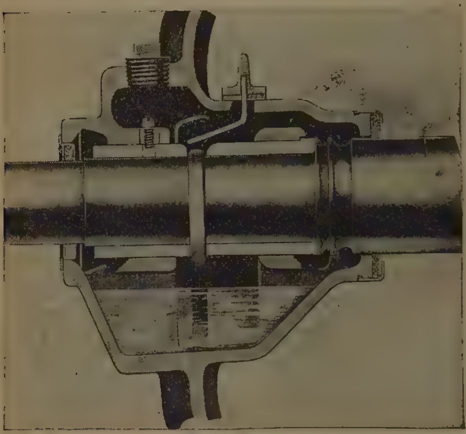


FIG. 70.—Ring Oiling Bearing. *The Westinghouse Elec. & Mfg. Co.*

83. Bipolar vs. Multipolar Construction.—The average voltage generated by the armature conductors depends upon the total flux cut per revolution. It makes no difference, so far as the average voltage is concerned, whether this flux is all concentrated in one magnetic circuit (bipolar machine) or is divided among several circuits (multipolar construction). Other factors, however, such as commutation and current per armature conductor, influence the number of poles best suited for a particular size of machine. The designer, therefore, selects a number of poles which will result in the lowest cost consistent with reliability. Both speed and size of a machine influence the number of poles most desirable; high-speed machines having fewer poles than slow-speed, and large machines having a greater number

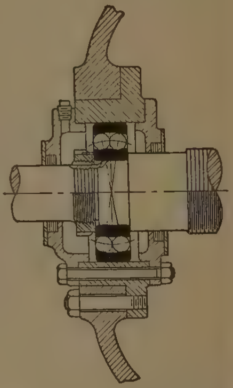


FIG. 71.—Ball Bearing for a Motor. *SKF Industries.*

of poles than small machines. The result is that belted machines usually have two or four poles, engine-driven generators six or more poles and turbine-driven machines two or four poles.

84. Armature Core.—Slotted armature cores are used in modern d.c. dynamos because this design provides better mechanical protection and decreases the length of air gap as compared with the smooth

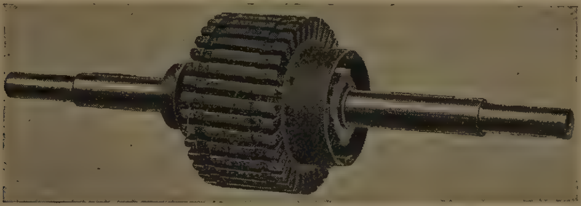


FIG. 72.—Armature Core and Shaft. *The General Electric Co.*

core type of construction. The core is built of thin sheet steel disks having rectangular notches in their outside edge. These disks are assembled either on the shaft or on a supporting spider in such a way that the notches in all the disks are in line, thus making a cylindrical, laminated armature core with slots parallel to the shaft (Fig. 72). Lamination of the armature core is necessary in order to prevent excessive eddy currents (Article 58). The laminations commonly

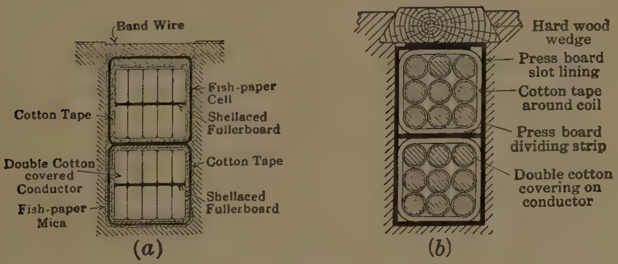


FIG. 73.—Arrangement of Conductors in Armature Slot.

used for d.c. dynamo armatures are about 0.025 inch thick. In all machines except those of small capacity, ventilating ducts are provided in the armature core by inserting spacers between the laminations at frequent intervals. (See Fig. 69.) These ducts allow air to circulate from the center of the armature core to the outside, due to the fanning action of the rotating armature. This air circulation is necessary in order to carry off the heat produced in the armature windings and core due to the losses. The laminations are made from a high grade of sheet steel in order that the losses in the armature core may be kept at a reasonable figure.

85. Arrangement of Armature Windings.—The individual copper wires forming the armature windings are generally covered with a cotton insulation. They are then assembled into groups, which are covered with tape and are then inserted in the slots in the core. These slots are lined with tough cardboard or similar material. The conductors are held in the slots by wooden or fiber wedges driven into the top of the slot (Fig. 73). The ends of the conductors which are not in the slots are held in place by band wires.

86. The Commutator.—The ends of the armature coils are connected to the commutator in such a way as to give the desired number of conductors in series. (See Article 79.) This commutator consists of a number of wedge-shaped segments of hard drawn or drop-forged copper, which are assembled together to form a cylinder. These bars are separated from each other by sheet mica and are insulated from their supporting rings by means of mica cones (Fig. 74). The segments or bars are held firmly in place by wedge-shaped clamping rings, which are drawn together by bolts in the larger machines or by threads on the shaft in small machines. The brushes bear upon the outside surface of the commutator. It is the custom in modern machines to undercut the mica between the commutator bars for a depth of about $\frac{1}{64}$ inch from the surface of the commutator. This causes the machine to run more quietly, gives better contact between brushes and commutator and results in less burning and sparking. The brushes used on undercut commutators are softer than those used with flush mica; they are self-lubricating and cause less wear on the commutator. The number of bars in a commutator depends upon the voltage and the number of poles. If the machine is to operate satisfactorily, without flashing over between brushes of opposite polarity, the average voltage between adjacent bars must not exceed about 15 volts for large, and 20 volts for small machines. Therefore, a high-voltage machine requires more bars than one of low voltage.

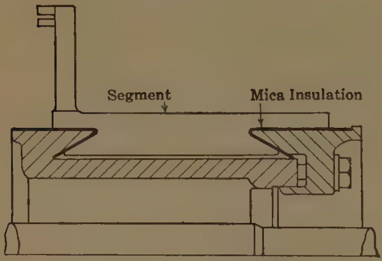


FIG. 74.—Construction of a Commutator.

87. Brushes and Brush Holders.—The brushes serve the double purpose of making contact with the rotating conductors and of changing the connections of these conductors with relation to the external circuit in such a way that direct current flows in the latter.[1] In modern machines, which have commutating poles, the brushes are so set that they make contact with the conductors which are exactly midway between the poles. In non-commutating pole machines, the brushes have to be shifted slightly from this mid-position (called the neutral). In motors, this shift is backward and in generators, forward in the direction of rotation.[2] There must always be at least one positive and one negative brush.

The brushes are supported in brush holders (Fig. 75) which are attached to brush studs bolted to the frame of the machine. In older types of machines, provision was made to move the brush studs a short distance in order that the amount of backward or forward shift of the brushes could be adjusted to meet operating requirements. With commutating pole machines, since it is seldom necessary to shift the brushes, they are firmly clamped in position during the process of testing at the factory. In all except very small machines, there are several brushes on a single stud because several small brushes will make better contact with the commutator than a single large brush. In multipolar machines, there are usually as many brush studs as there are main poles although small, wave-wound machines can be designed to operate with only two groups of brushes which, in this case, would be one pole pitch apart. (See Article 80.) Railway motors, which are usually four-pole, are frequently designed to use only two sets of brushes. The brush studs are of alternate + and − polarity and those of like polarity are connected together by bus bars as shown in Fig. 66. The connection would be the same for either a lap- or wave-wound armature.

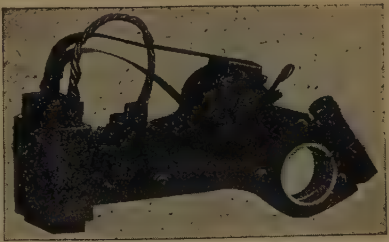


FIG. 75.—Brush Holder. *The General Electric Co.*

Machines with undercut commutators use brushes made of a mixture of carbon and graphite, the latter providing lubrication. Low-voltage machines use brushes made of copper gauze or of graphite mixed with a metallic powder. Carbon brushes are made in a large number of different grades to meet varying requirements. The grade best adapted for a particular machine is determined by the manufacturer and it is not safe to change the grade of brush used, except upon advice from the manufacturer or other competent source.

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[1] This was explained in Article 56.

[2] The reason for this is discussed under *armature reaction*, Chapter XV.

88. Field Excitation.—In all dynamos, except very small machines called magnetos, the necessary field flux is produced by current passed through windings on the field poles. These windings may consist of a large number of turns of fine wire requiring a small current, or a small number of turns of large wire, carrying a large current. The same flux can be produced by either winding, provided the product, amperes × turns, is the same. Direct-current machines are classified according to the type of field winding employed. A separately excited machine (Fig. 76) has the field winding supplied from a source separate from the armature. The field winding usually has a large number of turns and requires a relatively small current. The other types of machines, shunt, series, and compound are all self excited; that is, the power for excitation in the case of a generator is taken from the armature. These three types of windings are used on both motors and generators. In the shunt wound machine, the field circuit is connected in parallel with the armature (Fig. 77), and consequently is designed to have a large number of turns of relatively small size wire, thus giving a high resistance and requiring a small current. With generators, a field rheostat is always included in the shunt circuit so that the terminal voltage of the machine can be varied. This rheostat usually has a resistance about equal to that of the shunt field winding. With motors, a field rheostat is used only when the motor is designed to operate at different speeds. The series machine (Fig. 78) has a field winding which is connected directly into the

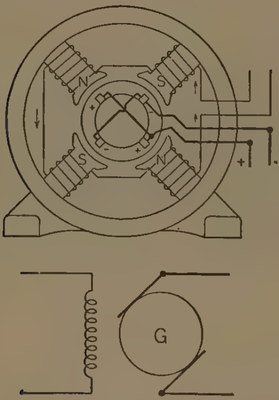

FIG. 76.—Connections of Separately Excited Generator.

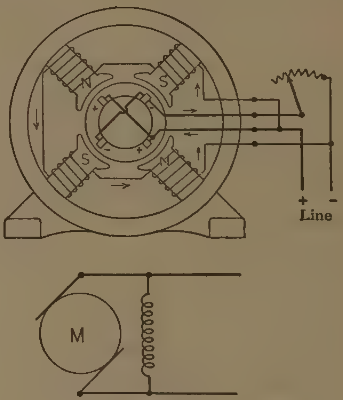

FIG. 77.—Connections of a Shunt Motor.

armature circuit. It therefore carries the entire armature current and is built with a small number of turns of large wire. Sometimes only a portion of the armature current is allowed to pass through the series field, by providing a shunt across the field winding (Fig. 108). The compound machine has both a shunt and a series winding placed on the main poles (Fig. 79). Ordinarily, for either a generator or a motor, the series winding is so connected that it aids the shunt winding as shown in Fig. 79. When so connected, the machine is said to be a cumulative compound machine. In some applications the series winding opposes the shunt; such a machine is said to be differentially compounded. When the shunt field of a compound machine is connected directly to the machine terminals (Fig. 79 *a*) it is said to be long shunt; when connected inside the series field, it is short shunt (Fig. 79 *b*). Practically there is very little difference in the two connections, although for a motor the long shunt

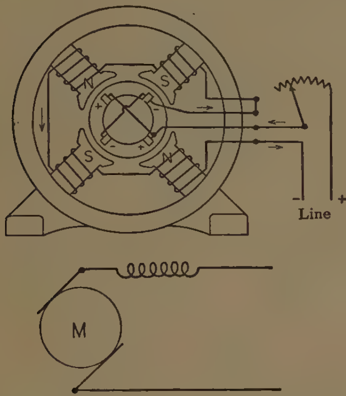

FIG. 78.—Connections of a Series Motor.

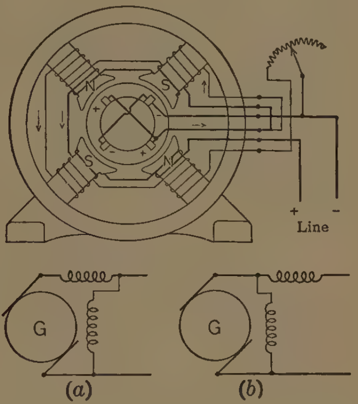

FIG. 79.—Connections of a Compound Generator.

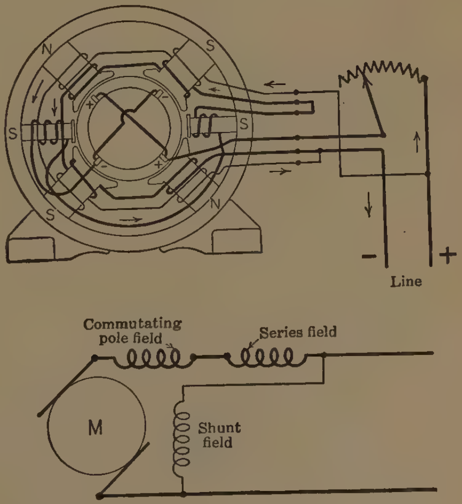

FIG. 80.—Connections of a Commutating Pole, Compound Motor.

connection makes it easier to reverse the direction of rotation. Short shunt connection is slightly better for generators, because the shunt field excitation rises slightly with load and hence not as much current is needed in the series field. The commutating poles always have a series type of winding regardless of the kind of machine upon which they are used. The connections for a compound machine of this kind are shown in Fig. 80. In the diagrams which follow, the commutating pole winding will be omitted, except where it has a bearing upon the result in question. It should be understood, however, that practically all modern machines now have this winding.

PROBLEMS ON CHAPTER XI

1–11. (*a*) Could a multipolar generator be designed to have the field windings around the yoke instead of on the poles?

(*b*) If so, show direction of current in the coils to give the correct polarity of the field poles.

(*c*) What disadvantages can you think of for this type of construction?

2–11. A bipolar d.c. generator has 1,120,000 lines of magnetic flux per pole. The machine has a ring-shaped yoke from which the two poles project. The allowable density in the poles is 90,000 lines per square inch and in the yoke 75,000 lines per square inch. What must be the cross-sectional area of the poles and the yoke?

3–11. If the same total flux as specified in Prob. 2–11 is to cut the armature, and four poles are to be used instead of two, calculate the area of each pole and of the yoke.

4–11. A ring-wound armature produces 125 volts when operated in a bipolar magnetic field which has 150,000 lines per pole. It is desired to place this armature in a four-pole field and to produce the same terminal voltage. What must be the flux per pole for the four-pole design, assuming the same percentage of polar arc?

5–11. A ring-wound armature operated in a bipolar field gives 125 volts with 2,300,000 lines per pole. If the same total flux is distributed in four poles, calculate the e.m.f. produced.

6–11. A four-pole, 125-volt generator has 87 bars in the commutator.

(*a*) What is the average voltage between adjacent commutator bars?

(*b*) If the machine were designed for 500 volts, how many bars would be required in the commutator to keep the average voltage between bars the same as in (*a*)?

7–11. A four-pole, 22.5-kw., 125-volt generator has four sets of brushes with three brushes in each set. Dimensions of brush in contact with commutator, $\frac{5}{8}$ inch × $1\frac{5}{8}$ inches. Calculate the amperes per square inch of brush contact area when the machine is carrying full rated load. (Neglect field current.)

8–11. A shunt-wound generator rated at 35 kw., 125 volts is to be converted into a series generator to produce the same terminal voltage at rated load. The machine has 6 poles and each field spool has 480 turns and carries 11 amperes. Calculate the number of turns on each spool of the series winding, neglecting drop in series field.

9–11. A 90-kw., 125-volt compound generator requires 17.6 amperes in the shunt field at full load.

(*a*) Calculate the current in the armature and in the series field if the machine is connected long shunt.

(*b*) Calculate the same for short-shunt connections.

10–11. A compound motor has a current input at full load of 23.4 amperes. The current in the shunt field is 0.48 ampere.

(*a*) What would be the current in the armature and the series field if the shunt field were connected directly to the line?

(*b*) What would be the armature and series field current if the shunt field were connected to the armature terminals?

11–11. The shunt field circuit of a 500-kw., 125-volt generator has a resistance of 4.8 ohms.

(*a*) What is the current in the field circuit?

(*b*) What amount of power is expended in field excitation?

(*c*) What percentage is this of the full-load output of the generator?

12–11. Why are the windings on the shunt fields of dynamos designed to be connected in series rather than in multiple?

13–11. Could a four-pole machine be so designed as to have only two field coils? If so, show by a diagram how the magnetic circuit and field coils would be arranged.

CHAPTER XII

D.C. GENERATOR CHARACTERISTICS

89. Fundamental E.M.F. Equation of a Dynamo.—It was shown in Article 57 that the e.m.f. generated in a conductor depends upon the rate at which it cuts magnetic flux. To calculate the e.m.f. generated in an armature conductor it is necessary, therefore, to determine the rate of cutting of flux.

Let e = the average e.m.f. generated in one armature conductor;
n = speed of armature in revolutions per second;
Φ = total flux for one pole of machine;
P = number of poles.

In one revolution the flux cut by one conductor is ΦP lines. The time required to make one revolution and cut this flux is $\frac{1}{n}$ second. Therefore, the average rate of cutting is flux÷time, i.e., $\Phi P \div \frac{1}{n}$ or ΦPn lines cut per second. The average e.m.f. is therefore

$$e = \frac{\Phi Pn}{10^8} \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

In any armature the total number of conductors N is divided into two or more parts, forming different paths through the machine. (See Article 80.) The e.m.f. at the brushes is the total e.m.f. generated in the conductors in any one path. The number of conductors in series in a machine having p paths is $N \div p$; therefore, the e.m.f. at the brushes is

$$E = \frac{\Phi PnN}{p \times 10^8} \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The e.m.f. of a particular conductor varies during each revolution, being zero when it is between the poles and a maximum while under the poles. Equation (26), however, gives the average voltage for the entire revolution. (See Article 57, Example 4.) The actual voltage at the brushes at any instant is equal to the sum of the voltages generated in all the conductors in any path at that instant, but, since there is always the same number of conductors under a pole at each instant, the sum of the average voltage of all the conductors in a path is the same as the sum of the instantaneous voltages induced in all the conductors in that path. This voltage is given by Equation (26).

Example 1.—A 6-pole generator runs at 750 r.p.m. and has a total flux of 3,190,000 lines per pole. There are 324 conductors on the armature, which is lap wound. The e.m.f. is therefore by Equation (26):

$$E=\frac{3{,}190{,}000\times 6\times 750\times 324}{6\times 60\times 10^8}$$

$$=129 \text{ volts.}$$

The e.m.f. of a dynamo may also be derived as follows: Assume that the flux density under the poles is B lines per square inch and that the density is uniform throughout. During rotation, a particular conductor would therefore generate a steady e.m.f. while under a pole and would generate no e.m.f. while between poles.

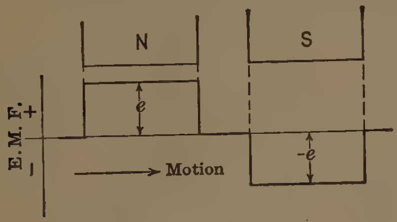

Fig. 81.—E.M.F. of a Conductor.

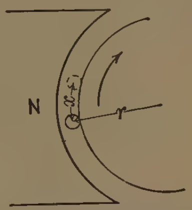

Fig. 82.—Calculation of Generated E.M.F.

This condition is represented in the curve, Fig. 81. As long as the conductor is under the influence of the N pole the e.m.f. will be a constant value in one direction, and while under the S pole, the e.m.f. will also be the same value but will be in the opposite direction. The curve repeats itself for each pair of poles. At any instant the e.m.f. at the brushes will be the sum of the e.m.f.'s generated in all the conductors in one path and since all paths are alike the e.m.f. of the machine can be determined if the total e.m.f. of one path is found.

Let l = length of active conductor parallel to shaft, i.e., the length of pole face;
P = number of poles;
n = revolutions per second;
Φ = total flux from one pole.

Referring to Fig. 82, suppose a conductor moves a distance of x inches while under the pole. Let the conductor be located at a radius of r inches from the center of the shaft. In one revolution a conductor moves a distance of $2\pi r$ inches and the time required to move this distance is $t=\frac{1}{n}$ second. The time to move a distance x is

$$t'=\frac{x}{2\pi r}\times t, \quad \text{or} \quad t'=\frac{x}{2\pi rn} \text{ second.}$$

The total flux cut in moving a distance x, is $\Phi' = Blx$ lines. Hence the rate of cutting is $\frac{\Phi'}{t'} = Blx \div t' = 2\pi rBln$ lines per second. The e.m.f. induced during the entire time t' is therefore

$$e' = \frac{2\pi rBln}{10^8} \text{ volts. . . . . } \qquad (27)$$

The conductor will continue to generate a steady e.m.f. of e' volts as long as it is under a pole; when midway between poles the e.m.f. is zero (Fig. 81).

Let y = the distance traveled by the conductor while under a pole. The total circumference of armature at the point where the conductor is located is $2\pi r$ inches. The portion of this circumference which is covered by the poles is $Py \div 2\pi r$ of this circumference. The number of conductors in one path is $N \div p$. At each instant $Py \div 2\pi r$ of these conductors are generating an e.m.f. of e volts and the remainder have zero e.m.f. Hence, at any instant the total e.m.f. of any path is the sum of the e.m.f.'s of the conductors in that path which is

$$\frac{N}{p} \times \frac{Py}{2\pi r} \times e' \text{ volts.}$$

Hence the e.m.f. at the brushes is

$$E = \frac{NPy}{2\pi rp} \times \frac{2\pi rBln}{10^8} \text{ volts}$$

$$= \frac{BlyPNn}{p \times 10^8} \text{ volts. . . . . } \qquad (28)$$

But the total flux for one pole is

$$\Phi = Bly \text{ lines.}$$

Hence Equation (28) becomes

$$E = \frac{\Phi PNn}{p \times 10^8} \text{ which is identical with equation (26).}$$

Example 2.—The machine of Example 1 has pole pieces having a length of 7 inches parallel to the shaft. The conductors are located at an average radius of 12 inches from the center of the shaft. The distance traveled by a conductor while under one pole is 9.5 inches. The flux density is 48,000 lines per square inch.

The number of conductors in one path is $N' = \frac{324}{6} = 54$.

While a conductor is under a pole it is cutting a uniform field of 48,000 lines per square inch. The e.m.f. induced in one conductor while it is under a pole is according to Equation (27).

$$e' = \frac{2\pi \times 12 \times 48{,}000 \times 7 \times 750}{10^8 \times 60}$$

$$= 3.16 \text{ volts.}$$

The poles cover 75.6 per cent of the armature circumference; hence, at any instant, 75.6 per cent of the conductors in one path are each generating an e.m.f.

of 3.16 volts and the voltage of the remainder is zero. Therefore, the total voltage induced in one path is

$$E_a = 54 \times 0.756 \times 3.16 = 129 \text{ volts.}$$

which is the same as calculated in Example 1.

Equation (26) can be used to calculate the e.m.f. of either a generator or a motor and applies to a toothed as well as a smooth-core armature. In the case of a toothed armature, most of the flux passes through the teeth because of the large reluctance of the path through the slots (Fig. 83). At first thought, it might seem as if the conductors would not cut the flux which passes through the teeth, but this is not the case. It is evident that a conductor, in moving from a to b (Fig. 83), must cut across all the flux from the pole. As a tooth leaves a pole, the flux entering this tooth must be transferred to the next tooth behind it, and, in the process of transfer, the lines may be considered to snap suddenly across the slot, thus cutting the conductors in that slot.

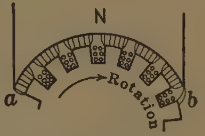

FIG. 83.—Flux in Toothed Armature.

For a particular machine, the number of poles P, the paths p, and the conductors N, are fixed; therefore, if we let

$$K = \frac{PN}{p \times 10^8},$$

Equation (26) becomes

$$E = K\Phi n. \qquad (29)$$

This simplified form of the e.m.f. equation is of great importance when studying the performance of dynamos.

90. Characteristic curves show the changes in voltage, speed, etc., caused by changes in flux or load. The common curves of this sort are:

(*a*) *Saturation or magnetization curves*, which show the variation in armature e.m.f. with varying field current, the armature current being kept constant.

(*b*) *External characteristics*, which show the variation of generator terminal voltage with variation of load or terminal current.

(*c*) *Total characteristics*, which show the variation of total voltage or generated e.m.f. with changes in total or armature current.

The shape of these curves will vary with the particular type of machine, i.e., whether shunt, series, or compound wound. Usually voltage characteristic curves like those just described are determined at constant speed, but they may be taken with varying speed to allow for variation in the speed of the prime mover.

91. Conditions Affecting the E.M.F. of a Generator.—From Equation (29) it will be seen that there are only two factors, Φ and n, which influence the e.m.f. generated by a particular machine. Evidently if the speed is constant, E is proportional to the flux; if the flux cut by the conductor is constant, E is proportional to the speed. The speed of a generator is controlled by the prime mover but the flux may be changed in the following ways:

(1) By changing the exciting current through either the shunt or series winding or both. This may be accomplished by a change in voltage at the terminals of the field circuit or by a change in the resistance of the field circuit by means of the field rheostat. This is a common method of changing the voltage of a generator.

(2) By changing the reluctance of the magnetic circuit, while the exciting current is held constant. This evidently would change Φ. In practice, this is done by moving the armature part-way out from its normal position under the poles, or by moving radially a portion of the field core. These methods, which are illustrated in Fig. 84, are used for motors only.

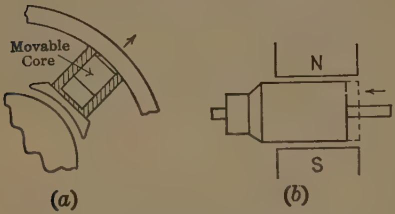

FIG. 84.—Methods of Changing the Flux of a Dynamo. (*a*) Movable field core (Stowe motor); (*b*) Shifting armature (Lincoln motor).

(3) By the magnetic effect of the armature current. When current flows in the armature of a dynamo, this armature becomes a magnet, and the effect of this is, in general, to change the strength and distribution of the main flux. This effect is called armature reaction. (See Chapter XV.) For present purposes it is sufficient to know that the armature reaction caused by the current in the armature decreases the main flux. This applies to both generators and motors under the usual operating conditions. At no load, the terminal voltage of a generator is practically the same as the generated e.m.f., but, when loaded, the terminal voltage is less because of the losses in the machine. Equation (26) and the statements made above apply strictly to the e.m.f. but are approximately true for the terminal voltage.

92. Magnetization or No-load Saturation Curve.—If a generator is operated at constant speed with no current in the armature, the generated e.m.f. will vary with changes in field current. The curve showing this variation is called a magnetization or no-load saturation curve (Fig. 85). The curve is plotted between exciting current I_f and terminal voltage E_t, but, since there is no armature current, E_t is the same as E, the generated e.m.f. With a shunt or compound machine, the saturation curve can be obtained with the shunt winding connected to the armature since the shunt field current is small and produces a negligible drop in the armature. A series machine would have its field supplied from a separate source. The curve (Fig. 85) does not start at zero, but there is a small voltage e_r when there is no current in the field winding, due to the residual magnetism in the field frame. The shape of the curve is somewhat like the ordinary magnetization curve for iron or steel. This is natural since it is, in effect, the magnetization curve of a magnetic circuit composed of the cast steel frame and poles, the laminated armature, and the air gap of the machine. Although the curve is plotted in terms of exciting current and induced voltage, it has exactly the same form as if it were plotted to magnetizing force H and flux density B since H is proportional to the field current (field turns being constant) and E_t is proportional to B, the number of poles, paths, and conductors being constant. At low values of field current, E_t varies proportionally, but as the current increases, the steel portion of the circuit becomes partly

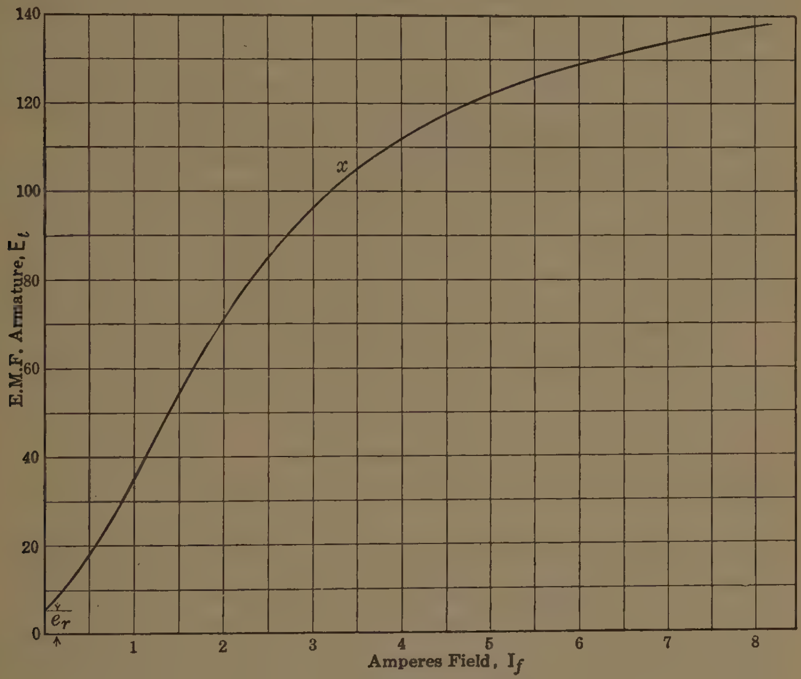

FIG. 85.—Saturation Curve.

saturated and the voltage increases more slowly. The abrupt bend at point x is called the knee of the curve. Shunt generators are designed to operate at a point somewhat above the knee, so that a slight change in speed will not cause a great change in voltage. Compound wound machines are operated at a lower point on the saturation curve in order that the necessary increase in voltage can be obtained without an excessively large series field winding. If the speed of the machine is not exactly constant while the readings of E_t and I_f are being taken, correction of E_t can be made to reduce all readings to the equivalent values at a constant speed, since E_t is proportional to speed according to Equation (29).

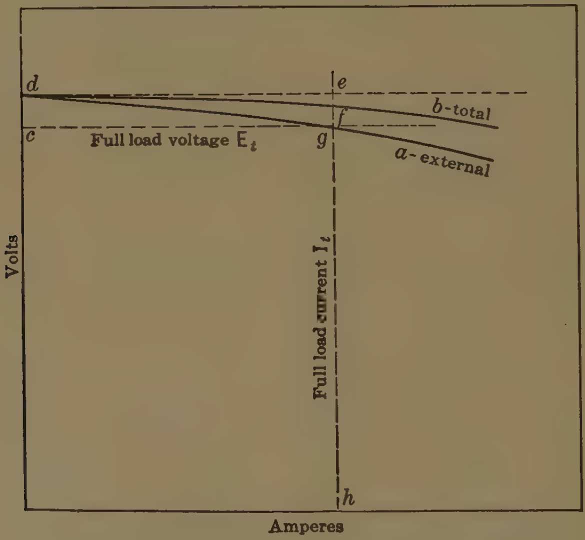

FIG. 86.—Characteristics of a Separately Excited Generator.

93. Performance of a Separately Excited Generator.—If this type of generator is operated at constant speed and the voltage adjusted to a particular value at no load, then as the load is thrown on (Fig. 86), the terminal voltage E_t will decrease as the load I_a increases, the field current I_f being held constant. The curve showing this change is called the external characteristic. The reasons for the drop in terminal voltage are:

(*a*) The IR drop due to the resistance of the armature winding and the brushes. This drop added to the terminal voltage E_t would give the generated e.m.f. E_a.

(*b*) The effect of armature reaction (Article 138). The load current flowing in the armature weakens the flux Φ which is cut by the armature; therefore, when carrying load, the actual e.m.f., E_a generated by the armature (according to Equation (29)) is less than at no load in spite of the fact that the exciting current I_f and the speed are constant. The total characteristic of a separately excited generator is found by adding the IR drop in armature and brushes to the terminal voltage to obtain E_a (Fig. 86).

94. Voltage Losses in a D.C. Generator.—The voltage drop in the armature of a d.c. generator can be determined if the armature current and the armature resistance are known. The loss in the brushes is due principally to a voltage drop caused by the resistance of the contact between brush and commutator. This contact resistance is not constant, but varies with the current carried by the brush, accordingly, for accurate work, curves are used showing contact drop for varying value of current density in the brush contact. For the usual range of current densities, however, the drop at full load is approximately 1 volt for the positive and 1 volt for the negative brushes. There is, however, a total drop of only 2[1] volts, regardless of the number of brushes, since all the brushes of like polarity are in parallel. For other loads, the total brush drop is approximately as follows:

$\frac{3}{4}$ load—1.75 volts
$\frac{1}{2}$ load—1.50 volts
$\frac{1}{4}$ load—1.25 volts

Brushes with considerable graphite in their composition have a somewhat lower voltage drop.

In the case of series or compound machines there is also a drop in the series winding so that the terminal voltage of the machine is less than the voltage at the brush studs. The commutating pole winding is always in series with the armature, and this also causes an additional voltage drop. This matter will be taken up in detail as the performance of each type of machine is studied.

95. Performance of a Shunt Generator.—If a shunt wound generator is operated under varying load conditions, it will be found that the terminal voltage E_t will decrease as the terminal or load current I_t is increased, provided the speed remains constant and there is no change in the resistance of the field circuit, that is, the setting of the field rheostat is not changed (Fig. 87). The reasons for this change in terminal voltage are in part the same as for the separately excited machine since there must be a fall of potential due to the resistance of the armature winding and the brushes, and usually a slight decrease of Φ due to armature reaction. In the case of the separately excited machine, the exciting current is constant: hence, the flux Φ is nearly constant although it may be decreased slightly by armature reaction. The total e.m.f. generated in the armature of a separately excited machine is therefore nearly constant. In the shunt machine, however, the field current decreases as the load increases since the shunt field circuit receives the voltage E_t and the resistance of the field circuit is constant. Therefore, the flux Φ will decrease as the load increases, due to this decreased excitation, and this will cause a still further decrease in E_t. A machine operated self-excited, as a shunt generator, will therefore fall off in voltage more than it would if separately excited, as shown by the curves in Fig. 87. The reasons for the drop in terminal voltage of a shunt machine can be summarized as follows:

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[1] Based on about 40 amperes per square inch.

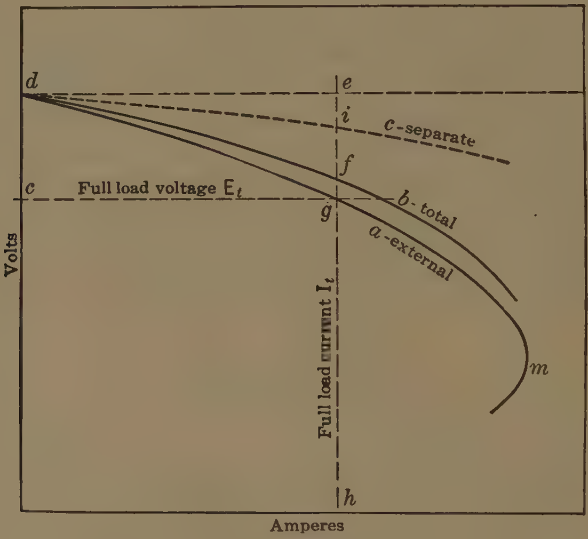

FIG. 87.—Characteristics of a Shunt Generator.

(1) IR drop in armature winding and brushes.
(2) Armature reaction, which weakens the flux.
(3) Decreased field excitation, due to decreased voltage at the terminals of field circuit, which still further decreases the flux.

Although a shunt machine will decrease in voltage as indicated in the curve (Fig. 87) if the field rheostat setting is not changed, it is, of course, possible to maintain normal voltage at full load by cutting out a portion of the field rheostat, thus increasing the excitation and producing the necessary increased e.m.f. to make up for the IR drop in the armature circuit. To maintain normal terminal voltage on a shunt machine when operating with varying load, continual adjustment of the field rheostat would be required as the load changed; so that a shunt generator is not suitable for use where a constant voltage must be maintained on a circuit in which the load varies frequently.

As a shunt machine is overloaded, the voltage falls very rapidly until a point m (Fig. 87) is reached when a further decrease in the load resistance causes a decrease in current and voltage, until, finally, if the terminals are short-circuited, the current drops to a lower value due to the residual magnetism. This self-regulating property of the shunt machine tends to protect it against injury due to short-circuits. Shunt generators are always provided with field rheostats so that the field current can be increased as the load is increased and the terminal voltage thereby maintained at the desired value.

The total characteristic of the shunt generator is plotted between the armature current I_a and the e.m.f. generated in the armature E_a.

$$I_a = I_f + I_t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where I_f is the shunt field current and I_t the load current.

$$E_a = E_t + I_a R_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where E_t is the voltage at the terminals of the machine and R_a is the resistance of the armature circuit including brushes. Usually it is more convenient to determine the brush drop separately, either from a curve or by the tabulation given in Article 94. This is added to the drop in the armature copper.

Example 1.—A 90-kw., 125-volt, shunt generator has an armature resistance, not including brushes, of 0.0025 ohm, and a shunt field resistance of 7.1 ohms. The full load terminal current is

$$I_t = \frac{90{,}000}{125} = 720 \text{ amperes.}$$

The field current is

$$I_f = \frac{125}{7.1} = 17.6 \text{ amperes.}$$

The armature current is

$$I_a = 720 + 17.6 = 737.6 \text{ amperes.}$$

The armature drop is

$$e = 737.6 \times 0.0025 = 1.85 \text{ volts.}$$

Allowing for brush loss as specified in Article 94, the e.m.f. generated by the armature at full load is

$$E_a = 125 + 1.85 + 2 = 128.85 \text{ volts.}$$

96. The Building up of voltage of a self-excited generator requires residual magnetism in the frame. Ordinarily a machine when operated at normal speed with the field circuit open will have sufficient residual flux to generate about 4 per cent of normal voltage. (See Fig. 85.) A shunt or compound generator is built up to normal voltage by decreasing the resistance of the field circuit by cutting out the field rheostat. If the connections are correct and the residual voltage normal, the terminal voltage will rise rapidly and can be adjusted to the required value by adjustment of the field rheostat. The building up is accomplished by the voltage due to the residual flux, causing a small current to flow through the shunt field winding. This flows in such a direction as to produce flux of the same polarity as the residual flux; hence, the generated e.m.f. increases and this increases the current through the field circuit. If the magnetization curve of the generator were a straight line, the e.m.f. would keep on building up indefinitely, but, since it is a curve, the resulting increase in e.m.f. due to a given increase in current becomes smaller and smaller as the voltage rises until finally a point is reached where the e.m.f. is just sufficient to force the required current through the field circuit and the voltage remains steady. The particular voltage to which the machine will build up, depends upon the resistance of the field circuit. If the resistance is low, the voltage will be high, the maximum value being that when all the rheostat is cut out and the shunt field winding receives the entire terminal voltage. The voltage with full field may be as much as 150 per cent of normal. If the resistance of the field circuit is high, caused by cutting in the field rheostat, the voltage will be lower. If the resistance of the field circuit is greater than a certain maximum, called the critical resistance, the machine will not build up. In order that the machine shall build up, it is necessary that the connections of the field shall be such that the residual voltage will send current through the field in the proper direction; otherwise, this field current would tend to demagnetize the machine. If a shunt generator which shows the proper residual voltage fails to build up, the connections of the shunt field should be reversed to see if the trouble is due to wrong connections. Reversal of direction of rotation will cause reversal of residual voltage; hence, the shunt field connections must be reversed when direction of rotation is changed. Sometimes failure to build up is due to high resistance in the shunt field circuit, due to poor brush contact or a loose connection. A shunt machine will not build up if the armature terminals are short circuited. Occasionally failure to build up is due to loss of the residual magnetism which may be caused by rough handling during transportation or, in the case of a compound generator, by a reversed current in the series winding caused by a feeding back of power from some external source. The remedy in the case of loss of residual magnetism is to separately excite the fields for a short time. If the machine has a series field, current from a storage battery or even a few dry cells can be passed through the series winding.

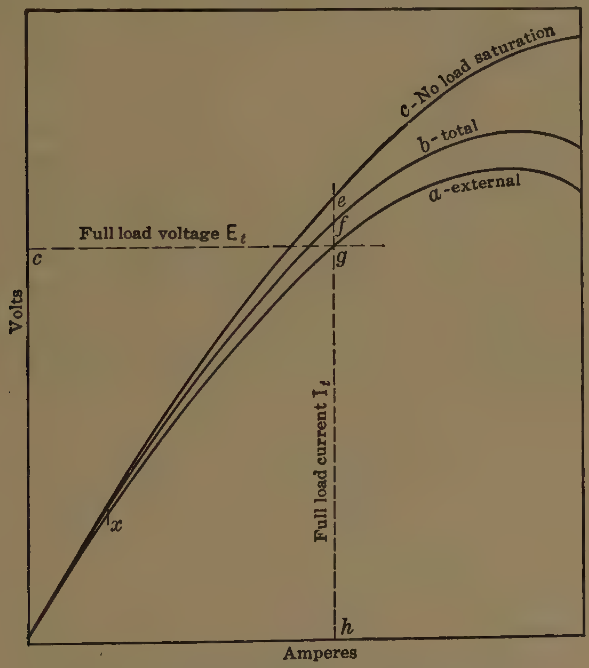

FIG. 88.—Characteristics of a Series Generator.

A series generator also depends upon residual flux to enable it to build up and the resistance of the external circuit must be below a certain maximum before it will build up. The relative polarity of field and armature must be correct for the same reason as in the case of the shunt machine, and failure to build up may be due to the same causes.

Generally, when a machine has been adjusted to build up with a certain polarity, it will continue to build up with the same polarity, although occasionally the residual flux may be reversed due to some external cause, when the machine will build up with reversed polarity.

97. Performance of a Series Generator.—At no load the terminal voltage of a series generator (Fig. 88) is, of course, only that due to the residual flux. The machine will not build up until the resistance of the external circuit is reduced to a certain value, when the voltage and current rise rapidly until equilibrium is established as in the shunt machine. This is point x on the curve. As the resistance of the external circuit is decreased, the terminal voltage increases for a time but finally decreases rapidly. It is evident that the field excitation increases with increase of load current, and while working on the lower part of the saturation curve there is a resulting increase in flux and, hence, increase in generated e.m.f. As the load current increases, however, and the magnetic circuit approaches saturation, there is less increase in e.m.f. and the curve tends to drop more. The total characteristic (Fig. 88 *b*) therefore rises rapidly for a time and then drops. If there were no armature reaction, the magnetization curve and the total characteristic would coincide. The terminal voltage is less than the generated e.m.f. by the drop in the armature and series field. If the external characteristic is determined by experiment, the total characteristic can be found by adding the voltage drop.

Example 1.—A 2.25-kw., 125-volt series generator has an armature resistance, not including brushes, of 0.04 ohm and a field resistance of 0.21 ohm. Calculate the e.m.f. at full load.

The full load current is

$$I_a = \frac{2250}{125} = 18 \text{ amperes.}$$

The drop in the armature is

$$e = 18 \times 0.04 = 0.72 \text{ volt.}$$

The drop in the field is

$$e' = 18 \times 0.21 = 3.78 \text{ volts.}$$

Hence the e.m.f. at full load is

$$E_a = 125 + 0.72 + 3.78 + 2 = 131.5 \text{ volts.}$$

98. Performance of a Compound Generator.—In general, it is necessary to maintain an approximately constant voltage on lighting or power circuits. If a shunt machine were used to supply these circuits, variations of load would produce large voltage variations unless the field current were adjusted as the load changed. By adding a series winding, making a compound machine, the required change in field strength can be secured automatically without adjustment of the shunt field rheostat. The curves of Figs. 89 and 93 show voltage characteristics of compound generators.

The series winding is usually so connected that it aids the shunt winding, thus tending to raise the voltage as the load increases. When connected in this manner, the machine is said to be a cumulative compound generator. The proper regulation of the voltage depends upon the strength of the series field. If the field is weak, the voltage drops somewhat with increase of load although not so much as if operated as a shunt machine. Such a machine is said to be under-compounded (Fig. 89 *c*). If the machine gives the same terminal voltage at no load and at full load, it is said to be flat-compounded.

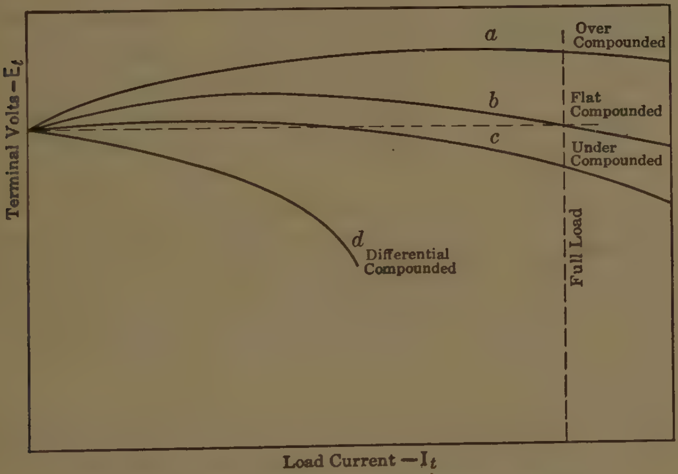

FIG. 89.—Characteristics of a Compound Generator.

If the voltage rises, the machine is overcompounded. For these three types of machines, the series field is cumulatively connected. If the series field winding were connected so it opposed the shunt, then the external characteristic would be below that when the machine is operated with the shunt winding alone (Fig. 89 *d*) and the voltage would drop off very rapidly. Such a machine is said to be differentially compounded.

When a compound generator is carrying a load, the terminal voltage E_t is less than the generated voltage E_a due to the voltage drop in the armature, brushes, and series field. This drop is proportional to the load current and can be compensated for by the proper number of turns on the series winding. In addition, there is a decrease of generated e.m.f. E_a due to the effect of armature reaction, but this is not proportional to the load as the same change in load produces a greater effect on the generated voltage at heavy load than at light load. If the voltage is to be maintained at the proper value at full load, enough series turns must be used to correct for IR drop and also for armature reaction. This will result in an excess of series ampere-turns at light loads, because the armature reaction has proportionally less effect on E_a, so that the terminal voltage rises above the normal, giving a " hump " in the external characteristic (Fig. 89 *b*). The deviation of this curve from a straight line or ideal condition may be from 2 to 5 per cent in standard machines.

The amount of compounding depends upon the ampere-turns of the series field and is a maximum for a particular series field winding, when the entire armature current passes through the series field winding. To secure less compounding, it is necessary only to shunt the series field by a resistance called a compounding shunt (Fig. 90). As this resistance is decreased, less current passes through the series field and the compounding is decreased. The compounding shunt is usually composed of German silver ribbon or a cast-iron grid resistance. This shunt is adjusted at the factory during the test of the machine and need not be changed unless the operating requirements change.[1]

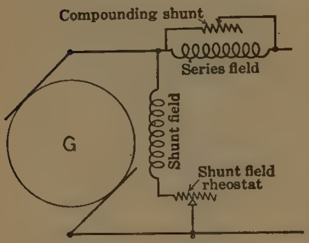

FIG. 90.—Method of Adjusting Compounding.

The total characteristic of a compound machine can be determined by finding the armature current and the e.m.f. generated by the armature. The method of doing this has already been described in the discussion of shunt and series generators.

Example 1.—A 90-kw., 125-volt compound generator (short shunt) has an armature resistance of 0.0025 ohm, a shunt field resistance of 7.1 ohms, and a series field resistance of 0.001 ohm. The full load terminal current is

$$I_t = \frac{90{,}000}{125} = 720 \text{ amperes.}$$

The drop in the series field is

$$E_s = 720 \times 0.001 = 0.72 \text{ volt.}$$

The current in the shunt field is

$$I_f = \frac{125.72}{7.1} = 17.7 \text{ amperes.}$$

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[1] See Article 102, for calculation of compounding.

The armature current is

$$I_a = 720 + 17.7 = 737.7 \text{ amperes.}$$

The armature drop is

$$e = 737.7 \times 0.0025 = 1.85 \text{ volts.}$$

Allowing 2 volts brush loss (Article 94), the e.m.f. generated at full load is

$$E_a = 125 + 0.72 + 1.85 + 2 = 129.57 \text{ volts.}$$

The full load point on the total characteristic would therefore be plotted with a value of 129.57 volts as ordinate and 737.7 amperes as abscissa. Other points could be obtained in a similar manner.

99. Voltage Regulation.—The external characteristic of a generator is sometimes called the voltage regulation curve of the machine. The voltage regulation of a generator is the change in terminal voltage occurring between no load and full load. Thus the regulation of the compound generator (Fig. 93) is 230 volts no load and 250 volts full load. The regulation, if expressed in per cent, is based on the full-load value.

Example 1.—The regulation of the compound generator (Fig. 93), is 8 per cent.

100. Effect of Operation of D.C. Generators at Voltages Higher or Lower than Normal.—The normal or rated voltage of a generator is always stamped on the name plate of the machine. It is important to operate at this rated voltage if the machine is to meet the manufacturer's guarantees.

For a **shunt generator,** operation at rated speed and at a voltage below the rated value would result in working the machine at a lower point on the saturation curve. Such a machine would, therefore, be more sensitive to fluctuations in speed and the voltage regulation would be poorer; i.e., the external characteristic would drop off faster. Operation above rated voltage would have the opposite effect; i.e., the regulation would be better. Since the ampere capacity of a machine is fixed by the armature winding, the rated kilowatt output could not be secured if the machine were operated at a voltage below normal without overheating the windings. On the other hand, if the machine were operated above rated voltage, the kilowatt output with rated armature current would be greater than normal so that a somewhat greater capacity of machine would result. There is a limit to the amount of this increased capacity, however, because the shunt field and armature core would be likely to reach an excessive temperature.

With **compound generators,** operation at reduced voltage makes the machine more sensitive to the effect of the series field so that the change in voltage between no load and full load would be greater. Operation above normal voltage would have the opposite effect. The safe output of the machine would be affected in the same way as for a shunt machine.

101. Effect of Operation of a D.C. Generator at Speeds Above or Below Normal.—A generator should be operated at the normal or rated speed as marked on the name plate; otherwise, the performance may not be satisfactory.

For a **shunt generator,** operation at a speed higher than normal, but at rated voltage, would result in the machine running at a lower point on the saturation curve. It would, therefore, be more sensitive to fluctuations of speed and would act the same as if operated at reduced voltage and normal speed. (See Article 100.) Operation below rated speed would have the opposite effect; i.e., the machine would be more nearly saturated and the voltage would be less sensitive to fluctuations in speed. If a machine is operated at reduced speed, however, it may be impossible to maintain rated voltage under load conditions because of the increased field excitation required. Furthermore, the field winding may overheat, due to the larger field current required.

For a **compound generator,** operation at a speed above normal requires a lower value of flux; hence, the machine is not as highly saturated so that the effect is the same as operation at reduced voltage and normal speed (Article 100). For this reason, the voltage regulation would be poorer. Operation below normal would have the opposite effect; the machine would be more stable, but the compounding effect of the series field would not be so great due to the higher saturation of the field circuit.

102. Determining the Strength of Series Field for a Compound Generator.—The required ampere-turns in the series field of a compound generator can be obtained by the help of an armature characteristic or field compounding curve (Fig. 91). This characteristic is plotted for field current (or field ampere-turns) and armature current, the terminal voltage being held at the desired value. If observations for flat compounding are required, the voltage would be held constant as the load is changed. For overcompounding the voltage would be adjusted as the load is increased, to give the desired increase in voltage. If the curve were a straight line, exact compounding or perfect regulation could be secured. The number of series turns required can be calculated from the armature characteristic curve. Assume that the ampere-turns per pole required at no load is ab (Fig. 91). Then the total ampere-turns required at full load is ac for flat-compounding and *ad* for overcompounding. When the machine is designed to operate as a flat-compounded generator (long shunt), the voltage at the terminals of the shunt winding is constant; therefore, this winding would supply the same number of ampere-turns at full load as at no load (*ab* in Fig. 91). The difference, or *cb*, must be supplied by the series winding. Hence, the number of turns on the series field can be determined when the amount of current in this winding is known.

Example 1.—A 90-kw., 125-volt generator requires a total of 5610 ampere-turns per spool at no load and 7670 ampere-turns at full load to maintain normal voltage.

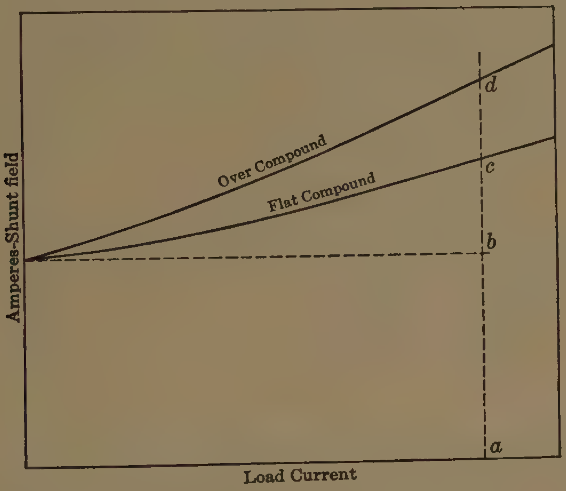

FIG. 91.—Armature Characteristics.

If 65 per cent of the load current is to pass through the series field, calculate the number of series turns per spool to produce flat compounding. The additional ampere-turns required to produce 125 volts at full load is

$$NI = 7670 - 5610 = 2060 \text{ ampere-turns.}$$

Neglecting the fact that the shunt field receives a slightly higher voltage at full load, the shunt field current is constant; hence, the excitation due to the shunt field is 5610 ampere-turns at full load as well as at no load. Hence, the series winding must supply 2060 ampere-turns at full load.

The line current is

$$I_t = \frac{90{,}000}{125} = 720 \text{ amperes.}$$

The series field current is

$$I_s = 0.65 \times 720 = 468 \text{ amperes.}$$

The turns per spool for the series winding would be

$$N_s = \frac{2060}{468} = 4.4.$$

Hence 4.5 turns must be used.

For an overcompound machine, the ampere-turns supplied by the shunt winding at full load is increased because the terminal voltage increases. Hence, in this case, the shunt ampere-turns is not *ab* (Fig. 91), but is greater than this in proportion to the increase of terminal voltage. The remainder of *ad* is supplied by the series winding.

Example 2.—The machine of Example 1 requires 8110 ampere-turns at full load to cause the voltage to rise from 125 volts at no load to 130 volts at full load, with the same current output (720 amperes). The voltage across the shunt field increases to 130 volts at full load; therefore, the excitation produced by the shunt field at full load is $N_1I_1 = 5610 \times \frac{130}{125} = 5830$ ampere-turns. The excitation required from the series winding is, therefore,

$$N_sI_s = 8110 - 5830 = 2280 \text{ ampere-turns.}$$

Assuming that the number of series field turns is to be kept the same as in Example 1, the current in the series field must be

$$I_s = \frac{2280}{4.5} = 507 \text{ amperes.}$$

This increased current would be secured by increasing the resistance of the compounding shunt.

103. Applications of D.C. Generators.—The particular kind of generator best adapted for supplying a certain load depends upon the system and the load requirements. The separately excited generator is used for low-voltage machines, such as electroplating generators, and for special boosting generators. **Shunt generators** are adapted for charging storage batteries since the voltage rises as the current decreases, thus preventing the battery from discharging back into the generator. Shunt generators are also used as "exciters" for furnishing the necessary field current to alternators. **Series generators** were at one time extensively used for series street-lighting circuits, but these machines are now obsolete since the series circuits, whether d.c. or a.c., are at present supplied from an a.c. source through special regulating transformers. Series generators are sometimes used as boosters for a long feeder. This is shown in Fig. 92 where a street

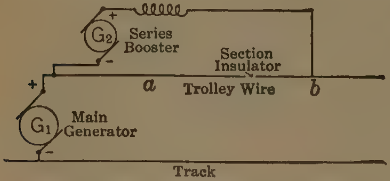

FIG. 92.—Application of a Series Booster.

railway system is to be supplied at the generator voltage except for one very long feeder upon which there is an excessive drop. In series with this feeder is a series generator. When there is a light load on the section *b*, there will be very little drop in the feeder, and a negligible voltage generated by the booster; hence, the voltage at *b* will be substantially the same as *a*. As the load on *b* increases, however, the increased drop in the feeder would require an increased voltage applied to the feeder, if the voltage at *b* is to be held approximately constant. The series booster gives an increased voltage as the load on *b* increases, and, by proper design, the voltage added by the booster can be made the same as the drop in the feeder so that the voltage at *b* would be approximately constant. **The compound generator** is the most commonly used machine for supplying a multiple system, because this system inherently requires a practically constant voltage or one which may increase slightly with the load to make up for line drop. The compound generator meets these requirements very well as can be seen by referring to the curve of Fig. 93. For lighting and for power systems where the feeders are not very long, a flat-compounded or slightly overcompounded generator is most desirable. For the usual isolated plant in office buildings, a machine giving 230 volts at no load and 250 volts at full load has been found very satisfactory. For supply of railway systems or for motor loads in industrial applications where long feeders are required, a greater overcompounding is necessary, although this would ordinarily not exceed 10 per cent. Generators for all these applications are of course connected for cumulative compounding. Differentially compounded generators are not so common but they are used for arc welding sets to prevent heavy short circuits when starting the arc (Article 165). They are also used in special forms of automobile lighting generators (Article 164). Another application is for battery charging so that there will be the proper increase of generator voltage as the battery becomes charged and the current decreases. Special types of generators and their applications are discussed in Chapter XX.

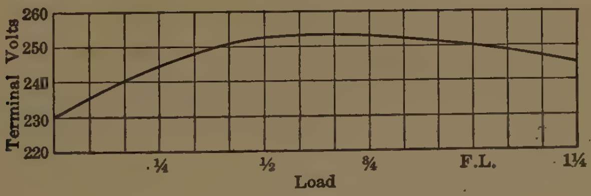

FIG. 93.—Compound Generator Regulation.

PROBLEMS ON CHAPTER XII

1–12. A four-pole generator running at 1200 r.p.m. has a flux per pole of 1,500,000 lines.

(*a*) What is the total flux cut by one conductor in one revolution?

(*b*) What is the average voltage induced in one conductor?

2–12. A 1-kw. two-pole generator has 228 conductors and runs at a speed of 2000 revolutions per minute. The flux per pole which is cut by the conductors is 1,600,000 lines. What is the e.m.f. generated?

3–12. The generator of Prob. 2–12 has conductors with an active length of 3 inches which are located 2.5 inches from the center of the shaft. A conductor travels 5 inches while cutting the entire flux under one pole. The density of the flux cut by the conductors is uniform under the entire pole and is 106,500 lines per square inch.

(*a*) What would be the time required for a conductor to move 1 inch?

(*b*) What is the total flux cut by one conductor in moving 1 inch under a pole?

(*c*) What is the value of e.m.f induced in the conductor while moving 1 inch as in (*b*)?

4–12. Referring to the machine of Prob. 2–12 and 3–12:

(*a*) Find the steady value of e.m.f. induced in a conductor while moving across a pole face. Determine by two methods.

(*b*) What will the induced e.m.f be when the conductor is between poles?

(*c*) What is the *average* e.m.f. of one conductor for an entire revolution?

5–12. Referring to Prob. 4–12 calculate the voltage between brushes. Calculate result by two methods.

6–12. An 8 kw. two-pole generator has 180 conductors. The flux per pole is 2,500,000 lines. At what speed (r.p.m.) must the generator be run to give an e.m.f.. of 125 volts?

7–12. A six-pole generator has a total of 218 conductors and is wave wound. The flux per pole is 1,190,000 lines. At what speed must the machine be run to generate 125 volts?

8–12. What would be the e.m.f. generated by the machine in Prob. 7–12 with a lap winding, and the same flux and speed?

9–12. A 1000-kw. sixteen-pole, lap-wound generator is to run at 90 r.p.m. There are 2304 conductors.

(*a*) What is the required flux per pole to give an e.m.f. of 500 volts?

(*b*) If the air-gap density is 8600 lines per square centimeter, what should be the area of each pole piece?

10–12. A four-pole, 125-volt flat-compounded generator has an armature wound with 47 coils, each coil having two turns. The coils are connected as a two-circuit or wave winding. The speed is 1800 r.p.m. What is the flux per pole at no load, normal voltage?

11–12. A given shunt generator has an induced e.m.f. of 125 volts when driven at 200 r.p.m. with a resistance of shunt field circuit of 5 ohms.

(*a*) If the speed were increased to 250 r.p.m. and the current in the field kept the same, what would be the e.m.f. generated?

(*b*) What must be the resistance of the shunt field circuit to keep the current constant as required in (*a*)?

12–12. The armature of an eight-pole generator has 1600 conductors. The area of the pole face is 500 square centimeters. Flux density in air gap 8000 gausses. Speed 600 r.p.m. What is e.m.f. induced, assuming a lap-wound armature?

13–12. A machine giving an e.m.f. of 220 volts at 1100 r.p.m. is to have its e.m.f. increased to 250 volts.

(*a*) If the speed were kept the same, how must the field flux be changed? What is this in per cent of normal?

(*b*) If the flux were kept normal how might the e.m.f. be changed? Express this in per cent.

14–12. A four-pole, 50-kw. generator gives an e.m.f. of 125 volts. There are 128 conductors in the armature, which is lap-wound.

(*a*) If the armature were reconnected for a wave winding, what would be the e.m.f. if the speed and flux were kept the same?

(*b*) What would be the current output of the generator with a wave-wound armature?

15–12. A 150-kw., six-pole, 250-volt, 450-r.p.m. generator has 496 armature conductors, lap-wound.

(*a*) If this were changed to a wave winding, what would be the terminal voltage?

(*b*) What would the current capacity and kilowatt rating be?

16–12. Data for a no-load saturation curve for a generator operating at 1800 r.p.m. are given below:

Amperes field	0	0.8	1.2	1.57	1.97	2.24	2.42	2.65
Volts armature	4	45	66.5	86	107	118	126	135

Calculate and plot values of volts armature for a speed of 1600 r.p.m.

17–12. The machine of Prob. 16–12 has 188 conductors and is four-pole, wave-wound. Each field spool has 842 turns, the area of pole face is 25 square inches. Calculate and plot values of flux density (lines per square inch) and ampere-turns per pole for a speed of 1800 r.p.m.

18–12. The following data apply to a separately excited generator:

	No load	Half load	Full load
Terminal volts	122	116.2	110
Terminal amperes	0	80	160

The resistance of armature including brushes is 0.05 ohm.

(*a*) Calculate the induced e.m.f. for the three different loads given above.

(*b*) If there were no armature reaction, what would be the terminal voltage for the three loads?

19–12. Referring to the machine of Prob. 18–12 what is the ratio of full-load to no-load flux.

20–12. The armature resistance of a 3-kw., 125-volt generator is 0.2 ohm, including brushes. The shunt field resistance is 70 ohms.

(*a*) What is the total e.m.f. required at no load and at full load to give rated terminal voltage?

(*b*) If the speed were kept constant, what change must be produced to keep the terminal voltage constant?

21–12. A 2-kw., 125-volt, shunt-wound generator has an armature resistance including brushes of 0.6 ohm. The resistance of the field circuit at full load is 100 ohms.

(*a*) What is the induced e.m.f. at full load?

(*b*) What is the percentage loss in the field?

22–12. A shunt-wound generator has an armature resistance of 0.12 ohm including brushes, and a field resistance of 30 ohms. The machine is adjusted to give 110 volts at no load, and then a load of 65 amperes is thrown on, which reduces the terminal voltage to 80 volts. What is the e.m.f. generated in each case?

23–12. The following data apply to a shunt-wound generator:

	No load	Half load	Full load
Terminal volts.......	151	141	125
Terminal amperes....	0	50	100

The shunt field current at no load is 3.6 amperes and the resistance of armature, not including brushes is 0.05 ohm.

(*a*) Calculate the generated e.m.f. for the three loads.

(*b*) What is the shunt field current for half and full load?

(*c*) If the shunt field current were adjusted to have the same value when machine is carrying full-load current, that it had at no load, what would the terminal voltage be at full load neglecting armature reaction?

24–12. Referring to the shunt generator of Prob. 23–12, assume that the shunt field current is changed so that the terminal voltage with full-load current is the same as the terminal voltage at no load:

(*a*) What would be the ratio of flux at full load to flux at no load?

(*b*) What would be the approximate value of the shunt field current required to produce a terminal voltage of 151 volts at full load?

(*c*) The observed shunt field current is 4.7 amperes instead of the calculated value found in (*b*). Give the reason for the difference.

(*d*) The generator has 515 turns on each shunt spool. How many additional ampere-turns per spool are required at full load to give 151 volts terminal voltage at full load?

25–12. A 2-kw., 125-volt series generator has an armature resistance of 0.6 ohm including brushes. The field resistance is 0.25 ohm.

(*a*) What is the total induced e.m.f. at full load?

(*b*) What is the drop across the field at full load?

(*c*) What is the loss of power in the field in per cent of full load output?

26–12. A series generator delivers the following voltages:

	No load	Half load	Full load
Terminal amperes....	0	57.5	115
Terminal volts.......	3	90	125

The resistance of the series field is 0.05 ohm and of the armature 0.05 ohm, not including brushes. Calculate the total voltage generated at half and full load,

27–12. A 7.5-kw., 125-volt, four-pole compound-wound generator has a shunt field resistance of 24 ohms, a series field resistance of 0.025 ohm and armature resistance of 0.042 ohm, not including brushes. The shunt field rheostat has a resistance of 10 ohms. Machine is flat compounded and connections are long shunt.

(*a*) What is the shunt field current a no load and at full load?
(*b*) What is the voltage at the brushes at full load?
(*c*) What is the total amount of power required for excitation at full load?
(*d*) Express (*c*) in per cent of power output of machine.

28–12. A 22.5-kw., 125-volt, four-pole, compound-wound generator has an armature resistance of 0.035 ohm, not including brushes, a series field resistance of 0.0325 ohm, and a resistance of shunt field circuit of 30 ohms. Shunt field is connected long shunt. The armature has 78 coils with two turns per coil, and is lap-wound. Speed 850 r.p.m.

(*a*) What is the flux per pole at no load for normal terminal voltage?

(*b*) What is the required flux per pole at full load assuming machine is flat compounded?

(*c*) What would be the required flux per pole at full load if the machine were over-compounded 10 per cent assuming the same load current?

29–12. A 110-kw., 550-volt compound-wound, long-shunt generator is over-compounded 10 per cent. Armature resistance 0.09 ohm, not including brushes, series field 0.04 ohm, shunt field circuit 100 ohms. Assuming that the external characteristic is a straight line, calculate values of the total characteristic for no load, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and full load current output.

30–12. The terminal voltage of a shunt generator at full load is 125 volts. When the load is thrown off the voltage rises to 135 volts with speed held constant. What is the voltage regulation of the machine?

31–12. Referring to the compound characteristic shown in Fig. 93.

(*a*) What is the voltage regulation?
(*b*) What is the per cent compounding?

32–12. A short-shunt compound generator rated at 1 kw., 125 volts, requires 1200 ampere-turns per spool at no load, normal voltage. When run at full load with the series field, there are required 1280 ampere-turns. What is the number of turns required for each series field spool?

33–12. The generator in Prob. 32–12 requires 1450 ampere-turns field excitation to give a terminal voltage of 137.5 volts at the same full load current as before. What number of turns is required in the series field to give 10 per cent overcompounding?

CHAPTER XIII

D.C. MOTOR CHARACTERISTICS

104. In a motor, electrical energy is converted into mechanical energy through the interaction of a magnetic field and of conductors carrying a current. Structurally a d.c. motor resembles a d.c. generator, and the same machine may be operated either as a generator or as a motor, although they usually differ in details of design because of different operating conditions. For example, generators are usually of the open type, with the armature and field windings exposed, whereas it is often necessary to use motors having a frame partially or entirely inclosed to protect the windings from injury.

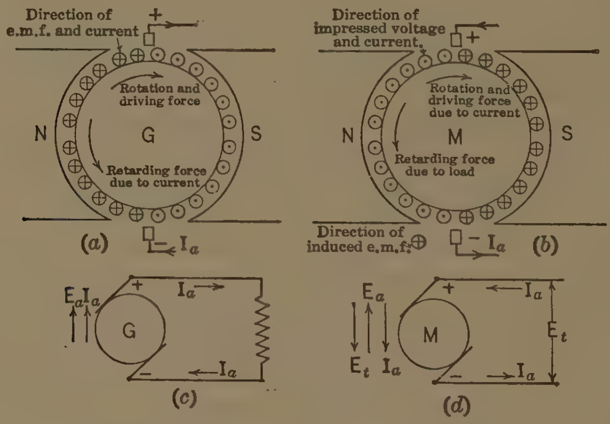
Direction of e.m.f. and current
+
Rotation and driving force
N
G
S
Retarding force due to current
(a)
I_a
Direction of impressed voltage and current
+
Rotation and driving force due to current
M
Retarding force due to load
Direction of induced e.m.f.
−
I_a
(b)
E_a I_a
+
I_a
G
−
I_a
(c)
E_a
+
I_a
E_t
M
E_t I_a
−
I_a
(d)

FIG. 94.—Generator and Motor Action.

105. Generator and Motor Action.—Assume that the machine in Fig. 94 *a* is driven by a steam engine in a clockwise direction. With the field polarity shown, the direction of the e.m.f. generated in the armature will be as indicated. If these conductors are connected in the usual manner through a commutator and brushes to an external load, current will flow in the load and the armature conductors. The direction of this current will be the same as the direction of the e.m.f., since the current is produced by this e.m.f. Because of this current, all the conductors under the influence of the field flux will have a force acting on them in a counter-clockwise direction. As long, therefore, as the external circuit is open and no current is flowing in the armature conductors, the driving force which must be supplied by the engine, in order to keep the armature turning, is only that necessary to overcome the friction and other losses in the armature. The direction of rotation will, of course, be the same as the direction of the turning force of the engine. As soon as current flows in the armature conductors, there is a force in the opposite direction, as explained above, and this force opposes the driving force and tends to slow down the machine. Normally, when this occurs, the engine governor admits more steam, and the engine produces a greater driving force to balance the increased retarding force due to the current. When the speed is not changing, the driving force due to the engine must equal the retarding forces due to the armature current and the losses in the machine. If the armature current increases, the retarding force increases, and the speed usually changes slightly until equilibrium has been re-established.

Suppose that a machine (Fig. 94 *b*), with field polarities as indicated, had its armature conductors supplied with current from an external source, and that this current were in the direction indicated by the symbols in the conductors under the poles. According to Article 45 the force on these conductors due to the current would tend to produce clockwise rotation of the armature. As soon, however, as the armature started to rotate, an e.m.f. would be generated in the armature conductors since they would be cutting flux. The direction of this e.m.f. would be the same as in Fig. 94 *a*, since the direction of rotation and polarity of field poles are the same. In other words, it makes no difference whether the conductors are rotated by a steam engine or by the force action of a current, the direction of induced e.m.f. is determined only by the field polarity and direction of rotation and follows the rule given in Article 55. In order, however, that the current shall flow in the direction indicated, the external voltage which produces this current must be in the same direction as the current. Hence, in this machine, there are two voltages, the impressed voltage E_t which forces the current through the armature and the e.m.f. generated in the armature E_a, which is in a direction opposed to the current. The latter is called a counter-electromotive force, abbreviated c.e.m.f. The machine in Fig. 94 *b* is, of course, a motor. The current and voltage relations in a motor and a generator are also shown in Fig. 94 *c* and *d*. It will be seen that in a generator there is a single voltage, the generated e.m.f. which acts in the direction of the current. In the motor, there are two voltages, the impressed or terminal voltage E_t, supplied from an external source, and the generated voltage or counter-electromotive force E_a, which acts in a direction opposed to E_t and the flow of current I.

106. Counter E.M.F. of a Motor.—It was shown in the preceding article that, in a motor, the induced e.m.f. in the armature E_a is opposed to the impressed voltage. In order that current shall flow through the armature in the proper direction to give motor action; i.e., produce rotation (Fig. 94 *b*), there must be a voltage available to overcome the resistance of the armature. This voltage is $e=I_aR_a$ where R_a is the resistance of the armature circuit, including brushes. We have seen that the only source of voltage of the proper polarity to force current through the armature in the required direction is the external voltage; hence, a portion of the impressed voltage must be available in order to overcome the armature resistance. If E_a were equal to E_t, they would exactly balance and no remainder would be left to overcome this resistance. If E_a were greater than E_t, then current would flow in the direction of E_a and the force action would be in the direction for a generator as in Fig. 94 *a*. If E_a were less than E_t, the remainder of the impressed voltage would force current in the direction indicated in Fig. 94 *b* and *d*, and the machine would be a motor. In any motor, therefore, the following equation must be satisfied,

$$E_t=E_a+I_aR_a=E_a+e. \quad \ldots \ldots \quad (32)$$

This is a very important equation and must always be satisfied, regardless of the speed or load conditions. It is apparent from this equation that e must increase and E_a decrease as the armature current increases, assuming a constant impressed voltage E_t. The resistance of the armature circuit of a motor is low, so that e is usually small and seldom exceeds 5 per cent of the terminal voltage. The counter e.m.f. E_a which is induced in a motor armature can be expressed by Equation (29) so that, for a particular motor,

$$E_a=K\Phi n. \quad \ldots \ldots \quad (33)$$

Example 1.—A 3-hp. shunt-wound motor takes a current of 23.4 amperes at 115 volts when carrying full load. The armature resistance, including brushes, is 0.31 ohm. The shunt field resistance is 150 ohms. The input to the motor at no load is 3.8 amperes. Calculate the counter e.m.f. at no load and at full load.

(*a*) At no load:

The field current is $I_f=\frac{115}{150}=0.77$ ampere.
The armature current is $I_a=3.8-0.77=3.03$ amperes.
The armature drop is $I_aR_a=3.03\times0.31=0.94$ volt.
The counter e.m.f. at no load is $E_a=115-0.94=114.06$ volts.

(*b*) At full load:

The armature current is $I_a=23.4-0.77=22.63$ amperes.
The armature drop is $I_aR_a=22.63\times0.31=7.02$ volts.
The counter e.m.f. at full load is $E_a=115-7.02=107.98$ volts.

107. Experimental Evidence of Motor Counter E.M.F.—With a motor, connected as shown in Fig. 95, the voltmeter will read the value E_t as long as switches *SS* are closed and power is furnished by the supply. As soon as these switches are opened, the armature current becomes zero as shown by the ammeter, but the voltmeter reading drops only slightly at first and then gradually decreases to zero as the machine slows down. At the instant that the switches are opened, the voltmeter reads E_a which would be nearly the same as E_t according to the preceding article.

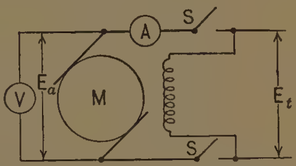

FIG. 95.—Experimental Evidence of Counter E.M.F.

Example 1.—If this test were applied to the motor of Example 1, Article 106, the voltmeter would read 115 volts while the switches were closed and the motor were carrying its full load. At the instant the switches were opened, the voltmeter reading would drop suddenly to 107.98 volts (E_a) and would then decrease slowly to zero as the motor slowed down.

108. Torque.—To produce rotation of a dynamo armature, a tangential force is necessary. This force, in the case of a motor, is developed inside the machine, due to the current, and, in the case of a generator, comes from an external source such as a steam engine or water wheel. The effect of the force is determined not only by its *magnitude* (pounds pull) but also by its arm or the radial distance at which it acts and is expressed by the product of force and arm. This product, called the torque, is represented by the equation

$$T=Fr, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where T is the torque expressed in pound-feet,[1] F is the force in pounds and r the radius in feet, at which the force acts. The horsepower resulting from a given torque depends upon the speed of the dynamo, so that

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[1] Not foot-pounds, which represents work.

$$\text{Horsepower} = \frac{2\pi n T}{550}, \qquad \ldots\ldots\ldots \quad (35)$$

where n is the speed in revolutions per second.

Example 1.—Two conductors (Fig. 49) are pivoted at a distance of 3.5 inches from the axis, and the force on each conductor, produced by the current, is 3 pounds. The torque for one conductor is, therefore,

$$T = \frac{3 \times 3.5}{12} = 0.88 \text{ pound-foot,}$$

and the total torque is 1.76 pound feet. If the conductors were rotating at 1200 r.p.m., the horsepower produced would be

$$\frac{2\pi \times 1200 \times 1.76}{60 \times 550} = 0.404 \text{ hp.}$$

109. Torque of a Motor.—The armature conductors for a motor are assembled in coils and connected to the commutator exactly as in a generator so that the current from the supply divides and passes through the several paths in the armature shown in Fig. 96 *a* and *b*. In the bipolar machine the current flows through two paths, the direction of current being opposite under the two poles. The forces produced by the current in the conductors are, therefore, all in the same direction as indicated by the arrows. When the conductor moves from one side of a brush to the other, the current in the conductor is reversed, and, since it then comes under the influence of the next pole which is of opposite polarity, the direction of the force remains the same. In the case of the lap-wound, four-pole machine (Fig. 96 *b*), the current divides into four parts since there are four paths through the armature. The current flows in opposite directions under poles of unlike polarity so that the total force due to the current

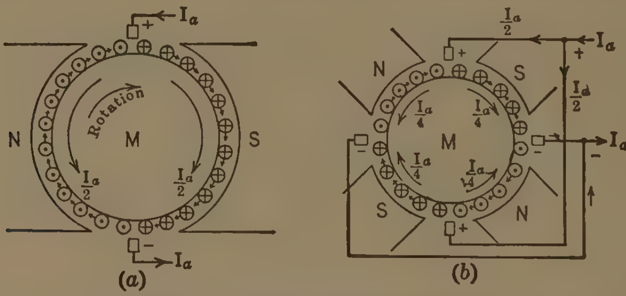

FIG. 96.—Current in a Motor Armature.

is the sum of the forces on the individual conductors, the same as for the bipolar machine. The total torque produced by the current can, therefore, be determined by adding the torque due to the individual conductors. By the use of Equation (17) it can be proved that the torque of a motor is expressed by the equation

$$T = K_t \Phi I_a, \qquad \ldots \ldots \ldots \qquad (36)$$

where Φ is the flux per pole and I_a is the total current supplied to the armature. The factor K_t includes the number of conductors, paths, and other factors which are constant for a particular machine.[2]

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[2] Equation (36) is derived as follows:

Let B = the average flux density in gausses;
L = length of pole face parallel to shaft, also length of active conductor in centimeters;
r = distance of conductor from center of shaft in feet;
p = paths in armature.

Then the current in one conductor is $I_a \div p$ amperes. The force on one conductor according to Equation (17) is, therefore,

$$F = \frac{BLI_a}{10 \times p \times 445{,}000} = \frac{BLI_a}{K'p} \text{ pounds,}$$

and the torque due to one conductor is

$$t = Fr = \frac{BLI_a r}{K'p} \text{ pound-feet.}$$

Let N = total conductors on the armature; and
K_1 = percentage of armature covered by the poles;
P = number of poles.

Then the circumference covered by one pole is $\frac{24\pi r K_1}{P}$ ins. The total flux per pole is

$$\Phi = \frac{24\pi r K_1}{P} \times \frac{L}{2.54} \times B \times 6.54 \qquad \ldots \ldots \ldots \qquad (37)$$

The total number of conductors under the poles at any instant is NK_1. Hence, the total torque due to the current is

$$T = NK_1 t = NK_1 \times \frac{BLrI_a}{K'p} \text{ pound-feet.} \qquad \ldots \ldots \ldots \qquad (38)$$

But

$$BLr = \frac{\Phi P}{60.96\pi K_1}.$$

Hence the torque is

$$T = \frac{N\Phi P I_a}{60.96\pi K'p} \text{ pound-feet,} \qquad \ldots \ldots \ldots \qquad (39)$$

and since for a particular machine N, P, and p are constants,

$$T = K_t \Phi I_a.$$

It should be noted that torque is independent of speed and, hence, is the same at standstill as when the motor is running, provided the current and flux are the same. Whenever the speed of a motor is not changing, the torque due to the armature current exactly equals the retarding torque caused by the losses in the motor, plus the external mechanical load. We shall see, in later articles, how a motor adjusts itself to care for varying load conditions.

110. Motor Action under Varying Loads.—Assume that a motor is supplied with a steady voltage E_t, that the field flux is constant, and that there is no external mechanical load on the motor. Then the retarding force, tending to stop the motor, is small and is due only to the friction and other losses in the motor armature. In order that the armature may continue to run at a fixed speed, there must be a torque produced by the armature current which is just equal to the retarding torque and, since this is small, the motor requires only a small armature current when running at no load. The amount of this current is determined by Equation (32). If an external mechanical load be applied to the motor, an additional retarding torque is produced and the motor at once tends to slow down because the torque due to this small armature current is sufficient to overcome only the friction load. Since we have assumed the field flux to be constant, it is evident, according to Equation (36), that the armature current must increase if sufficient torque is to be produced to balance the increased retarding torque due to the increased load. The only way in which more current can flow in the motor armature, since the impressed voltage E_t is constant, is for the counter e.m.f. E_a to decrease because I_aR_a must be greater with increased current. This follows from Equation (32). Therefore, in all cases when a motor must produce increased torque with constant flux and constant impressed voltage, the counter e.m.f. must decrease in order to allow more current to flow in the armature. According to Equation (33), $E_a = K\Phi n$; and, if the flux is constant, the only way in which E_a can be changed is to change the speed. Therefore, when additional load is thrown on the motor it slows down, E_a decreases, and this allows more armature current to flow. The motor will continue to slow down, and the armature current to increase, until enough current is flowing in the armature to produce the required larger torque to satisfy Equation (36). If the load is decreased, there is more torque than is required, and the motor increases its speed. This at once increases the counter e.m.f., which decreases the armature current sufficiently to produce the torque required for the smaller load. The motor then continues to run at a constant speed until the load changes again. In general, it may be said that any d.c. motor regulates its armature current for varying load requirements by variations in the speed. The required speed variation to accomplish this result depends upon the type of motor, as we shall see in later articles.

In the preceding discussion, we assumed that the field flux was constant. While this is not always the case, it is still a fact that the motor adjusts its counter e.m.f. to suit the load requirements, decreasing in value for increasing loads and, thereby, allowing more armature current to flow.

Example 1.—The motor of Example 1, Article 106, develops 3.27 hp. at full load when the armature current is 22.63 amperes, the counter e.m.f. 107.98 volts and speed 1150 r.p.m. The torque at full load according to Equation (35) is:

$$T_f = \frac{3.27 \times 550 \times 60}{2\pi \times 1150} = 14.9 \text{ pound-feet.}$$

At no load, the motor develops 0.34 hp. the counter e.m.f. is 114.06 volts, the armature current 3.03 amperes and the speed 1200 r.p.m. The torque is

$$T_0 = \frac{0.34 \times 550 \times 60}{2\pi \times 1200} = 1.5 \text{ pound-feet.}$$

If the load requirements are such as to call for a fixed armature current, then E_a will be constant. But $E_a = K\Phi n$ (Equation 33); hence, if Φ is decreased, the motor speed n must increase proportionally in order to keep E_a constant; similarly, increasing Φ would decrease the speed. It should be noted that in all cases the speed will adjust itself to produce the proper counter e.m.f E_a.

111. Power Relations in a Motor Armature.—The power delivered to a motor armature is $P_t = E_t I_a$ (Fig. 97 *a*). Evidently this power must be consumed by the losses in the armature or it must appear as mechanical power at the motor pulley. If we multiply Equation (32) by I_a we have:

$$E_t I_a = E_a I_a + I_a^2 R_a$$

$$P_t = E_a I_a + I_a^2 R_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Evidently the term $I_a^2 R_a$ represents the power lost as heat in the armature circuit and this is an electrical loss. Since this is the entire loss in the electrical circuit of the armature, the remainder of the power or $E_a I_a$ must be converted into mechanical power. The various steps are shown by the diagram (Fig. 97 *b*). The product of the armature current and the counter e.m.f. represents the total power converted from the electrical to the mechanical form. A portion of the mechanical power thus converted must be used to overcome the friction, hysteresis, and eddy current losses in the motor armature since these losses produce a mechanical drag on the armature. The remainder of the power, which is left after subtracting these losses, appears as power available at the pulley or shaft for driving the external load. At no load, the output at the pulley is zero, and the $I_a^2R_a$ loss is small so that practically the entire input to the motor armature is converted into mechanical power and furnishes the driving force to keep the armature rotating by balancing the retarding force due to the friction and other losses specified. In Article 110, it was shown that the motor automatically adjusts the current to produce the required torque. Looking at the matter from a power standpoint, the motor automatically adjusts itself so that the product of armature current and counter e.m.f. shall be just sufficient to supply the external mechanical load, plus the losses (friction, etc.) in the motor. We have stated that the total mechanical power developed in a motor armature is equal to E_aI_a. Actually, it is the product of the armature current and the portion of the impressed voltage E_t which is used to balance the counter e.m.f.; but, since this is equal numerically to E_a, the power is calculated by using the value of E_a.

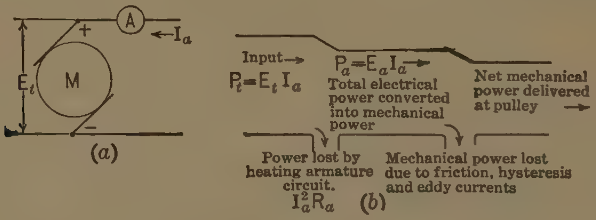
A
+
$\leftarrow I_a$
E_t
M
−
(a)
Input→
$P_t=E_tI_a$
$P_a=E_aI_a\rightarrow$
Total electrical power converted into mechanical power
Net mechanical power delivered at pulley →
Power lost by heating armature circuit.
$I_a^2R_a$
Mechanical power lost due to friction, hysteresis and eddy currents
(b)

FIG. 97.—Power Relations in a Motor Armature.

Example 1.—The motor of Example 1, Article 106, at full load has an input to the armature of

$$P_t=115\times22.63=2600 \text{ watts.}$$

The loss in brushes and armature winding is

$$P_e=(22.63)^2\times0.31=160 \text{ watts.}$$

The total mechanical power developed is

$$P_m=107.98\times22.63=2440 \text{ watts or } 3.27 \text{ hp.,}$$

or it is

$$P_m=2600-160=2440 \text{ watts.}$$

112. Summary of Motor Equations.—There are three important equations which are necessary when studying the performance of motors; these are repeated below for convenient reference.

(*a*) The voltage equation, (32), Article 106.

$$E_t = E_a + I_a R_a.$$

(*b*) The counter e.m.f. equation, (33), Article 106.

$$E_a = \frac{\Phi P N n}{p \times 10^8}$$

$$= K\Phi n.$$

(*c*) The torque equation, (36), Article 109.

$$T = K_t \Phi I_a.$$

THE SHUNT MOTOR

113. Running Performance.—If the load on a shunt motor is varied, the speed will decrease slightly as the load increases. In Fig. 98 is given a speed-load characteristic typical of shunt motors. It will be seen that the speed changes from 1720 r.p.m. at no load to 1655 r.p.m. at full load; that is, a drop of 3.9 per cent. In general, the change in speed between no load and full load is small, being usually about 5 per cent and seldom exceeding 10 per cent. The shunt motor, when running, would be connected as in Fig. 77. The equations of Article 112 can be used to determine the reason for the speed change shown on the curve (Fig. 98).

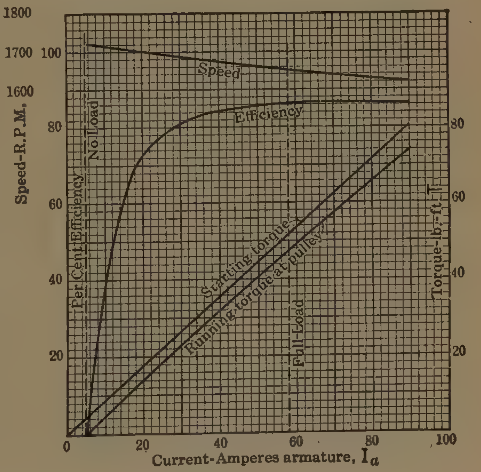

FIG. 98.—Performance Curves for a Shunt Motor, 15 hp., 220 volts.

Since the field winding is connected directly across the constant potential supply (Fig. 77), the field current is constant and, therefore, we should expect the flux to be constant. Assume, for the moment, that this is the case. An increased mechanical load requires that the motor shall develop an increased torque and, with the flux constant, according to Equation (*c*) of Article 112, this increased torque can be secured only by an increase of armature current. An increased armature current can be secured only by a decrease of the counter e.m.f.

E_a, according to Equation (a), Article 112, since E_t is constant. If the flux is constant, the required change in E_a can be accomplished only by a decrease in speed according to Equation (b), Article 112, and this speed change will be directly proportional to the required change in counter e.m.f.

Example 1.—The 15-hp., 220-volt motor of Fig. 98 has a no-load speed of 1720 r.p.m. and a no-load current input to motor of 6.07 amperes. The full-load current input to the motor is 59.27 amperes, and the field resistance is 173 ohms. Resistance of armature, including brushes, is 0.165 ohm. Assuming the flux constant, calculate the full-load speed.

The shunt field current is

$$I_f = \tfrac{220}{173} = 1.27 \text{ amperes.}$$

The armature current at full load is

$$I_a = I_t - I_f = 59.27 - 1.27 = 58 \text{ amperes.}$$

The armature drop is

$$I_a R_a = 58 \times 0.165 = 9.56 \text{ volts.}$$

The full-load counter e.m.f. is

$$E_a = E_t - I_a R_a = 220 - 9.56 = 210.44 \text{ volts.}$$

The no-load counter e.m.f. is

$$220 - (4.8 \times 0.165) = 219.2 \text{ volts.}$$

The full-load speed would be

$$\frac{210.44}{219.2} \times 1720 = 1648 \text{ r.p.m.}$$

But, according to the actual test curve of this motor (Fig. 98), the full load speed is 1655 r.p.m.; hence, it is apparent that our assumption of constant field flux was not exactly correct and the flux decreases slightly with increase of load so that the required decrease of counter e.m.f. from 219.2 volts at no load to 210.26 volts at full load is actually accomplished partly by a decrease in field flux Φ and partly by a decrease in speed. This decrease in flux is due to armature reaction (Article 138) which weakens the field; hence, in a shunt motor, armature reaction tends to raise the speed and make the full-load speed more nearly equal to the no-load speed. Even with commutating poles, there is a tendency for the flux to decrease with increase in load so that inherently these machines have less speed change than the non-commutating pole type. With commutating pole machines it is sometimes customary to use a stabilizing winding, which is a light series winding on the main poles, so connected as to aid the shunt field. This increases the excitation slightly with increase of load, thus requiring a greater decrease in speed, in order to satisfy Equation (*b*), Article 112. This obviates any tendency of the motor to have unstable speed at heavy loads.

The change in speed of a motor from no load to full load is called the speed regulation. If expressed in per cent, it is

$$\text{Per cent speed regulation} = \frac{\text{No-load speed} - \text{full-load speed}}{\text{No-load speed}} \times 100.$$

Example 2.—Referring to the curve, Fig. 98, the speed regulation is

$$\frac{1720 - 1655}{1720} \times 100 = 3.9 \text{ per cent.}$$

114. Starting Torque.—When starting a shunt motor, the field is directly across the line and a resistance (R_S, Fig. 99) is connected in series with the armature. The field flux is, therefore, at full strength, except for the slight change which may be produced by armature reaction. The starting torque, therefore, is practically proportional to the armature current according to Equation (*c*), Article 112. This is shown by the curve (Fig. 98). The current which flows in the armature at standstill is determined by the resistance of the armature circuit, that is, $R_S + R_a$, since there can be no counter e.m.f. when the armature is stationary. The field flux, with full-load current in the armature, is the same whether the motor is at a standstill or is running and carrying its full load; hence, the starting torque with full-load current is the same as the total full-load running torque. To produce a starting torque 50 per cent greater than the full-load running torque would require a current 50 per cent greater than the full-load current.

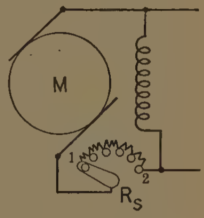

FIG. 99.—Starting a Shunt Motor.

115. The Starting Resistance.—The armature of a shunt motor has such a small resistance that an excessive current would flow if it were connected directly to the supply when the armature was not turning. Since there is no counter e.m.f. generated until the machine starts, the current is determined entirely by the armature drop I_aR_a.

Example 1.—Referring to the 15-h.p., 220-volt motor of Example 1, Article 113, the armature resistance is 0.165 ohm. If thrown directly on the line, without external resistance, the current would be $\frac{220}{0.165} = 1330$ amperes or about 23 times full-load current, which would probably cause the machine to flash over at the commutator and produce a short circuit. To prevent an excessive rush of current, additional resistance R_S must be inserted temporarily in the armature circuit (Fig. 99.) The amount of additional resistance is so chosen as to limit the starting current to a safe value; usually about 150 per cent of full-load current.

Example 2.—If the motor of Example 1 is to have the starting current limited to 150 per cent of full-load current, find the required external resistance R_S.

The full-load armature current, according to Example 1, Article 113. is 58 amperes

The allowable starting current is

$$58 \times 1.50 = 87 \text{ amperes.}$$

The total armature resistance required is

$$\tfrac{220}{87} = 2.52 \text{ ohms.}$$

Hence the external resistance is

$$R_S = 2.52 - 0.165 = 2.36 \text{ ohms.}$$

Shunt motors not larger than about $\frac{3}{4}$ hp. may be thrown directly onto the circuit without a starting rheostat as their resistance is relatively high and there is not as much danger of damage to the machine. As soon as the motor starts to rotate, a counter e.m.f. is developed, and the armature current is decreased. In order, therefore, to maintain the starting current at approximately constant value until the motor and its driven load have been brought to normal speed, the resistance R_S (Fig. 99) must be decreased as the motor speeds up and the counter e.m.f. increases. This is accomplished by a motor starter or starting box which contains the required armature resistance R_S arranged with a number of contacts in such a manner that the resistance can be cut out in steps as the speed increases. The motor is started by closing the line switch and moving the lever to the first contact (1, Fig. 99). As the motor speed increases, the arm is moved slowly to the right. When running, the arm is on point 2, where all the external resistance is cut out.[2]

116. Speed Adjustment by Field Control.—The speed of a shunt motor which is driving a definite load can be adjusted by changing either the field flux or the voltage applied to the armature. This follows from the equations in Article 112. If the load requirements are such that the armature current is constant, then, according to Equation (*a*) the counter e.m.f. E_a must be constant. Hence, by Equation (*b*), the speed n must change inversely as Φ in order that E_a shall remain constant. Therefore, the speed of a shunt motor increases when the field flux is decreased, and the speed decreases when the field strength is increased. The common method of changing the strength of the field flux is by varying the shunt field current, using a rheostat in series with the field winding (Fig. 100). A 15 per cent speed adjustment can be secured on ordinary shunt motors designed for constant speed operation. Motors, especially designed for the purpose, may be secured to give as much as a 4-to-1 variation between maximum and minimum speeds. When the speed of a shunt motor is adjusted in this way, the speed regulation is nearly the same for different values of field current, as will be seen from Fig. 101. This constant-speed characteristic of a shunt motor, when speed adjustment is secured by field control, is of importance in many applications where it is necessary to change the speed to suit different operating conditions, but where, having once adjusted the speed, it is important to maintain nearly constant speed from no load to full load. For example, in the case of a lathe, it is necessary to adjust the speed to suit the diameter of the piece being machined, but it is also important that the speed shall not change greatlv when the depth of cut is varied.

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[2] Motor starters and controllers are described in Chapter XXI.

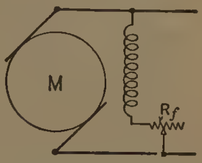

FIG. 100.—Speed Adjustment by Field Control.

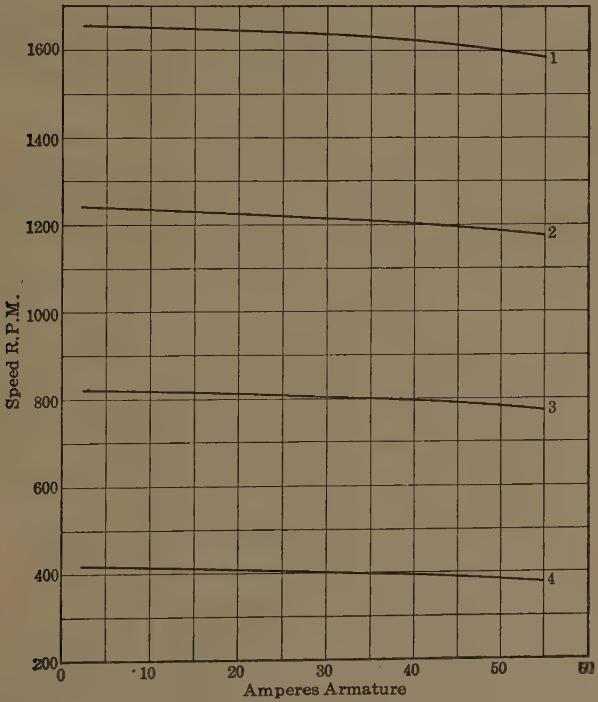

FIG. 101.—Speed Characteristics for an Adjustable Speed Shunt Motor.

The available horsepower output of an adjustable speed motor is the same at all speeds. This can be seen from the following consideration. The allowable ampere input to the motor armature depends upon the heating and is nearly constant at all speeds, being slightly greater at high speed, because of the greater air circulation. But, according to Equation (*c*), Article 112, the torque will be less at the high speed because Φ is decreased. The horsepower, however, depends upon speed as well as torque, and since Φ varies inversely as the speed, the horsepower is the same at high or low speeds. The motor must be built large enough to deliver the rated horsepower at the low speed and, hence, its weight and cost is greater than for a motor designed to deliver the same horsepower at the highest speed only.[3]

Example 1.—A 10-hp., 220-volt motor has an armature current of 39.3 amperes when delivering rated load at a speed of 400 r.p.m. Assume that the field flux is halved by changing the field excitation and that the armature current remains the same

(*a*) Calculate the speed with the weak field.

(*b*) Calculate the torque and horsepower at the two speeds.

(*a*) Since I_a is assumed to remain constant the armature drop is constant and the counter e.m.f. E_a will be the same at the two speeds. (See Equation (*a*), Article 112.) Hence, if E_a is to be constant, the speed will be double or 800 r.p.m. if the flux is halved. (See Equation (*b*), Article 112.)

(*b*) The horsepower at 400 r.p.m. is given as 10 hp. and the torque at that speed is

$$T=\frac{\text{hp.}\times 550}{2\pi\times\text{r.p.s.}}=\frac{10\times 550\times 60}{2\pi\times 400}=131\text{ pound-feet.}$$

At 800 r.p.m. the flux is halved and therefore the torque is halved since the armature current is the same. (See Equation (*c*), Article 112.) Hence the torque at 800 r.p.m. is 131 ÷ 2 = 65.5 pound feet. The horsepower at 800 r.p.m. is

$$\frac{2\pi\times 800\times 65.5}{550\times 60}=10\text{ hp. as before.}$$

The speed of a shunt motor can also be changed by varying the reluctance of the magnetic circuit, while the exciting current is kept constant. This changes the flux and gives the same effect as a corresponding change in field current. In practice, the magnetic reluctance is varied either by sliding the armature part way out from between the field poles (Fig. 84) or by moving a portion of the field cores radially. Either method requires that the motor have special mechanical features which are expensive. Since the same result can be secured much more simply and easily by means of a field rheostat, the variable reluctance type of motor is seldom used.

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[3] The effect of speed on cost and weight of a motor is discussed in Chapter XXXIX.

117. Speed Adjustment by Armature Control.—If a rheostat were connected in series with the armature (Fig. 102) of a shunt motor, the speed could be varied because the voltage across the armature terminals would be changed. The shunt field would be connected directly to the supply, and, therefore, the field excitation would be constant, and Φ also would be constant except for the small change due to armature reaction. Assuming Φ constant and I_a also constant, the speed would be proportional to the voltage impressed on the armature (Equations (*a*) and (*b*), Article 112).

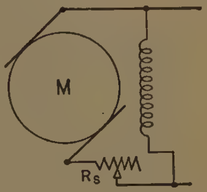


FIG. 102.—Speed Adjustment by Armature Control.

Example 1.—A 10-hp., 220-volt motor running at 1600 r.p.m. takes an armature current of 42 amperes and a field current of 0.3 ampere. The armature resistance, including brushes, is 0.38 ohm. Calculate the resistance which must be inserted in the armature circuit to reduce the speed to 800 r.p.m. with the same armature current.

The field strength remains constant since I_a is constant, therefore, if the speed is halved, the counter e.m.f. will also be halved.

The drop in the armature circuit is

$$I_aR_a = 42 \times 0.38 = 15.5 \text{ volts.}$$

The counter e.m.f. at 1600 r.p.m. is

$$E_a = E_t - I_aR_a = 220 - 15.5 = 204.5 \text{ volts.}$$

The counter e.m.f. at 800 r.p.m. is

$$204.5 \div 2 = 102.3 \text{ volts.}$$

The drop in the armature is the same as at 1600 r.p.m. since the current is the same; hence, the external resistance, R_s (Fig. 102), must be sufficient to consume the remainder of the terminal voltage. The drop across this resistance is, therefore,

$$e = 220 - (102.3 + 15.5) = 102.2 \text{ volts.}$$

Therefore,

$$R_S = \frac{102.2}{42} = 2.44 \text{ ohms.}$$

When the speed of a shunt motor is adjusted by armature control, the speed regulation is very poor if there is considerable additional resistance in the armature circuit.

Example 2.—Referring to the 10-hp., 220-volt motor of Example 1, the no-load armature current is 2.2 amperes and the no-load speed is 1656 r.p.m. when there is no external resistance (R_s, Fig. 102) in the armature circuit. Calculate the speeds at one-half, and one-quarter of full-load current and also at no-load current when a resistance, $R_s = 2.44$ ohms, is in the armature circuit.

The total resistance in the armature circuit is

$$R_a + R_S = 0.38 + 2.44 = 2.82 \text{ ohms.}$$

For half load or 21 amperes, the drop in the circuit is

$$21 \times 2.82 = 59.2 \text{ volts.}$$

Hence the counter e.m.f. must be

$$220 - 59.2 = 160.8 \text{ volts,}$$

and since the field flux is practically constant, the speed will be proportional to the counter e.m.f.'s.

The speed at half load is then

$$S = \frac{160.8}{204.5} \times 1600 = 1260 \text{ r.p.m.}$$

For one-quarter load or 10.5 amperes, the drop is

$$10.5 \times 2.82 = 29.6 \text{ volts.}$$

The counter e.m.f. is

$$220 - 29.6 = 190.4 \text{ volts,}$$

and the speed at one-quarter load is therefore

$$S = \frac{190.4 \times 1600}{204.5} = 1490 \text{ r.p.m.}$$

For no-load or 2.2 amperes the drop is

$$2.2 \times 2.82 = 6.2 \text{ volts.}$$

The counter e.m.f. with the resistance in the armature circuit is

$$220 - 6.2 = 213.8 \text{ volts.}$$

The counter e.m.f. with 2.2 amperes and no external resistance is

$$E_a = 220 - (2.2 \times 0.38) = 219.16 \text{ volts.}$$

We will use this value and the corresponding speed of 1656 r.p.m. rather than 204.5 volts and a speed of 1600 r.p.m. because the former represents the same no-load flux conditions; whereas, the latter corresponds to full-load flux which is slightly different in value. The speed at no load with the external resistance is then

$$S = \frac{213.8}{219.2} \times 1656 = 1610 \text{ r.p.m.}$$

Comparing these results, we see that, without external armature resistance, the speed regulation is good, being 1600 to 1656 or 3.4 per cent; whereas, with an external resistance of 2.44 ohms, the regulation is very much poorer, being 800 to 1610 r.p.m. or 50 per cent. The curves (Fig. 103) show the variation at other loads. The adjustment of full-load speed to 800 r.p.m. could have been made by increasing the shunt field current (field control) instead of by inserting armature resistance. If this were done, the speed regulation would be very much better, as is seen by the curve (Fig. 103 *b*). In this case the regulation is 800 to 820 r.p.m. or 2.5 per cent, as contrasted with 50 per cent with armature control.

A comparison of the methods of speed adjustment of a shunt motor which have just been described shows the superiority of the field control method. The adjustment is accomplished by means of a small rheostat in the field circuit; whereas, the armature control method requires a bulky rheostat which wastes a large amount of power. Hence, the field control method is much more efficient if rated horsepower is to be delivered by the motor at the different speeds.

Example 3.—With field control of the 10-hp. motor of Examples 1 and 2, above, the shunt field current is 0.3 ampere at 1600 r.p.m. and 0.66 ampere at 800 r.p.m. Compare the efficiency of the motor when carrying the full-load armature current of 42 amperes with the speed reduced to 800 r.p.m. by the two methods. The input to the motor is substantially the same with either method, being as follows:

Field control, 800 r.p.m.,

$$220(42+0.66)=9385 \text{ watts.}$$

Armature control, 800 r.p.m.,

$$220(42+0.3)=9306 \text{ watts.}$$

With field control, the flux is doubled to give 800 r.p.m.; hence, the torque is doubled and the output is 10 hp. the same as at 1600 r.p.m. With armature control the field flux is constant, corresponding to 0.3 ampere field and, hence, the torque is the same at 800 r.p.m. as at 1600 r.p.m. But, since the speed is halved, the output is only 5 hp. in this case.

For field control, the efficiency at 800 r.p.m. is

$$\frac{746\times 10}{9385}\times 100=79.5 \text{ per cent.}$$

For armature control, the efficiency at 800 r.p.m. is

$$\frac{746\times 5}{9306}\times 100=40 \text{ per cent.}$$

With armature control, the efficiency is low because a large proportion of the total power is lost in the external resistance R_S.

The field control method also gives much better speed regulation as will be seen by comparing the curves (Fig. 103) and the results in Example 2. For these reasons, field control is more generally used and armature control is employed only when an occasional decrease of speed is required or where the load drops off rapidly with decrease in speed, as is the case with fans and blowers. A combination of field and armature control is common for motors driving fans.

Instead of an armature resistance, a multi-voltage supply system is sometimes employed in order to make possible variation in armature voltage without the low efficiency and poor speed regulation which occurs when a resistance is used. This system is described in Article 163.

118. Shunt Motor Characteristics.—Summarizing the conclusions arrived at in the preceding articles, we can say that the shunt motor is essentially a constant speed device and that the speed can be adjusted over a considerable range and still maintain its constant speed characteristic. The motor can be made a varying speed machine (i.e., speed varying widely with changes in load), but this is accomplished at reduced efficiency. The starting torque is practically proportional to the load current so that the machine is adapted for starting fairly heavy loads.

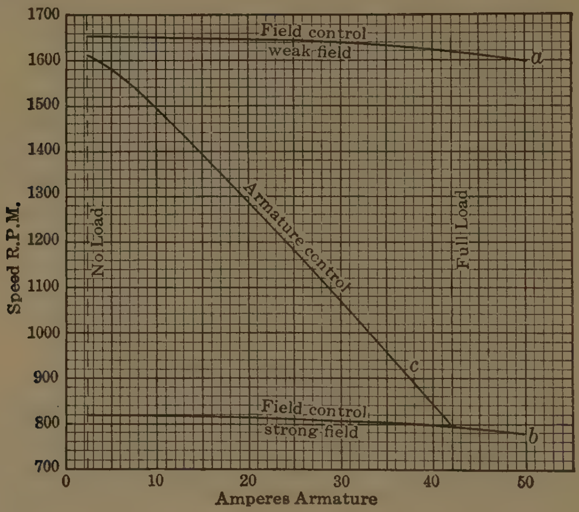


FIG. 103.—Comparison of Methods of Speed Control.

THE SERIES MOTOR

119. Running Performance.—A series motor, when running, is connected as shown in Fig. 104. Evidently, therefore, the series field must carry the armature current and, like the series generator, consists of a small number of turns of relatively large size conductor. As the load on a series motor changes, the speed varies widely, an increased load causing a decrease in speed (Fig. 105). Because of its speed characteristic, the series motor is called a varying speed machine. The reason for the performance shown on the curve can be derived by the aid of the equations in Article 112. With increased load on the motor, the current must increase, both because greater torque is required and because a greater power input to the motor is necessary; and, since the voltage is constant, an increased input necessitates increased current. To allow the required additional current to flow in the armature, the motor counter e.m.f. must decrease, according to Equation (*a*), Article 112. If the field flux remained constant, the necessary decrease in E_a could be accomplished by a slight decrease in speed, as in the case of the shunt motor. The flux, however, does not remain constant but increases almost proportionally to the increase in

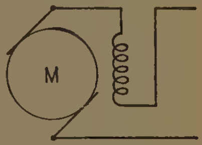


FIG. 104.—Connections of a Series Motor.

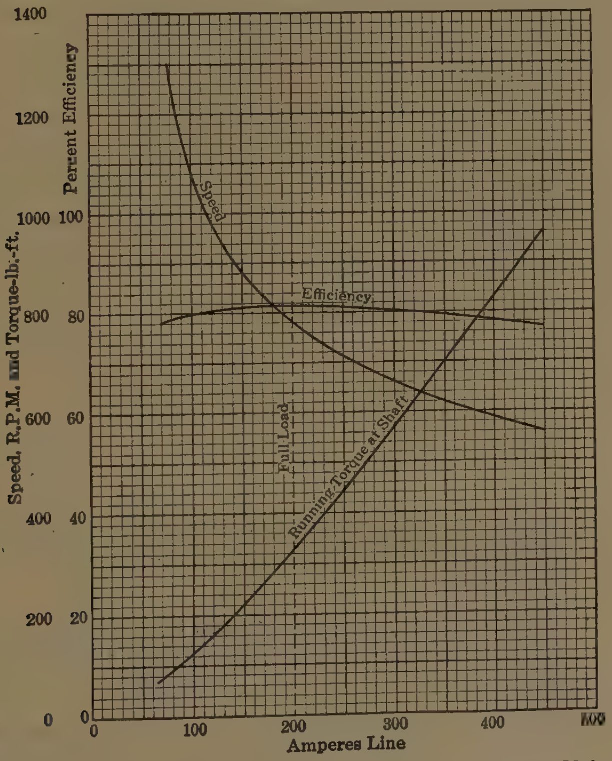


FIG. 105.—Performance of a Series Motor. 50 Hp., 230 Volts.

current. Hence, the speed must decrease still further to make up for the increase in flux.

Example 1.—A 25-hp., 500-volt series motor runs at 1150 r.p.m. when the current is 15 amperes. The resistance of the armature, including brushes, is 0.44 ohm and of the series field is 0.56 ohm.

(*a*) If the flux were constant, calculate the speed when the machine is carrying 45 amperes.

The total resistance is

$$R = 0.44 + 0.56 = 1.0 \text{ ohm.}$$

The counter e.m.f. with 15 amperes is

$$E_a = 500 - (1 \times 15) = 485 \text{ volts.}$$

The counter e.m.f. with 45 amperes is

$$E'_a = 500 - (1 \times 45) = 455 \text{ volts.}$$

If the flux were constant, the speed at 45 amperes would be

$$\tfrac{455}{485} \times 1150 = 1080 \text{ r.p.m.}$$

(*b*) But we know that the field flux of a series motor increases with increased load. If the flux at 45 amperes is 60 per cent greater than at 15 amperes, the speed must drop below 1080 r.p.m. in order to keep the counter e.m.f. at 455 volts and allow 45 amperes to flow.

Using Equation (*b*), Article 112, we have,

$$485 = K\Phi \times 1150;$$

also

$$455 = K \times 1.6\Phi \times n.$$

Hence,

$$n = \frac{455 \times \Phi \times 1150}{485 \times 1.6}$$

$$= 675 \text{ r.p.m.}$$

Therefore, with this series motor, when the flux increases 60 per cent the speed must drop to 675 r.p.m. in order to decrease the counter e.m.f. sufficiently to allow 45 amperes to flow.

When the load on a series motor decreases, the field strength decreases, and the motor speed must increase in order to generate the required counter e.m.f. When the load becomes very small, the speed rises to such a point that there is danger of the armature windings being thrown out of the slots and the machine being wrecked; hence, a series motor should never be belted to its load as it would reach a dangerous speed if the belt should break or slip off the pulley. Instead, a series motor is always direct-connected or geared to its load.

120. Starting Torque.—A resistance must be used to limit the current when starting a series motor (Fig. 106), because the armature resistance is low and there is no counter e.m.f. until the armature begins to turn. The requirements in this respect are, in fact, the same as for a shunt motor (Article 115). Unlike the shunt motor, however, the field current of the series motor varies with the load, and is nearly proportional to the armature current until the magnetic circuit approaches saturation. But torque is proportional to flux and armature current, and, therefore, in a series motor the torque is approximately proportional to I_a^2, hence it increases rapidly with increased current. This is shown by the curve (Fig. 105). Full-load current in the motor produces the same flux and, therefore, the same torque at starting as is obtained when the machine is running at normal speed and carrying full load. To obtain a starting torque 50 per cent greater than full-load torque requires only about 1.3 times full-load current; whereas, a shunt motor requires 1.5 times full-load current to produce 1.5 times full-load torque. For this reason a series motor is especially well adapted for starting heavy loads. A comparison of the starting torque of a series

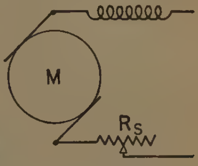


FIG. 106.—Connections for Starting a Series Motor.

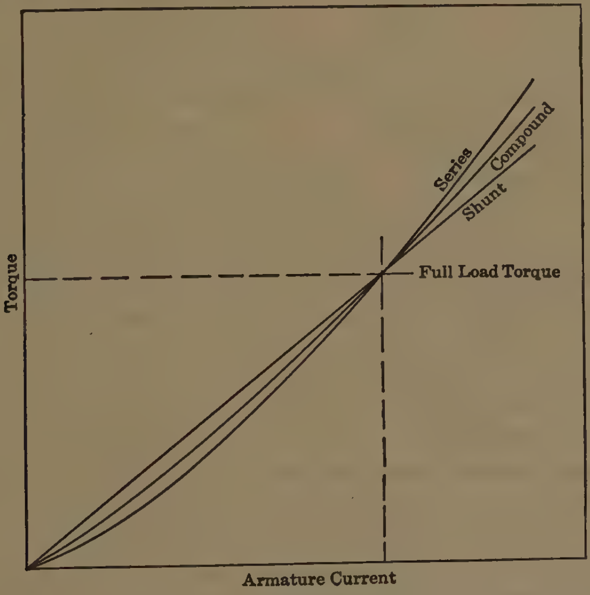
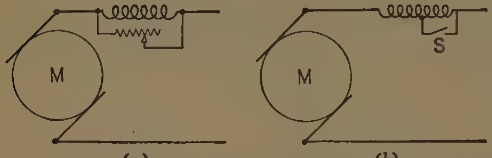


FIG. 107.—Comparative Starting Torque of D. C. Motors.

and a shunt motor, each having the same horsepower and speed rating, is given in Fig. 107. The torque for a given current is the same for both motors at full load because they have the same rating. At heavy loads, the torque for a given current is greater in the case of the series motor. The starting resistance is arranged to be cut out in steps, the same as for a shunt motor. (See Article 115.) Series motors as large as 5 to 8 horsepower may be successfully started by throwing them directly on the line without a starting resistance.

121. Speed Adjustment.—A series motor may have its speed for a particular load changed by armature control, using a series resistance (Fig. 106). Increasing the resistance decreases the speed the same as for a shunt motor since, for a constant armature current, the field strength is constant and, therefore, the speed must be reduced to generate the required counter e.m.f. (See Article 110.) The speed of a series motor carrying a particular load may be increased by field control in two ways. In Fig. 108 *a* the field flux is decreased by shunting off a portion of the current, thus decreasing the excitation and weakening the flux. The motor must then speed up in order to generate the necessary counter e.m.f. Instead of decreasing the current in the field, the number of turns through which the load current passes may be decreased (Fig. 108 *b*). Closing switch *S* weakens the field and causes the motor to run at a higher speed. Armature control is commonly used to adjust the speed of crane or railway motors, and field control is used to a limited extent for increasing the speed of railway motors.



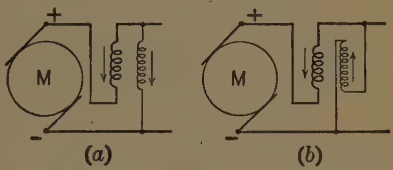
(*a*) (*b*)

FIG. 108.—Methods of Adjusting Speed of a Series Motor by Field Control.

122. Series Motor Characteristics.—Summarizing the conclusions of the preceding articles, we can say that the series motor is a varying speed device with no definite no-load speed. The motor is capable of developing a large starting torque, which is approximately proportional to the square of the current. The series motor is commonly used for crane, hoist, and railway service because of these characteristics. A shunt motor would not be well adapted for railway service because of the increased torque required when the car is climbing a hill. A shunt motor, if used on an electric car, would produce the large torque required at practically constant speed so that the car would run at nearly the same speed up hill that it did on the level. This would cause a very heavy demand on the power house and would require a large motor. The series motor, however, slows down considerably when the car is climbing a hill so that the required torque is developed at a much lower speed. This makes the demand on the power house smaller and permits the use of a smaller motor.

THE COMPOUND MOTOR

123. Running Performance.—The usual arrangement of connections for this motor is such that the series winding aids the shunt winding (Fig. 109 *a*) so that the field flux increases with increase of load. This arrangement is known as a cumulative compound motor; whereas, if the series field opposes the shunt, it is a differential motor. The latter arrangement is seldom used, so that the term "compound motor" is generally understood to apply to the cumulative connection. A typical speed-load curve for such a motor is shown in Fig. 110, curve *a*. The characteristic for a shunt motor of the same horsepower output and full-load speed is also shown (curve *b*). It will be seen that the speed regulation of the compound motor is poorer than the shunt motor although the variation is not so great as for a series motor. The reason for the difference between the shunt and compound characteristic can be found by the aid of the equations of Article 112. At full load both motors have the same field flux, which is practically the same as at no load in the case of the shunt motor as was explained in Article 113. In the case of the compound motor, the full-load excitation is equal to that of the shunt motor, but it is produced partly by the series winding and partly by the shunt winding. Hence, at no load, when there is practically no excitation due to the series winding, the flux is considerably decreased, and we have found (Article 116) that if the field flux of a motor is decreased the speed will increase. Therefore, the compound motor at no load is running with a weaker field than the shunt motor, and hence the speed of the compound motor is higher at no load, as can be seen from Fig. 110. On overloads, the speed of a compound motor falls off more rapidly than a shunt motor, which is an advantage in some motor applications.



(*a*) (*b*)

FIG. 109.—Connections of a Compound Motor. (*a*) Cumulative; (*b*) Differential.

A differential compound motor (Fig. 109 *b*) would have a closer speed regulation than a shunt motor because, as the load increased, the field would be weakened instead of strengthened, and this would tend to hold the speed more nearly constant. A differential connection is seldom used because a shunt motor will give regulation sufficiently close for most requirements, and because the differential motor has a tendency to unstable speed when heavily loaded.

124. Starting Torque.—In starting a compound motor, the shunt field would be directly across the line (Fig. 111) and a resistance R_S would be placed in the armature circuit. The starting would be the same as has already been described for the shunt and series motors. According to Equation (*c*), Article 112, the torque depends upon the flux and the armature current. The flux Φ evidently increases as the current increases because of the effect of the series winding. Hence, the starting torque would increase more rapidly than it did for the shunt motor but less rapidly than for the series motor. The curves of Fig. 107 give this comparison. They represent the performance of three motors having the same horsepower rating at the same speed; hence, the torque with full-load current flowing will be the same for

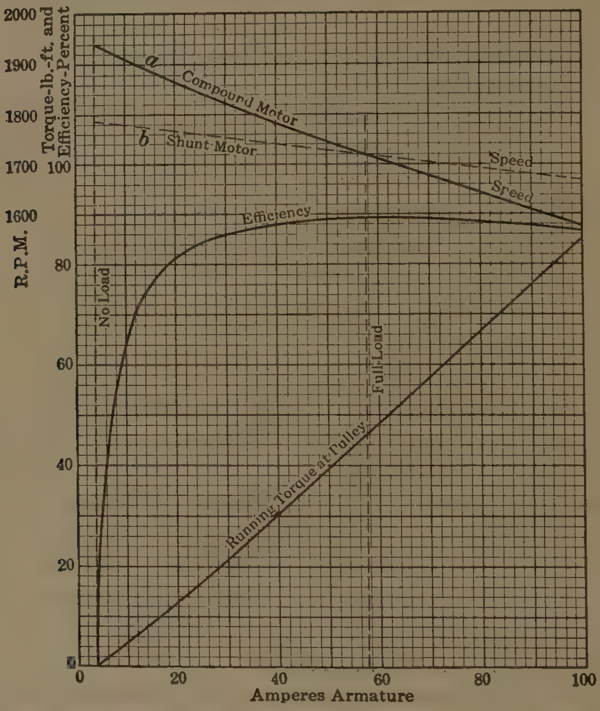


FIG. 110.—Performance of a Compound Motor 15 Hp.. 220 Volts.

all three machines. The starting torque with full-load current in the armature will be the same as the full-load running torque so that point *a* on the curves represents the starting torque for full-load current. It has already been shown that the starting torque of a shunt motor is proportional to the load current; hence, its torque is represented by a straight line (Fig. 98). The series motor has a weak torque for small current, but it increases approximately as the square of the load current. It is less than the shunt motor for currents less than full load, but greater than the shunt motor for larger currents. The torque curve for the compound motor lies between the shunt and series (Fig. 107), since, at less than full load, the strength of field for a given armature current is less than the shunt but more than the series. Above full-load current, the compound motor has a field weaker than the series but stronger than the shunt.

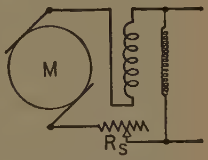


FIG. 111.—Starting a Compound Motor.

125. Speed Adjustment.—The speed of a compound motor can be adjusted either by armature control or by field control. The method of doing this was fully described in connection with the shunt motor (Articles 116 and 117). With field control, approximately the same speed regulation is secured after the speed has been adjusted, as in the case of the shunt motor. With armature control, there is a lower efficiency and a poorer speed regulation as there was with the shunt motor.

126. Compound Motor Characteristics.—Summarizing the preceding paragraphs, we find that the compound (cumulative) motor is, essentially, a compromise between a shunt and a series motor. Its speed varies more than a shunt motor, but, on the other hand, it has a definite no-load speed so that the entire load can be thrown off, if necessary, without danger to the motor. The amount of speed change from no load to full load depends upon the strength of the series field; a motor with a weak series winding, having a speed regulation similar to that of a shunt motor, while a machine with a strong series winding would approach more nearly the characteristics of a series motor. The starting torque obtainable with a compound motor is greater than for a shunt motor; hence, the compound motor is employed where large starting torque is required, but where there must be a limit to the running speed as is the case with elevators.[1]

127. The Multivoltage System of Speed Control.—In some cases, the range of speed adjustment possible with field control (see Article 116) is not sufficient to meet the requirements, and a multivoltage supply is used. This system is ordinarily applied to shunt or compound motors. The shunt winding may be kept permanently connected to a source of constant potential while the armature is supplied by a varying voltage to meet the special requirements. Evidently, in this case, the speed for a given load current will be approximately proportional to the impressed voltage for the reason already stated in Article 106. Hence, any speed within the mechanical limits of the machine can be secured by applying the proper voltage to the armature. In some cases, this voltage is obtained from a special generator and can be varied gradually over the entire range. This arrangement, known as the Ward-Leonard system (see Article 163), is very flexible and gives a wide speed range, but is expensive because a special generator is required to supply the motor. It is used, however, for reversing mill drives in steel mills and sometimes for large mine hoists.

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[1] See Chapter XXXIX for applications.

128. Effect on a Motor of Change in Supply Voltage.—The motors described earlier in this chapter are all intended to operate from a constant voltage supply. In practice, however, there may be considerable variation in this supply voltage due to the varying voltage drop on the feeders, caused by fluctuations in the load. The effect of these voltage variations upon the performance of motors is, therefore, of practical importance.

As the terminal voltage of a shunt motor decreases, the shunt field current decreases, thus decreasing the flux. But according to Equation (*a*), Article 112, if the terminal voltage E_t decreases, then the counter e.m.f. E_a must decrease, since the armature drop I_aR_a is constant for a given load. Because I_aR_a is small, the required decrease in E_a is nearly proportional to the decrease in E_t. If the flux decreased proportionally to the decrease in terminal voltage, then, since $E_a = K\Phi n$ (Equation (*b*), Article 112), the counter e.m.f. would decrease proportionally to the change in terminal voltage. Hence, the speed would not have to change in order that the equation $E_t = E_a + I_aR_a$ shall be satisfied under the changed voltage conditions. But because of the saturation of the magnetic circuit, the flux does not decrease quite as rapidly as the terminal voltage decreases; hence, the speed must decrease to some extent to make up the deficiency. In the usual shunt motor, a change of 5 per cent in terminal voltage results in considerably less than 5 per cent speed change, the change being nearer 3 to 4 per cent. An increase of terminal voltage above normal would give an increase of speed. It is apparent, therefore, that a moderate change in terminal voltage would not seriously affect the speed of a shunt motor. However, if the motor must carry the *same horsepower* at reduced voltage, then the current input will be greater than for normal voltage, and the motor may overheat. Similarly, if the motor is operated above normal voltage, although the speed change may not be of importance, the increased field current may overheat the field coils and ultimately cause them to burn out. Motor manufacturers recommend that a shunt motor should be operated at a voltage not more than 10 per cent above or below normal.

For a **series motor,** the flux Φ remains constant as long as the load current is constant. A decrease in the terminal voltage E_t would require a decrease in E_a which is practically proportional, since I_aR_a is small. Hence, the speed will vary almost proportionally to the impressed voltage. The motor may overheat, however, if it is operated at rated horsepower but at reduced voltage, for the same reason as in the case of the shunt motor.

129. Effect of Heating on Speed of a Motor.—For any type of motor, as long as the field strength remains constant, the effect of increased temperature of a motor armature would be that the motor would run at a slightly lower speed than when " cold " because an increased temperature increases the armature resistance and it was shown in Article 117 that an increase in armature resistance decreases the speed. The amount of decrease of speed due to heating is small (usually less than 1 per cent).

The effect of heating of the shunt field of a motor is, however, of greater importance. Heating of the field, assuming constant terminal voltage, would cause a decrease in the strength of field, which would cause the motor to have a higher speed when hot than when cold. This effect is less with a compound machine than with a shunt; in fact, the heating of the series field would increase the resistance of the armature circuit which would tend to counteract the effect of heating of the shunt field. For a shunt motor, this change in speed may be as much as 10 per cent, which is not objectionable for many applications, but may cause trouble in some cases. For example, with certain types of centrifugal pumps, the load increases very rapidly if the pump is operated slightly above rated speed. Hence, the motor may be overloaded if it is designed to give normal speed of the pump when operating " cold." In weaving, a constant speed is very important and fluctuations of speed as indicated would be a serious disadvantage. By special design of the motor, the effect of field heating can be reduced.

PROBLEMS ON CHAPTER XIII

1–13. A motor has an armature resistance of 0.023 ohm, including brushes. Calculate the voltages necessary to force currents of 600, 300, 150, and 60 amperes through the armature when it is standing still.

2–13. If the motor of Prob. 1–13 is shunt wound and running from a 230-volt supply: Calculate the counter e.m.f.'s which must be generated when currents of 600, 300, 150, and 60 amperes are taken by the motor armature.

3–13. A 115-volt shunt motor has an armature resistance, not including brushes of 0.05 ohm. When the armature is generating a counter e.m.f. of 109.5 volts, calculate the current which would flow in the armature circuit. Brush drop 2 volts.

4–13. The motor referred to in Probs. 1–13 and 2–13 runs at 850 r.p.m. when the armature current is 600 amperes and the supply voltage 230 volts.

(*a*) If this machine were operated as a generator at a speed of 850 r.p.m. with a load of 600 amperes in the armature, calculate the terminal voltage. Assume that the shunt field current and armature reaction is the same as when running as a motor.

5–13. The generator of Prob. 27–12 operates at 1200 r.p.m. Assume that it is operated as a shunt motor from a 125-volt supply and that the armature current is the same as when operating as a generator at full load.

(*a*) Calculate the speed when operating as a motor under these conditions.

(*b*) If it were operated as a motor at 1200 r.p.m. and the same armature current as in (*a*), what would the counter e.m.f. be?

(*c*) Compare the strength of field for parts (*a*) and (*b*).

6–13. When a motor is delivering a certain load the pull in the belt due to the armature current is 50 pounds. The pulley is 10 inches in diameter.

(*a*) What is the torque at the pulley due to the armature current?

(*b*) If the armature conductors are located at a distance of 15 inches from the center of the shaft, what force in pounds must be exerted to produce 50 pounds force at the pulley?

7–13. The conductors of a certain motor have an active length of 11 inches and are in a field having an average density of 55,000 lines per square inch. There are 264 conductors on the armature which has an eight-pole lap winding. The total input to the motor armature is 1000 amperes.

(*a*) Calculate the torque for one conductor when it is in the density specified.

(*b*) If 70 per cent of the conductors are covered by the poles, calculate the total torque developed in the armature.

(*c*) Calculate the total mechanical power (horsepower) developed in the armature if it runs at a speed of 400 r.p.m.

(*d*) If 2 per cent of the mechanical power developed is lost in friction and windage, determine the net horsepower at the pulley.

8–13. A four-pole motor has a wave-wound armature with a total of 650 conductors. The pole faces are 3 inches long in a direction parallel to the shaft. The armature conductors are located at an average distance of 3 inches from the center of the shaft. Average flux density of the field surrounding the armature conductors is 53,000 lines per square inch. The poles cover 63 per cent of the armature surface. The input to the armature is 18 amperes.

(*a*) Calculate the average force in pounds acting on each conductor under a pole.

(*b*) What is the total force tending to produce rotation?

(*c*) What is the total torque with 18 amperes flowing?

9–13. Referring to the motor of Prob. 8–13, what would the total torque be for 9, 4.5, and 2.25 amperes, assuming constant strength of field?

10–13. The armature current for the motor of Prob. 8–13 is held constant at 18 amperes and the total flux per pole is varied. Calculate the torque for:

(*a*) A flux per pole one-half and one-quarter the value in Prob. 8–13.

(*b*) A flux 10 per cent less than value in Prob. 8–13.

11–13. The motor of Prob. 8–13 has its armature current changed to 9 amperes and the flux per pole is 3 per cent more than with 18 amperes. Calculate the torque with 9 amperes. (Compare with Prob. 9–13.)

12–13. The motor of Probs. 1–13 and 2–13 operates at 850 r.p.m. when the armature current is 600 amperes. Assuming that the flux per pole does not change with change in armature current, calculate the speed when the load changes so that the required armature currents are 300, 150, and 60 amperes.

13–13. A motor (Probs. 1–13 and 12–13) operates at 850 r.p.m. with 600 amperes in the armature. The voltage impressed on the armature is 230 volts and the armature resistance is 0.023 ohm, including brushes. Assume that the flux per pole does not change as the armature current changes (armature reaction neglected).

(*a*) Calculate the armature current which would flow for a 5 per cent and a 10 per cent increase in speed.

(*b*) At what speed would the armature have to run to make the armature current zero?

(*c*) What would the current be if the armature were brought to a standstill?

14–13. The motor of Probs. 1–13 and 2–13 runs at 850 r.p.m. with 600 amperes in the armature and 900 r.p.m. with 60 amperes, and an armature resistance of 0.023 ohm.

(*a*) Calculate the speed for 600 and for 60 amperes with double the amount of armature resistance.

(*b*) Calculate the speeds for 600 and for 60 amperes if the armature resistance were ten times as great as in Prob. 1–13.

15–13. Referring to Prob. 12–13, assume that the armature reaction with 300 amperes flowing in the armature causes a 2 per cent change in field strength as compared with the field strength with the current at 600 amperes and that for 150 or 60 amperes the change is proportional.

Calculate the speeds with 300, 150, and 60 amperes and compare with the results in Prob. 12–13.

16–13. Referring to the motor of Probs. 1–13 and 2–13:

(*a*) Calculate the power input to the armature for currents of 600, 300, 150, and 60 amperes.

(*b*) Calculate the total mechanical power developed when running, for currents as in (*a*). Express in horsepower.

(*c*) Calculate the power lost in the armature circuit, for currents as in (*a*).

17–13. The motor of Prob. 8–13 runs at 1000 r.p.m. when 18 amperes flows in the armature.

(*a*) Calculate the counter e.m.f. of the machine.

(*b*) Calculate the total mechanical power developed in the armature.

18–13. If the mechanical losses in the motor of Prob. 17–13 are 8 per cent of the total mechanical power developed, calculate the horsepower available at the pulley, with 18 amperes in the armature.

19–13. If the motor of Prob. 8–13 operates at 1000 r.p.m. with 18 amperes in the armature, calculate the total mechanical power developed, based upon the torque derived in Prob. 8–13. Compare the result with the power found in Prob. 17–13 (*a*).

20–13. A 115-volt shunt motor has an output of 10 hp. and runs at 970 r.p.m. The armature resistance, including brushes, is 0.083 ohm. Assume a friction and windage loss of 5 per cent of the output at the pulley.

Calculate the necessary armature current to produce the required torque.

21–13. A certain shunt motor is designed to operate at speeds of 600, 1200 and 1800 r.p.m., according to the adjustment of the field rheostat.

(*a*) Calculate the torque at the pulley for each speed when the motor delivers 30 hp.

(*b*) If the pulley is 11 inches in diameter, what is the tangential pull at the pulley face for each case in (*a*)?

22–13. Referring to the motor of Prob. 8–13, calculate the horsepower developed in the armature at 2000, 1000, 500 r.p.m. and at standstill, assuming that the input to motor armature is 18 amperes in each case and that the flux per pole is constant.

23–13. The motor of Probs. 8–13 and 17–13 is operated at a constant speed of 1000 r.p.m.

(*a*) Calculate the required input of current to the armature when the load at the pulley is varied so that the total horsepower which must be developed by the armature is 2.5 and 6 hp. Neglect armature reaction.

(*b*) For a constant supply voltage and constant speed (1000 r.p.m.), what is the relative value of the counter e.m.f. for the two loads given in (*a*)?

24–13. Calculate the e.m.f. generated in the armature of the motor of Prob. 8–13 when it is operated at the speeds specified in Prob. 22–13.

25–13. A 25-hp. shunt-wound motor is connected to 230-volt mains. The field takes 1.66 amperes and at no load the total input to the motor is 4.36 amperes. The rated full-load input to motor armature is 88 amperes. The no-load speed is 365 r.p.m. The resistance of armature, including brushes, is 0.13 ohm.

(*a*) What would be the counter e.m.f. with 88 and with 44 amperes in the armature?

(*b*) If the flux per pole remained constant, calculate the speed for 88 and for 44 amperes.

26–13. The actual speed of the motor of Prob. 25–13 with full-load current (88 amperes) is 355 r.p.m. instead of the speed calculated in Prob. 25–13.

(*a*) Account for the discrepancy between these two results.

(*b*) What is the ratio of flux at full load to flux at no load?

27–13. For the motor of Prob. 25–13, calculate:

(*a*) The current which would flow if the armature were connected directly to the line when motor was not running.

(*b*) The amount of starting resistance required to give full-load torque at starting.

28–13. A bipolar shunt motor is connected to a 220-volt supply and takes 52.2 amperes. The field resistance is 110 ohms and the armature resistance 0.12 ohm, including brushes.

(*a*) What is the counter e.m.f.?

(*b*) If the flux per pole is 2,720,000 lines and the number of conductors 300, what is the speed in r.p.m., assuming brushes on the neutral?

29–13. What is the speed regulation for the motor of Prob. 26–13?

30–13. (*a*) If the brushes of the motor of Prob. 25–13 are shifted so as to produce a reduction of 2 per cent in the flux at full load as compared with the no-load flux, calculate the speed with full-load current (88 amperes in armature).

(*b*) What would the speed regulation be?

31–13. Referring to the motor of Prob. 25–13:

(*a*) What would the starting torque be with 88 amperes in the armature and 1.66 amperes in the field?

(*b*) What would the starting torque be for a current 1.5 times full-load current?

(*c*) How much additional resistance must be inserted in the armature circuit to give the starting torque determined in (*b*)?

32–13. Assume that the motor with starting resistance required for Prob. 31–13 (*b*) and (*c*) has a load such that when running at constant speed the current input to the armature is 88 amperes.

(*a*) If the starting resistance (Prob. 31–13, *c*) were left in circuit until the motor stopped accelerating in speed what would be the value of this speed?

(*b*) If the starting resistance were halved what would be the steady speed?

(*c*) If all the starting resistance were cut out what would be the speed?

33–13. The no-load speed of the motor of Prob. 25–13 is 365 r.p.m. and the full-load speed is 355 r.p.m.

(*a*) If the field flux were made one-half that in Prob. 25–13, what would be the speeds for the same armature currents as at no load and full load, Prob. 25–13?

(*b*) What would be the percentage speed regulation with the weak field? Compare this with the regulation with full field, Prob. 29–13.

34–13. The motor in Prob. 25–13 has the resistance of the shunt field circuit doubled and the no-load speed is observed to rise from 365 to 570 r.p.m. What is the ratio of the field flux at the high speed to the flux at the low speed?

35–13. With full field on the motor of Prob. 25–13, the speed with 88 amperes in the armature is 355 r.p.m. With the field flux one-half this value, and 88 amperes in the armature:

(*a*) Calculate the total mechanical horsepower developed at the two speeds?

(*b*) What is the total torque developed at the two speeds?

36–13. For the motor of Prob. 25–13:

(*a*) Calculate the additional resistance required in the armature circuit of the otor to give a speed of 200 r.p.m. with an armature current of 88 amperes.

(*b*) If the load were reduced to one-half that in (*a*), or 44 amperes, calculate the speed with the resistance the same as in (*a*).

(*c*) Calculate the speed for one-quarter of full-load current.

(*d*) Calculate the speed if all load were thrown off, assuming no-load current same as in Prob. 25–13.

(*e*) What is the speed regulation with the resistance as in (*a*)? Compare with Prob. 29–13.

37–13. For the motor of Probs. 25–13 and 36–13.

(*a*) Calculate the total power input to armature circuit for a full-load speed of 355 r.p.m. and for 200 r.p.m.

(*b*) What is the total mechanical power developed in the armature. Account for the difference in result for the two speeds.

38–13. A series motor operating from a 550-volt supply has a field resistance of 1.0 ohm and an armature resistance, including brushes, of 0.4 ohm.

The speeds with varying loads are as follows:

5 amperes	794 r.p.m.
10 amperes	455 r.p.m.
20 amperes	290 r.p.m.
40 amperes	220 r.p.m.

(*a*) Calculate the strength of field flux for 10, 20, and 40 amperes in terms of flux for 5 amperes.

(*b*) Calculate the speed with 40 amperes, assuming that the flux remains the same as at 5 amperes.

39–13. (*a*) Referring to the motor of Prob. 38–13, calculate the total torque developed in the motor at 5 amperes and at 40 amperes.

(*b*) Compare the ratio of torques with the ratio of currents for part (*a*).

40–13. If the field winding of the motor of Prob. 38–13 were shunted so that one-half the armature current passes through the field, calculate the speed when the machine is taking 40 amperes.

41–13. (*a*) How much resistance must be inserted in series with the motor of Prob. 38–13, in order to reduce the speed to 290 r.p.m. when the motor takes 10 amperes from the line?

(*b*) What would the speed be with this resistance in circuit if the load were decreased to 5 amperes?

42–13. In a certain compound motor 20 per cent of the total field flux at full load is produced by the series winding. The motor operates on a 230-volt circuit and takes 150 amperes in the armature at full load. Shunt field is connected to the motor terminals. The resistance of the series winding is 0.09 ohm and of the armature, including brushes, is 0.046 ohm. The speed at full load is 590 r.p.m. and at no load, 800 r.p.m.

(*a*) Calculate the full-load speed with the series field cut out.

(*b*) Calculate the speed regulation with and without series field.

(*c*) Calculate the speed for full-load current with the series field reversed, giving a differential motor.

(NOTE.—This must be calculated on the assumption that the decrease in flux due to differential connection is the same as the increase due to cumulative connection. This is not exactly true, due to the saturation of the magnetic circuit.)

43–13. Referring to the motor of Prob. 42–13:

(*a*) Calculate the total torque developed with full-load current in the armature and series winding connected cumulatively.

(*b*) Calculate torque for full-load current without the series winding and also with the winding reversed.

44–13. The 230-volt shunt motor of Prob. 25–13 has an armature resistance of 0.13 ohm, including brushes. The full-load speed is 355 r.p.m. If the motor were operated on a voltage 10 per cent less than normal:

(*a*) Calculate the speed with full-load current on the assumption that the field flux changes proportional to the change in voltage.

(*b*) Calculate the speed with full-load current if the field flux decreases 4 per cent for 10 per cent decrease in voltage.

45–13. Referring to the series motor of Prob. 38–13, calculate the speed when the motor is operating from a line 10 per cent below normal voltage and taking 40 amperes.

46–13. The shunt motor of Prob. 25–13 is to be operated with its field winding connected to a 230-volt supply and the voltage of the armature is to be varied.

(*a*) Calculate the speed with 88 amperes in the armature for voltages of 230, 115, and 57.5 volts.

(*b*) Calculate the total power developed in the armature with 88 amperes and the voltages given in (*a*).

CHAPTER XIV

COMMUTATION

130. Fundamental Requirements.—It has been shown in a previous article that a commutator is necessary for all d.c. machines in order that the connections of an armature coil may be reversed at the proper instant during the rotation of the armature. Reversal of coil connections results in a reversal of the direction of the current in the coil. This reversal of the connections of a coil and the direction of current in the coil is called commutation. In Fig. 112 *a* the commutation of coil *B* is starting since, at this point, the brush first makes contact with bar 2. Until this occurs, coil *B* is carrying the same current as coil *A* and other coils at the left of the brush. This current is assumed to be 10 amperes in a positive direction. As soon as bar 2 makes contact with the brush, coil *B* is short-circuited by the brush, and the current in coil *B* begins to change. Under proper conditions, the current in coil *B* should decrease at a uniform rate until, at position *b* (Fig. 112), which is midway in the period of commutation, the current in coil *B* should be zero. The current should then increase in a reversed or negative direction at a uniform rate until point *c* is reached, when bar 3 is ready to break contact

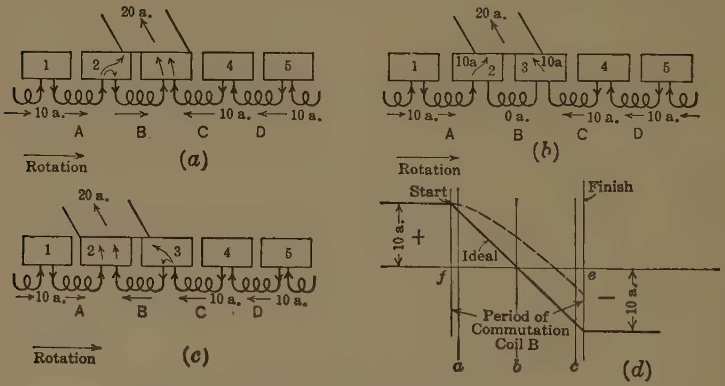

FIG. 112.—Commutation, Ideal Conditions.

with the brush and coil *B* is about to be cut into the armature circuit on the opposite side of the brush. The time during which coil *B* is short-circuited by the brush is the period of commutation. The changes which occur in the current in coil *B* under ideal conditions are shown in Fig. 112 *d*. The fundamental requirement for proper commutation is that the current shall be completely reversed before the coil is again cut into the circuit, after being short-circuited by the brush, so that at the instant when the bar 3 breaks contact with the brush there shall be no current flowing from bar 3 to the brush. If there is current flowing between brush and bar when this break occurs there will be sparking between the tip of the brush and the bar. With perfect commutation there should be no sparking between brushes and commutator.

The ideal conditions illustrated in Fig. 112 *d* can be secured only approximately in practice, due principally to the self-induction of the armature coils, which resists the tendency to change the current, so that at the instant when the break occurs (Fig. 112 *c*), the current is not completely reversed and there is still some current flowing between bar 3 and the brush. This condition is shown by the dotted line in Fig. 112 *d*. Under these conditions the current must rise suddenly to 10 amperes when the break occurs, and this causes sparking.

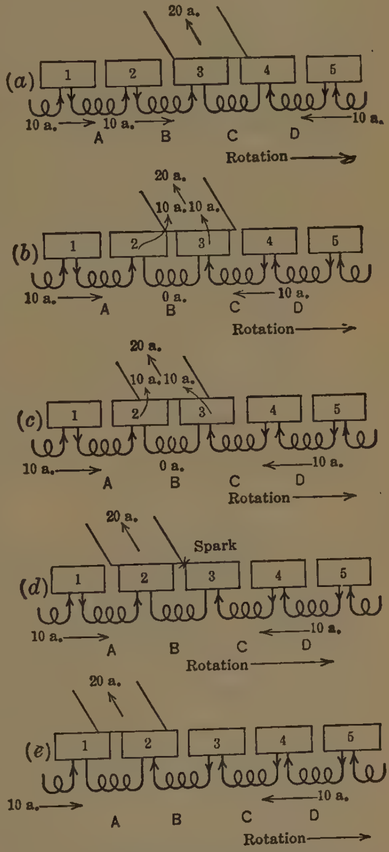

FIG. 113.—Commutation with Copper Brushes on Neutral.

131. Commutation with Copper Brushes on the Neutral.—The contact resistance between copper brushes and the commutator is very low so that when this type of brush is used the change in current is very different from the ideal requirements shown in Fig. 112 *d*. Suppose that the copper brushes are so located that they short-circuit coils which are not cutting any flux, i.e., are in the neutral plane.

The changes in current in coil *B* during the process of commutation will then be as shown in Figs. 113 and 114. At the instant shown in (*a*), coil *B* is carrying a load current of 10 amperes in the positive direction. When the brush makes contact with bar 2, however (Fig. 113 *b*), the contact resistance is so low that the current from coil *A* flows directly to the brush and the current in coil *B* drops very quickly to zero. The current in coil *B* continues to remain zero until the brush is about to break contact with bar 3, thus cutting coil *B* into the negative side of the armature. At this instant, shown in (*d*) (Fig. 113), the entire negative current of 10 amperes is flowing to the tip of the brush from bar 3 and no current is flowing in coil *B*. There must, therefore, be a very sudden increase of current in a negative direction in coil *B* at the instant of break, but the self-induction of the coil tends to prevent this sudden change of current in coil *B*, so that, for an instant, the current continues to flow across the break between bar 3 and the tip of the brush, and hence, a heavy spark occurs. Due, therefore, to the very low contact resistance, copper brushes, when set on the neutral so that the commutating coil does not cut any of the main flux, will spark very badly and they are, therefore, not operative in that position.

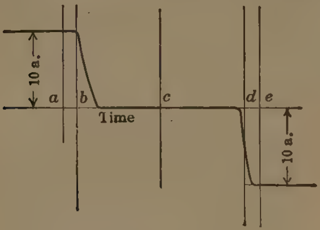

FIG. 114.

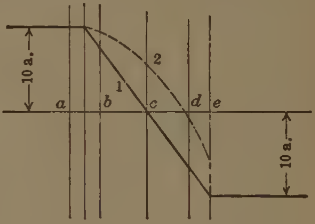

FIG. 115.

FIG. 114.—Current in Coil During Commutation. Copper Brushes.

FIG. 115.—Current in Coil *D* During Commutation. Carbon Brushes. Curve 1, self-induction neglected. Curve 2, effect of self-induction.

132. Commutation with Carbon Brushes on the Neutral.—Carbon brushes have a much higher contact resistance than copper brushes; so, when these are used, the process of commutation is very different. This is shown in Fig. 115, neglecting the effect of self-induction in the coil. In Fig. 116 *a*, coil *B* is carrying 10 amperes (the load current for one path) in a positive direction. When bar 2 makes contact with the brush, the contact resistance between bar 2 and the brush is higher than between bar 3 and the brush because a much smaller area of bar 2 is in contact. The result is that only a portion of the current of 10 amperes from coil *A* flows from bar 2 to the brush, while the remainder flows through coil *B* (Fig. 116 *b* and *d*). When there is an equal area of bars 2 and 3 in contact with the brush as in (*c*) (Fig. 116), the contact resistances are equal and the current in the coil is zero since it is cutting no flux and self-induction in the coil is neglected. As rotation continues, the contact resistance for bar 3 increases and for bar 2 decreases so that more and more of the current from coil *C* tends to flow through coil *B* and bar 2. This current is in a negative direction. At the instant when bar 3 is about to break contact with the brush, the contact resistance at this point is very high so that practically the entire 10 amperes from coil *C* will be flowing through coil *B* and no current will be flowing between bar 3 and the brush. Under these conditions, there would be no sparking when the brush breaks contact with bar 3. It will be seen, therefore, that with carbon brushes, much better commutating conditions can be obtained than with copper brushes. There is, however, an appreciable amount of self-induction in each of the armature coils which retards the reversal of current in the coil. This is shown by the broken line in Fig. 115. Thus, at point *e* when the brush is about to break contact with bar 3 the current is not 10 amperes as it should be, but has reached a value of only 6 amperes. Hence, 4 amperes must flow from bar 3 to the brush, and there will be a spark when contact is broken. Due, therefore, to the self-induction of the coils, there will be some sparking even with carbon brushes if they are in a neutral zone where the commutated coil does not cut any of the main flux. The sparking will, however, not be so severe as with copper brushes, as can be seen by comparing Figs. 114 and 115, since with the carbon

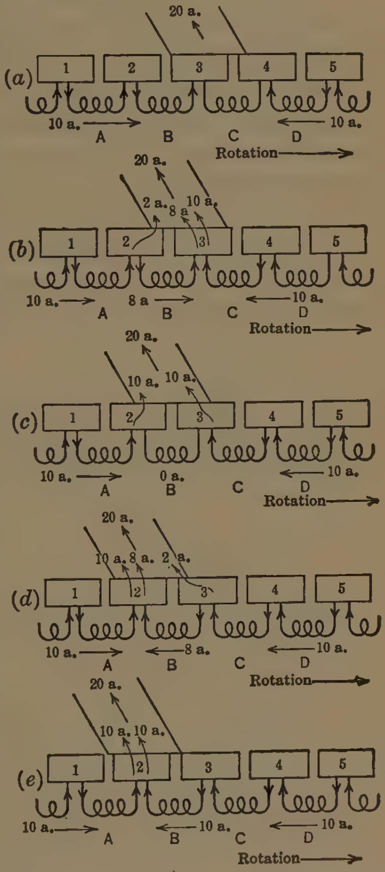

FIG. 116.—Commutation with Carbon Brushes on the Neutral.

brush there is a change from 6 to 10 amperes, whereas, with the copper brush the change is from 0 to 10 amperes. The method of overcoming the effect of self-induction is explained in the next paragraph.

133. Commutation with Brushes Shifted from the Neutral.—Suppose that a machine with carbon brushes has these brushes shifted from the neutral so that the coil undergoing commutation is under the influence of flux from the main poles. If the machine were carrying no load, there would be no current in the armature circuit, neglecting the current needed for excitation. But the coil short-circuited by the brush would now have an e.m.f. induced in it and, since there would be a closed path through the brush, a current would flow in the short-circuited coil even when the machine is carrying no external load. The amount of this current would depend upon the strength of field cut by the coil and upon the resistance of the circuit made up of coil and brush. This resistance would be low, however, so that a weak field would be sufficient to cause a large current to flow in the short-circuited coil. This current could be increased by shifting the brushes nearer the main poles. Suppose the brushes were shifted until a current of 4 amperes were flowing in the coil at the instant when bar 3 is about to break contact with the brush. This is shown in Fig. 117 *a*. Under these circumstances, at no load there would be a current of 4 amperes circulating through coil *B* in a *negative* direction, while the current in the other coils would be zero since there is no load on the machine. If, now, a load of 20 amperes per brush is thrown onto the machine, the load current at this instant would be as shown in Fig. 117 *b*, due to the self-induction of the coil as was explained in the previous article. The load current through the coil tends to be 6 amperes, whereas it should be 10 amperes for sparkless commutation. The e.m.f. generated in coil *B*, however, tends to set up a current of 4 amperes in a direction to *aid* the load current to flow in coil *B*, whereas the e.m.f. of self-induction tends to set up a current of 4 amperes in a reversed (or positive) direction.

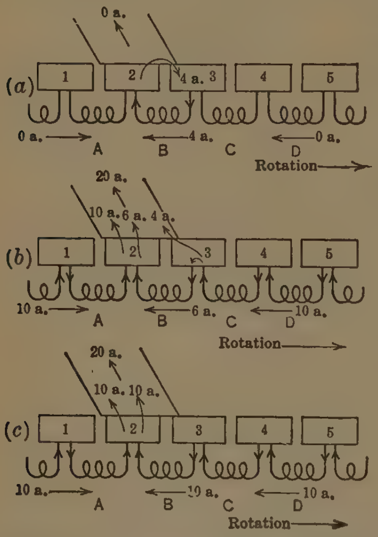

FIG. 117.—Commutation with Carbon Brushes Shifted from the Neutral.

Hence, the e.m.f. set up in coil *B*, due to its cutting the main field (Fig. 117), is equal and opposite to the self-induced e.m.f. of the coil, which is therefore neutralized and a current of 10 amperes will flow in the coil at the instant of break, as shown in Fig. 117 *c*. Hence, no current will flow from bar 3 to brush and sparking will be eliminated. It should be apparent that sparking of copper brushes could also be eliminated by shifting them the proper amount.

134. Direction and Amount of Brush Shift.—Examination of Fig. 116 will show that, if the brushes are shifted to produce an e.m.f. in the coil undergoing commutation, the direction of this shift must be such as to give an e.m.f. of the proper direction to *oppose* the e.m.f. of self-induction. In other words, the short-circuited current which tends to flow due to this commutating field must be in the same direction as the current in the coil due to the load current. Thus, referring to Fig. 116 *d*, the short-circuited current should tend to flow in a negative direction since it is desired to build up a current in coil *B* in a negative direction before it is cut into the armature circuit on the right-hand side of the brush. This is also shown in Fig. 118 *a*. Since this machine is a generator, the direction of induced e.m.f. and current in the armature is as indicated, and the conductors pass the positive brush from left to right; hence, the short-circuit current in the conductor should be the same as the direction of current under the S pole. For that reason, the brushes should be shifted forward to a position such as line *ab* so that the short-circuited coil will cut flux from the S pole. In the case of a motor (Fig. 118 *b*), the direction of the *induced* e.m.f. would be the same as that shown in Fig. 118 *a*, but the direction of *current* would

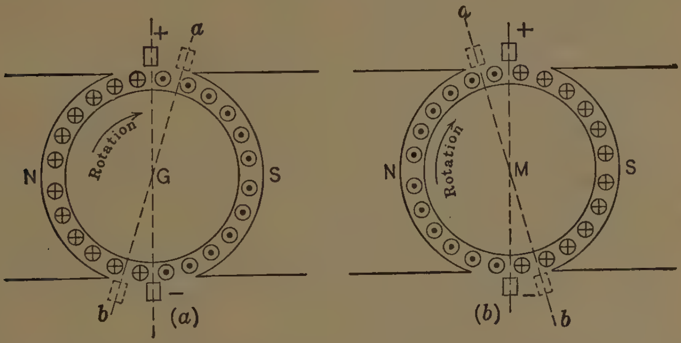

FIG. 118.—Position of Brushes for a Dynamo. (*a*) Generator; (*b*) Motor.

be in the opposite direction. Therefore, in order that current in the short-circuited conductor shall flow in the correct direction (away from observer in case of conductors passing the positive brush), the brushes must be shifted backward so that the short-circuited conductors under the positive brush will cut flux from an N pole.

If the brushes of a generator were given a backward shift instead of a forward shift, the direction of induced e.m.f. and of short-circuit current would be such as to oppose the reversal of current instead of aiding it, as is necessary. The sparking would, therefore, be very violent if the machine were loaded. The same would be true for a motor with a forward shift.

Most d.c. dynamos of the older type, without commutating poles, require a slight shift of brushes, forward for a generator and backward for a motor, even where carbon brushes are used. Motors which are designed for reversing service, such as railway, crane and hoist motors, are operated with brushes on the mechanical neutral. (See Article 135.) Such motors, when without commutating poles, require careful designing to make them operative without excessive sparking. It would seem that the required amount of brush shift would depend upon the load current, since a larger current would require a larger e.m.f. induced in the commutated coil. With carbon brushes, however, it is possible with a well-designed machine to find one position of the brushes which will give practically sparkless commutation at all loads. With copper brushes, however, it is generally necessary to change the brush position as the load changes. Copper brushes are, therefore, seldom used in modern machines. It is now standard practice to secure the required commutating field, not by shifting the brushes, but by producing the desired field directly over the short-circuited coils, by means of small poles called commutating poles or interpoles, located between the main poles. This scheme is described in Chapter XVI.

PROBLEMS ON CHAPTER XIV

1–14. A 125-volt generator runs at 1050 r.p.m. and has 109 commutator bars. The brushes are wide enough to cover two bars at once. What is the period of commutation for this machine?

2–14. The conditions illustrated in Fig. 117 apply to a generator with a forward shift of the brushes. Draw similar diagrams showing the current in the coils if the brushes were given a back shift sufficient to cause 4 amperes to flow in the short-circuited coil when there is no load current.

3–14. What would be the effect on the commutation of shifting the brushes of a motor forward?

4–14. Assuming that the load current per brush of a generator is 40 amperes, draw a diagram similar to Fig. 117 showing the conditions for proper commutation.

5–14. Assume that the brushes of a generator were shifted forward sufficiently to produce 8 amperes in the short-circuited coil, when there is no load current. Draw diagrams similar to Fig. 117, showing the currents when the machine is carrying a load current of 20 amperes per brush.

CHAPTER XV

ARMATURE REACTION

135. Distribution of Flux from Main Field.—When the main field of a dynamo is excited, with no current in the armature, the flux at the air gap is uniformly distributed except at the pole tips, where there is some fringing (Fig. 119 *a*). At the points *ab*, midway between the pole tips, the flux is practically zero. These points are called the geometrical or mechanical neutral of the machine. In a multipolar machine there would be a mechanical neutral point midway between each pair of poles. The mechanical neutral is also called the no-load neutral. The position of the neutral point or point of zero flux at the surface of the armature core changes when there is current in the armature so that the "load neutral" is not on the line *ab*. For this reason, when reference is made to the neutral point or neutral plane, the no-load or geometrical neutral is meant unless specifically stated otherwise.

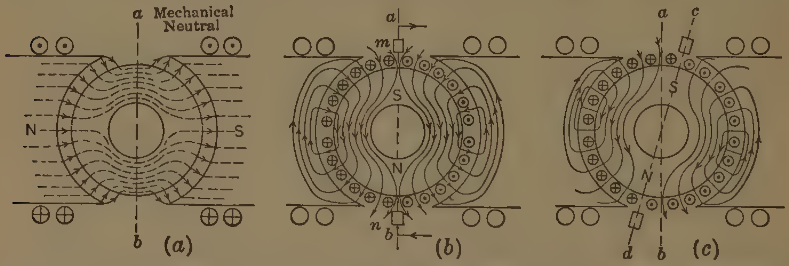

FIG. 119.—Flux Distribution. (*a*) Field alone; (*b*) Armature alone, brushes on neutral; (*c*) Armature alone, brushes shifted.

136. Distribution of Flux Produced by Armature Current.—If current is passed through the armature of a bipolar machine when there is no current in the field circuit, the flux produced by the armature current will be as shown in Fig. 119 *b*, when the brushes are on the neutral line. The path for the flux will be at right angles to the axis of the field magnets, and this armature flux will pass across the two main poles. Due to the arrangement of conductors in the armature winding, the current from the external source will divide and flow by two paths in the armature. The current in the conductors on the right-hand side of the neutral line *ab* will be flowing in the opposite direction to the current on the left, as is indicated by the symbols on the armature conductors. If the field produced by this armature current were explored by a compass, it would be found that there is an N pole at *b* and a S pole at *a*, so that the armature winding acts like a solenoid and produces magnetic poles which are in line with the brushes. In a multipolar machine, the same effect occurs, there being an N or S pole in line with each brush group. This is true for either a lap or a wave winding. If the brushes are shifted from the neutral line *ab*, the resulting poles shift also as shown in Fig. 119 *c*.

The path for the flux produced by the armature current is across the main poles as shown by the flux lines in Fig. 119 *b*. The armature current, therefore, tends to magnetize the main poles at right angles to the direction of the field flux and is called a cross-magnetizing effect. The strength of the cross-magnetizing effect of the armature current can be determined by calculating the ampere-turns acting on one of the closed magnetic paths through one of the poles. Consider the path *mn*. The total magnetic effect, along the path *mn*, is found by summing up the effect of all the conductors enclosed by this path. (See Article 44.) Each of the conductors so enclosed constitutes one turn, and it is not necessary to know where the other side of the turn is located, except to know that it is outside the path *mn*.[1]

Example 1.—Suppose there are a total of 100 conductors on the armature and that the current from an external source is 100 amperes; then, since there are two paths between brushes, the current in each conductor is 50 amperes. There is a total of 50 conductors enclosed by the path *mn*, and the total magnetizing effect along the path *mn* is $50 \times 50 = 2500$ ampere-turns. If the reluctance of the path *mn* were known, the flux density could then be calculated.

137. Combined Effect of Armature and Field Flux with Brushes on Neutral.—When there is current in both armature and field of a dynamo, as would be the case when the machine is carrying load, the flux at the air gap is the resultant of the armature and main field fluxes. With the brushes on the neutral, it would be a combination of Figs. 119 *a* and *b*. This is shown in Fig. 120. The main field acts from left to right along path *xyz*; the armature field, along path *mn*. At the upper pole tip on the N pole and the lower tip on the S pole the armature flux is in the same direction as the main flux; hence, they add, and the resulting flux is greater than without the armature current. At the lower tip on the N pole and the upper tip on the S pole, the armature and field flux are opposed so that the resulting flux is the difference of the two; hence, these two tips have less flux density than before. The result is a distortion of the main flux in the air gap, with a strong tip or point of high flux density and a weak tip or point of low flux density, for each pole (Fig. 120). At no-load, the flux density at the air gap is uniform, and this is represented by the ordinates of curve *a* (Fig. 121). This corresponds to conditions shown in Fig. 119 *a*, where the flux is produced entirely by the current in the field winding. When there is current in the armature winding, also (Fig. 120), the flux density varies as shown by the ordinates of curve *b* of Fig. 121. The brushes are kept on the geometrical neutral, but the actual neutral zone is now shifted slightly to the right as is indicated in curve *b*.

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[1] Actually, of course, the other side of each turn is on the opposite side of the armature.

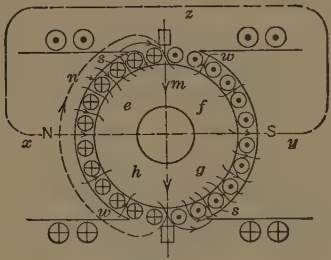

Fig. 120.

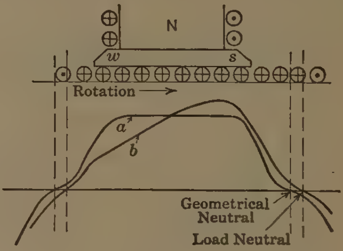

Fig. 121.

Fig. 120.—Flux Distribution. Combined effect of armature and field flux, brushes on neutral.

Fig. 121.—Flux Density in Air Gap for a Non-commutating Pole Machine. Showing effect of load upon flux density. (*a*) No load; (*b*) Load. (See also Fig. 126.)

The effect of the armature and field currents in producing the resultant flux can be determined by the aid of Fig. 120. The current in the conductors in quadrant *f* tends to strengthen the main field, while that in quadrant *e* tends to weaken the main field. The number of conductors in each of these quadrants is the same, and the current is the same in all the armature conductors; hence, the two quadrants neutralize each other. Therefore, when the brushes are on the neutral, there is no magnetizing or demagnetizing effect due to the armature current but only a distorting effect, resulting in different flux densities in different parts of the pole face. On this basis, the total flux per pole would remain the same from no load to full load, and this is approximately true in practice. However, due to the increased density in a portion of the pole face, the total flux is decreased slightly so that armature reaction, with brushes on the neutral, causes a slight decrease of field strength as well as a distorting effect.

138. The combined effect of armature and field current with brushes shifted from the neutral is shown in Fig. 122, which is the resultant of Fig. 119 *a* and *c*. In this case, there exists a cross-magnetizing effect similar to that described in Article 137 which distorts the main flux and produces strong and weak tips on each pole. Referring to Fig. 122, let the brushes be shifted to the line *cd* at an angle β from line *ab*. Draw line *fi* β degrees the other side of *ab*. Considering the main magnetic circuit, *xyz*, the conductors between *k* and *i* produce a magnetic effect which is equal and opposite to that produced by the conductors between *c* and *l*; hence, the resulting

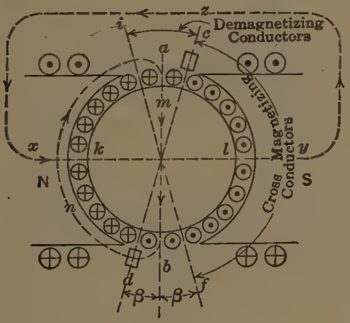

FIG. 122.

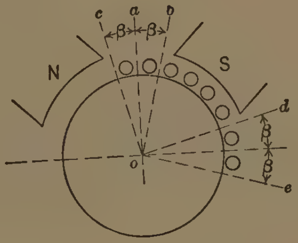

FIG. 123.

FIG. 122.—Armature reaction. Brushes shifted from neutral.
FIG. 123.—Armature Reaction for a Multipolar Machine.

effect on the main flux is zero. The conductors between *i* and *c*, however, have a magnetic effect opposed to the main field winding and, therefore, they tend to demagnetize the main field. Hence, when the brushes are shifted from the neutral, armature reaction causes a cross-magnetizing or distorting effect on the main flux and also causes a demagnetizing effect which acts directly to decrease the flux per pole. The magnitude of this demagnetizing effect evidently increases with an increased brush shift. Therefore, the brushes should be set as closely as possible to the neutral line. With modern machines having commutating poles, it is possible to operate with the brushes exactly on the geometrical neutral. With machines without commutating poles, it is necessary to shift the brushes slightly from the neutral position in order to prevent sparking.[1]

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[1] The reason why this is necessary was explained in Chapter XIV.

For a generator, the shift is forward or in the direction of rotation; for a motor, it is backward or against the direction of rotation.

139. Armature Reaction for Multipolar Machines.—The method of calculating the cross-magnetizing and demagnetizing effect in multipolar machines is the same as for bipolars. Thus, if the brushes of a multipolar machine are shifted β degrees from the neutral (Fig. 123), the cross-magnetizing and demagnetizing conductors are as shown. The total demagnetizing conductors acting on any one of the four paths through the field frame are the conductors between lines *b* and *c*. The cross-magnetizing conductors acting on one magnetic circuit through a single pole face are the conductors between *b* and *d*.

Example 1.—A 400-kw., 250-volt ten-pole compound generator has 800 armature conductors. Assume a brush shift of 2° from neutral.

(*a*) Calculate the demagnetizing ampere-turns per magnetic circuit with 168 amperes per armature conductor.

(*b*) Calculate the cross-magnetizing effect for one pole face.

(*a*) The demagnetizing conductors per circuit are

$$\frac{2+2}{360}\times 800 = 8.9 \text{ conductors.}$$

The demagnetizing effect is

$$8.9\times 168 = 1490 \text{ ampere-turns per circuit.}$$

(*b*) The cross-magnetizing conductors are

$$\frac{36-4}{360}\times 800 = 71 \text{ conductors.}$$

The cross-magnetizing effect is

$$71\times 168 = 11{,}920 \text{ ampere-turns per pole.}$$

PROBLEMS ON CHAPTER XV

1–15. A bipolar motor has an armature current of 20 amperes. There are 950 conductors in the armature and 75 per cent of the armature is covered by the poles. Calculate the cross-magnetizing ampere-turns acting along a path through the tips of one pole.

2–15. A bipolar machine has a field polarity as shown in Fig. 119*a* and current in the armature conductors in the opposite direction to that shown in Fig. 119*b*. The brushes are on the neutral. Plot the resultant field and indicate the strong and weak pole tips.

3–15. Referring to the machine of Prob. 2–15, plot the resultant field and show the strong and weak pole tips with brushes shifted in a clockwise direction.

4–15. Assume a bipolar dynamo with the S pole on the left and clockwise rotation of the armature. Plot resultant field and indicate cross-magnetizing and demagnetizing conductors under the following conditions:

(*a*) Generator with brushes shifted ahead of neutral.
(*b*) Generator with brushes shifted back of neutral.
(*c*) Motor with brushes shifted ahead of neutral.
(*d*) Motor with brushes shifted back of neutral.

5–15. A bipolar armature has 420 conductors and each field pole subtends an angle of 120 degrees.

(*a*) What percentage of the armature periphery is covered by the poles?

(*b*) If the brushes were set in line with the pole tips, calculate the cross-magnetizing and demagnetizing ampere-turns when the current at the armature terminals is 100 amperes.

6–15. Referring to the machine of Prob. 7–13, calculate the cross-magnetizing ampere-turns acting along a path through the tips of one pole.

7–15. Assume that the brushes of the machine of Prob. 7–13 are shifted 3 degrees from the neutral. Calculate the demagnetizing ampere-turns per magnetic circuit.

8–15. A 4-pole, 50-kw., 125-volt generator, with lap-wound armature, has poles each spanning 60 degrees. There are 100 slots with two conductors each. The brushes are in line with the pole tips.

(*a*) Calculate the cross-magnetizing effect per pole.

(*b*) Calculate the demagnetizing effect per magnetic circuit?

9–15. A 4-pole, 100-kw., 500-volt generator has a lap-wound armature with 500 conductors. The brushes are set with a forward lead of 10 degrees.

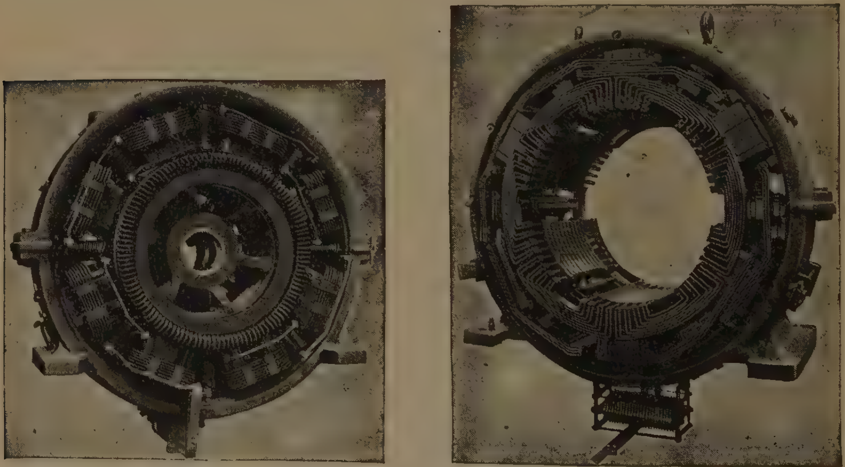
(*a*) Calculate the armature demagnetizing ampere-turns per magnetic circuit.

(*b*) If 0.868 of the current output flows through the series field, determine the number of turns required on each series field winding to overcome the armature reaction at full load.

CHAPTER XVI

COMMUTATING POLES AND COMPENSATING WINDINGS

140. Commutating Poles.—It was shown in Article 133 that, for successful commutation, the coil short-circuited by a brush must be in a magnetic field in order to assist in reversing the current in the coil. This field can be secured by shifting the brushes near the tips of the main poles, but, when this is done, the armature reaction is increased. (See Article 138.) In modern machines, the required commutating flux is secured, not by shifting the brushes, but by locating auxiliary poles on the geometrical neutral. These auxiliary poles are called interpoles or more correctly commutating poles (Fig. 124 *a*). When these are used, the brushes are set on the geometrical neutral and the required commutating flux is secured by properly proportioning the exciting winding on the commutating poles. It is apparent that, for ideal results, the commutating flux should be



(*a*) (*b*)

FIG. 124.—Arrangement of Field Windings. (*a*) Commutating pole machine; (*b*) Compensated machine. *The Westinghouse Elec. & Mfg. Co.*

proportional to the armature current. In practice, this is approximately realized by placing the commutating field winding in series with the armature, thus causing the flux to increase as the current increases. Due to saturation of the magnetic circuits, however, the flux is not exactly proportional to the current at heavy loads. The construction of commutating poles has been described in Article 81. In general, the brushes of commutating pole machines should be set exactly on the neutral. They must be set much more accurately than is necessary for non-commutating pole machines; otherwise, there may be instability of speed in a motor or difficulty in parallel operation in case of a generator.

141. Compensating Windings.—The overload which a commutating pole machine can carry, without excessive sparking, is limited by the saturation of the commutating poles so that, at very heavy loads, the commutating pole flux is no longer proportional to the armature current and sparking results.[1] To meet the severe load requirements sometimes encountered in practice, it is necessary to employ a compensating winding in addition to the commutating pole winding (Fig. 124 *b*). The compensating winding is placed in the faces of the main poles and is connected in series with the armature so that the current through it is proportional to the load current. The current in the compensating winding flows in a direction opposite to that in the armature conductors (Fig. 125) and thereby neutralizes the distorting effect of the armature current. Since the brushes are on the neutral, there can be no demagnetizing effect so that armature reaction is completely neutralized by means of the compensating winding. In the case of a commutating pole machine, the field is distorted by armature reaction, the same as for a non-commutating pole machine (Fig. 126 *a*), but when a compensating winding is used, distortion is eliminated (Fig. 126 *b*) and the flux density at different points in the air gap is not affected by changes in load. Commutating poles are also used on compensated machines to provide the necessary commutating flux, but in this case the flux is more nearly proportional to the load so that compensated machines can carry heavier loads without sparking than standard commutating pole machines.

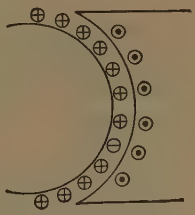


FIG. 125.—Compensating Conductors.

142. Applications.—The use of commutating poles on both motors and generators has become standard practice except in sizes below about 2 kw. By their use, it has been possible to employ a soft graphite brush and this reduces commutator wear. The use of a compensating winding increases considerably the cost of a machine and is not justified for usual application. For severe service, however, such as is found in steel mill practice, and for very high-voltage machines such as are used on the high-voltage railway systems, the compensating winding is required. It is also used where very close speed regulation is necessary as in drives for paper mills.

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[1] The usual manufacturers' guarantee for commutating pole machines is that they will carry a 50-per cent overload momentarily without excessive sparking.

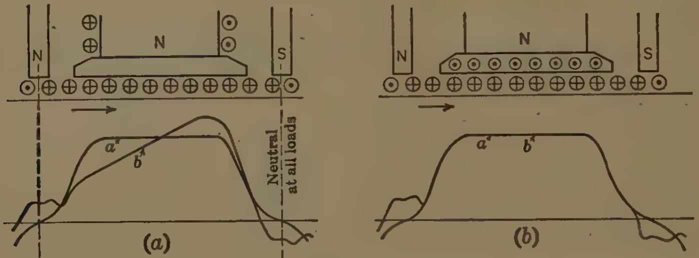

FIG. 126.—Flux Density in Air Gap. (*a*) Commutating-pole machine loaded and at no load; (*b*) Compensated machine, loaded and at no load.

PROBLEMS ON CHAPTER XVI

1–16. (*a*) What would be the effect on the commutation of a machine if the commutating poles were too weak?

(*b*) What would be the effect if they were too strong?

2–16. What would be the effect of shunting the commutating-field winding of a machine?

3–16. Name three ways in which the amount of commutating flux could be adjusted for a fixed load current.

4–16. What would be the effect of reversing the connections of a commutating-pole winding with respect to the armature?

5–16. A four-pole compound generator runs in a counter-clockwise direction. Assign polarities to the main poles and indicate the corresponding polarities for the commutating poles.

6–16. The machine in Prob. 5–16 is changed to a compound (cumulative) motor running in the same direction. Indicate polarities of main and commutating poles.

CHAPTER XVII

LOSSES AND EFFICIENCY OF D.C. DYNAMOS

143. Nature and Location of Losses.—The losses in d.c. machines may be grouped under three headings: copper losses, which occur in the electrical circuits; core losses, which occur in the magnetic circuit; and mechanical losses. In every case, these losses cause heating of the machine, and hence they must be kept as low as possible; otherwise the rating of the machine is reduced and the efficiency of conversion of power from a mechanical to an electrical form, or vice versa, will be low. A tabulation of these losses is given below:

Distribution of Losses in a d.c. Dynamo

- Input
  - Output
  - Losses
    - —Armature copper losses
      - (*a*) Armature, $I_a^2R_a$
      - (*b*) Brushes, I_aE_b
    - —Field copper losses
      - (*a*) Shunt field, $I_f^2R_f$
      - (*b*) Series field, $I_s^2R_s$
      - (*c*) Commutating field, $I_a^2R_c$
    - —Mechanical losses
      - (*a*) Bearing friction
      - (*b*) Brush friction
      - (*c*) Air friction (windage)
    - —Core losses
      - (*a*) Hysteresis
      - (*b*) Eddy current

It should be particularly noted that the counter e.m.f. of a motor does not cause a loss; in fact, it is a measure of the useful power developed by the motor. (See Article 111.) Neither does armature reaction directly produce a power loss; it only causes a decrease in field flux. Indirectly, however, if the field flux is changed by armature reaction, the core losses will be changed.

144. The copper losses are calculated from the resistance and the current in the circuit. They are sometimes called the I^2R losses. The loss in brush-contact resistance can best be calculated from the product of current and brush drop.[1] Losses in field windings should include losses in field rheostat.

Example 1.—A 250-kw., 250-volt compound-wound generator has an armature resistance of 0.005 ohm, a series field resistance of 0.0008 ohm, a commutating pole field resistance of 0.0005 ohm, and a shunt field resistance of 33.3 ohms; connections are long shunt. Find the resistance losses at full load.

The line current is

$$I_t=\frac{250{,}000}{250}=1000 \text{ amperes.}$$

The shunt field current is

$$I_f=\frac{250}{33.3}=7.5 \text{ amperes.}$$

The armature and series field current is

$$I_a=I_S=1000+7.5=1007.5 \text{ amperes.}$$

The armature loss is

$$P_a=1007.5^2\times 0.005=5018 \text{ watts.}$$

The brush loss is

$$P_b=1007.5\times 2=2015 \text{ watts.}$$

The shunt field loss is

$$P_f=7.5\times 250=1880 \text{ watts.}$$

The series field loss is

$$P_S=1007.5^2\times 0.0008=813 \text{ watts.}$$

The commutating field loss is

$$P_c=1007.5^2\times 0.0005=502 \text{ watts.}$$

The shunt field loss is, evidently, practically independent of the load on the machine since the terminal voltage is nearly constant. The copper losses in the armature circuit, including series and commutating field, are proportional to the square of the current in the armature circuit and, therefore, are approximately proportional to the square of the load. Copper losses are independent of the speed.

145. Mechanical Losses.—When a machine is running, there is friction in the bearings, air friction or windage due to the fanning action of the armature, and friction between the brushes and commutator. To overcome this friction requires a continual expenditure of power and, since this power is consumed by the machine, it is called the mechanical loss. This loss increases with increase of speed but is independent of the load as long as the speed is constant. Mechanical losses are not easily calculated and are usually determined by test.

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[1] See Article 94, for values of this drop.

146. Core Losses.—The armature core of a d.c. dynamo, when running, is subjected to a reversal of flux each time it passes a pole, and this causes a hysteresis loss. The amount of this loss depends upon the flux density, the quality of the material, and upon the frequency with which the reversals occur. In the case of the armature core, hysteresis loss is proportional to the speed and the 1.6 power of the flux.

The eddy current loss which occurs in the armature core is caused by the voltage induced in the iron core due to its rotation in the magnetic field. This voltage produces a current in the core, as was explained in Article 58. Eddy current loss depends upon the quality of the material, the speed, and the flux density. For a particular machine, this loss is proportional to the square of the speed and of the flux. The hysteresis and eddy current losses heat the armature core and produce a mechanical force opposing rotation. The energy necessary to supply these losses is furnished from the total mechanical energy developed in the machine and is not supplied directly from the electrical circuit, as is the case with the copper losses. Change of load does not change the core losses except as the speed or the flux per pole is changed; therefore, for generators which run at constant speed and nearly constant flux, the core losses are nearly independent of load. This is true also of constant speed motors.

147. Efficiency.—The efficiency of a machine is the ratio $\frac{\text{output}}{\text{input}}$. Efficiency is usually expressed as a percentage, and can be written as follows:

$$\text{Per cent efficiency} = \frac{\text{output}}{\text{input}} \times 100 = \frac{\text{output}}{(\text{output}+\text{losses})} \times 100$$

$$= \frac{(\text{input}-\text{losses})}{\text{input}} \times 100.$$

A typical efficiency curve is shown in Fig. 98. It will be seen that the efficiency is fairly high, from one-half to full load, but drops rapidly below half load. This is due to the fact that the mechanical losses and the core losses are nearly independent of load and, hence, are a larger percentage at light loads. In general, the efficiency is determined by measuring the losses, instead of measuring output and input.

Example 1.—Referring to the generator of Example 1, Article 144, determine the full-load efficiency if the core loss is 3400 watts and friction loss, 5400 watts. The losses are as follows:

Armature copper	5,018 watts
Brush contact	2,015 "
Shunt field	1,880 "
Series field	813 "
Commutating field	502 "
Core loss	3,400 "
Friction loss	5,400 "
Total	19,028 watts

The full-load efficiency is

$$\frac{250{,}000\times 100}{250{,}000+19{,}028}=92.9 \text{ per cent.}$$

An idea of the efficiency of machines can be gained by reference to the tabulation below:

EFFICIENCIES OF D.C. MOTORS IN PER CENT

Horsepower	½ Load	¾ Load	Full Load
½	52	58	65
1	58	65	73
10	83	85.5	86.5
50	85	88.5	89.5
100	88.6	91	91.5

The efficiency of d.c. generators would be about the same.

148. Stray Power Loss.—The mechanical losses and the core losses, taken together, are called the stray power loss. Sometimes the name constant loss is applied to these since they are practically independent of load. For a constant speed machine, since the mechanical losses vary with the speed, and the core losses with speed and flux, the stray power will vary with speed and with the e.m.f. generated by the armature. Hence, in a generator, operated at constant speed, the stray power loss will be slightly higher at full load than at no load. With a shunt motor, the stray power at full load is slightly less than at no load since the speed and flux both decrease with load. With series and compound motors, the change of stray power with load is greater because the speed change is greater. The stray power of either a generator

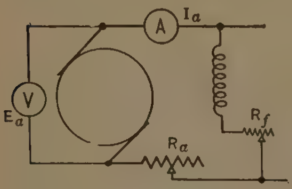

FIG. 127.—Connections for Stray Power Test.

or a motor can be determined by running the machine as a motor without external mechanical load. The connections are as shown in Fig. 127. In this test, the current in the armature I_a, and the voltage at the armature terminals E_a are measured. Then the power input to the armature is

$$P_a = E_a I_a \text{ watts.}$$

This power must all be lost in the armature because the power output is zero. The power P_a determined in this way is practically equal to the stray power loss because the current I_a and consequently the copper loss is small under the conditions existing in this test. In making this stray power test, the machine is operated at the correct speed by adjusting the field rheostat R_f. The correct armature voltage E_a is secured by adjusting the rheostat R_a. If it is desired to find the stray power loss which occurs when a certain machine is carrying full load, the rheostat R_f is adjusted to give full-load speed, and the voltage E_a is made the same as the voltage which the machine generated when it was carrying full load. Then the product $E_a I_a$ is the full-load stray power of the machine.

Example 1.—Referring to the generator of Example 1, Article 144, the resistance of the armature circuit is

$$R = 0.005 + 0.0008 + 0.0005 = 0.0063 \text{ ohm.}$$

The drop at full load is

$$1007.5 \times 0.0063 = 6.3 \text{ volts.}$$

The e.m.f. generated at full load is

$$E_a = 250 + 6.3 + 2 = 258.3 \text{ volts.}$$

A stray power test is made by running this generator as a shunt motor with a voltage at the armature terminals of 258.3 volts and a speed of 400 r.p.m., which is the full-load speed. The input to the armature is 34.2 amperes. The stray power at full load is then

$$P_a = 258.3 \times 34.2 = 8800 \text{ watts.}$$

149. Efficiency of a Motor from Name Plate Marking.—A motor always has marked upon its name plate the horsepower output and the amperes input, at normal supply voltage. The full-load efficiency, therefore, can easily be determined since the input is the product of rated volts and amperes.

Example 1.—The name plate of a d.c motor is marked 75 hp., 230 volts, 265.5 amperes. Calculate the full-load efficiency.

The input is $265.5 \times 230 = 61{,}100$ watts.
The output is $75 \times 746 = 55{,}900$ watts.
The full-load efficiency is

$$\frac{55{,}900 \times 100}{61{,}100} = 91.5 \text{ per cent.}$$

150. Determining Power Required to Drive a Machine.—It is frequently necessary to determine the amount of power required to drive a machine under particular operating conditions. This can be found by driving the machine with a motor and measuring its input. The approximate power required to drive the machine would be determined by measuring the current taken by the motor, with and without the machine. The difference in the current, multiplied by the motor voltage, would be the approximate power for the machine. The connections are as in Fig. 128. This method neglects the difference in the copper losses with and without the machine. A more accurate method is to measure the armature current and determine the armature resistance by the fall of potential method. The machine is driven by the motor, and readings of E_a and I_a obtained. The power, $P_a = E_a I_a$, is the input to the motor armature and includes the armature copper losses, the stray power, and the power required to drive the machine. The armature copper loss can then be calculated and the stray power loss determined by the method described in Article 148. Subtracting these from P_a gives the output of the motor or the power required to drive the machine.

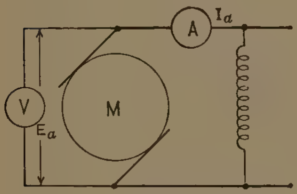

FIG. 128.—Connections for Determining Power to Drive a Machine.

Example 1.—A 230-volt shunt-wound motor has an armature resistance, including brushes of 0.259 ohm. When driving an unknown load, the input to the motor armature is 28.9 amperes.

The counter e.m.f. of the motor is therefore

$$E_a = 230 - (28.9 \times 0.259) = 222.5 \text{ volts.}$$

Running the motor at no-load output, an armature voltage of 222.5 volts, and a speed the same as when carrying the unknown load, the input to the armature is found to be 2.51 amperes. The stray power, according to Article 148, is

$$2.51 \times 222.5 = 558 \text{ watts.}$$

The armature copper loss, when loaded, is

$$28.9^2 \times 0.259 = 217 \text{ watts.}$$

The input to armature is

$$230 \times 28.9 = 6650 \text{ watts.}$$

The power required to drive the load is

$$6650 - (558 + 217) = 5875 \text{ watts or } 7.9 \text{ hp.}$$

PROBLEMS ON CHAPTER XVII

1–17. A 2-kw., 110-volt series generator has an armature resistance, not including brushes of 0.37 ohm and a field resistance of 0.5 ohm.

(*a*) What is the total I^2R loss in the machine at full load, including brush loss?

(*b*) What percentage is this of the output?

2–17. A 9-kw., 125-volt, compound-wound (long-shunt) flat-compound generator has an armature resistance, not including brushes, of 0.056 ohm, a shunt field circuit resistance of 45 ohms, and a series-field resistance of 0.022 ohm.

(*a*) At full-load output, what is the armature, I^2R loss, including brushes? What is the total field loss?

(*b*) At half load and constant terminal voltage, what is the total armature I^2R loss? What is the total field loss?

(*c*) Express results of (*a*) and (*b*) in per cent of output of generator.

3–17. The generator in Prob. 2–17 has four sets of brushes with two brushes per set. Dimensions of each brush in contact with commutator $1\frac{1}{8}$ inches × $\frac{1}{2}$ inch. Calculate the I^2R loss in the brushes at full-load output.

4–17. The generator in Prob. 2–17 has a core loss at full load of 600 watts and a friction and windage loss of 400 watts. What is the efficiency of the generator at full load?

5–17. A 100-kw., 118-volt, long-shunt, flat compound-wound generator has the following losses at full load: Core loss 1700 watts, I^2R armature and brushes 5770 watts, I^2R shunt field circuit 1462 watts, I^2R series field 1028 watts and commutating field 635 watts. Brush friction 415 watts. Bearing friction and windage 259 watts.

(*a*) What is the efficiency at full load?

(*b*) What is the efficiency at half load, assuming constant terminal voltage?

(*c*) What horsepower would be required to drive this generator at no load?

6–17. (*a*) What would be the approximate full-load current of a 60-hp., 230-volt motor?

(*b*) What would the current be at three-fourths load?

7–17. A 10-hp., 115-volt motor when carrying a certain load has an input of 63 amperes. What is the horsepower output of the motor?

8–17. What horsepower would be required to drive a 75-kw. generator at full load.

9–17. (*a*) What is the stray power of the generator of Prob. 4–17?

(*b*) What would be the efficiency of this machine at three-fourths load?

10–17. (*a*) What would be the full load stray power of the motor in Prob. 25–13?

(*b*) What would be the efficiency of this motor at full load?

11–17. The name plate of a motor reads: 45 hp., 115 volts, 330 amperes.

(*a*) Calculate the full-load efficiency.

(*b*) What would be the approximate horsepower output when the current input is 295 amperes and 380 amperes?

12–17. A 2000-kw., 575-volt 300-r.p.m., compound-wound (long-shunt) generator was operated as a shunt motor at 300 r.p.m. Resistance of armature not including brushes 0.00191 ohm, resistance of series field 0.00137 ohm, resistance of shunt-field circuit 28.3 ohms. When driven as a motor with an e.m.f. of 588.5 volts, the armature current was found to be 129 amperes.

(*a*) What would be the stray-power loss of this machine when running as a generator at full load?

(*b*) What would the full-load efficiency be?

13–17. A 230-volt motor has an input of 183 amperes when driving a certain load. When the load is disconnected, the input is 13 amperes. Calculate the approximate power required to drive the load.

14–17. The motor in Prob. 25–13 is used to measure the friction of a line shaft. The motor is driven from 230-volt mains and runs at a speed of 360 r.p.m. with the shafting connected. The input to the motor armature is 75 amperes. The motor is then disconnected from the shafting and operated at the same speed, by adjustment of the field rheostat, and the current input to armature is found to be 3.5 amperes. What horsepower is required to drive the line shaft?

CHAPTER XVIII

RATINGS AND GUARANTEES

151. Rating.—With a few exceptions, the rating of electrical machinery is a statement of the power which a machine will deliver (kilowatts for generators and horsepower for motors) without exceeding specified safe operating limits. Usually these limitations on output are heating, or commutation, or possibly both, but the limitation may be one of speed regulation for a motor or voltage regulation for a generator. In general, electrical machines will deliver more than their rated output, but, if so loaded, they may overheat, or may spark excessively. The rating of the machine is based upon operation at a particular speed, terminal voltage, frequency for a.c. machines, and duration of the load; i.e., whether continuous or intermittent. These quantities which form the basis of the kilowatt or horsepower rating are, therefore, sometimes called *rated speed*, *rated voltage*, etc. The manufacturer's rating, and the rated speed, voltage, etc., are stamped on a name plate attached to the machine.

The operation of a machine at a speed or voltage different from its rated value will, in general, change the heating and sometimes the commutation when carrying rated load. This is referred to in Chapters XII and XIII.

152. Heating of Electrical Machinery.—It was shown in Chapter XVII that there are losses in the copper and the iron core, and friction losses in bearings and on the commutator. All of these losses are dissipated in the form of heat so that the machine must necessarily be warmer than the surrounding air, which acts as a cooling medium. The temperature of a machine carrying a steady load will reach a steady value when the heat given off by the machine is equal to the heat produced in the machine due to its losses. The temperature which the windings of the machine can withstand is, however, very definitely limited if the machine is to continue in operation; therefore, in order that the machine may carry as large a load as possible, without exceeding a safe operating temperature, it is necessary to provide for free circulation of cool air through the machine and to have the temperature of this cooling air as low as is feasible. Sometimes, particularly in large turbine-driven alternators, forced ventilation is necessary, produced either by means of separate fans or by fans on the generator shaft.

The manufacturer specifies that a machine will carry its rated load under rated conditions of voltage, speed, etc., without exceeding a certain temperature. This guarantee is made on the assumption that the temperature and circulation of the air are both normal. Thus, if a machine is operated in a very small room or is boxed in so that the heated air cannot escape, or if the air passages are allowed to become clogged by dirt or dust, the machine will overheat and may burn out, even when carrying rated load. Furthermore, a machine can carry a heavier load for a short time, one-half hour for example, than it could continuously, so that in the statement of the rating the duration of the load is always stated.

153. Temperature Rise.—The temperature rise is the difference between the temperature of the cooling air and that of different parts of the machine. The life of insulating materials depends upon the temperature at which they operate. If the temperature is too high, the material may char, or become brittle, and finally be destroyed by the vibration of the machine. As a result of a large number of experiments, it has been found that the maximum safe operating temperatures for insulating materials are as follows:[1]

Class A—Cotton, silk, paper, treated or immersed in oil; also enameled wire	105° C.
Class A—Materials not treated or oil-immersed	90° C.
Class B—Insulation composed principally of mica or asbestos	125° C.

The temperatures given apply to the hottest point in the winding which, in general, is inside the winding. If the temperature is to be measured by thermometers, these must be placed on accessible portions of the winding where the temperature is lower than it is in the interior of the coil. It is standard practice to allow a difference of 15 degrees so that for insulation having a "hottest spot" or limiting temperature of 105° C., the temperature as measured by a thermometer placed on the surface of the winding must not be more than 90° C. It is standard practice to base the temperature rise on the surrounding air at a temperature of 40° C. (104° F.) which, therefore limits the maximum temperature rise as measured by thermometer to 50° C. for Class A insulation, as specified in the above tabulation.

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[1] Standardization Rules of the American Institute of Electrical Engineers.

154. Standard Ratings.—At the present time there are two standard ratings of electrical machinery, which differ in respect to their temperature guarantees. The 40-degree rating is the older of the two and has been standard for a number of years. At present, however, the 50-degree rating is quite commonly used. Under the 40-degree rating the manufacturer guarantees that no part of the machine (except the commutator which is allowed to run somewhat hotter), will have a temperature rise exceeding 40° C. when the machine is operated continuously at rated full load.[1] It is further guaranteed that the machine will carry a 25 per cent overload for two hours without exceeding a temperature rise of 55° C. It will be seen, therefore, that on the basis of Class A insulation, which is commonly used for moderate voltage machines, the actual temperature under full load, assuming 40-degree air, would be 80 degrees for the outside of the coil and 95 degrees for the hottest spot; while on overload the temperature of the hottest spot would be 110° C., which is slightly above the allowable temperature for this class of insulation. An occasional overload would, however, not permanently injure the insulation. Under the 50-degree rating, the machine has an allowable rise under full-load conditions of 50° C. with no overload guarantee. Both the 40-degree and 50-degree ratings guarantee momentary overloads of 50 per cent without excessive sparking. Such overloads are of so short a duration that the heating is unimportant, and the limit is set by the commutating ability of the machine.

The 50-degree rated motor, when fully loaded, would reach a temperature of 105° C. in the "hottest spot" so that it does not have the factor of safety that the 40-degree motor possesses. Advantage may be taken of the margin in the 40-degree motor to ensure continuous operation or to allow for a possible increase of load due to a change in the service requirements. If necessary, however, the same margin can be secured with a 50-degree motor if it is properly chosen. A 50-degree motor, when operated at about 0.83 times its rating, will give about 40° rise and approximate the temperature performance of a 40-degree motor.

In the usual industrial application, however, the loads are accurately known, and it is not necessary to allow this margin so that a 50-degree motor can be used instead of a 40-degree motor of the same horsepower rating with a considerable saving in cost and weight.

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[1] For detailed performance guarantees, see the Handbook of the Electric Power Club, an association of the principal motor manufacturers in the United States.

The heating of a machine is affected by operation at voltages or speeds above or below normal. The effect of this was discussed in Chapters XII and XIII. The effect of speed upon the rating of a particular size of machine is explained in Chapter XXXIX.

155. Effect of Operation at Speeds above or below Rating.—If a machine is operated at a speed lower than the rated value, the ventilation is poorer, and the machine may overheat. If a motor is operated at rated horsepower output and reduced speed, the machine will require a current larger than normal, and thus the armature may be overloaded. Operation above rated speed would give better ventilation and require less current to carry rated load, but, if the speed increase is considerable, the centrifugal strains may endanger the safety of the armature windings. It is the practice of manufacturers to guarantee that constant speed d.c. motors will operate successfully at rated load at a speed not more than 5 to 7.5 per cent above normal. Operation above rated speed may also have some effect on the commutation.

156. Standard Voltage Ratings.—The voltage ratings which have been adopted as standard by manufacturers in the United States are as follows:

D.c. motors	115, 230, and 550 volts
D.c. generators	125, 250, and 600 volts

The difference between the corresponding generator and motor voltage allows for voltage loss on the feeders.

PROBLEMS ON CHAPTER XVIII

1–18. The name plate of a certain compound motor reads as follows: 40 hp., 1150–1300 r.p.m., 230 volts, 145 amperes.

(*a*) What is the rating?
(*b*) What is the full-load speed?
(*c*) What is the no-load speed?
(*d*) What is the speed regulation?
(*e*) How much current does the motor require at full load?
(*f*) What is the full-load efficiency?

2–18. A compound-wound generator has the name plate marked as follows: 150 kw., 250 volts, 600 amperes, 600 r.p.m. What is the normal terminal current and voltage when the machine is delivering rated load?

3–18. The type and speed of a 10-pole, 550-kw., 550-volt generator (90 r.p.m.) are such that a rise of temperature of 85° C. in the armature dissipates 2.5 watts per sq. in. If the available radiating surface of the armature is 12,150 sq. in., the resistance 0.0125, and the core loss 11,000 watts, calculate the temperature rise of the armature at full load. Neglect field current.

4–18. A 150-kw., 250-volt generator has an armature surface of such a character that it radiates 0.04 watt per sq. in. per degree Centigrade rise in temperature. The armature resistance is 0.008 ohm, hysteresis loss in teeth 620 watts and in body 1140 watts; eddy loss in teeth 70 and in body 130. Radiating surface 2500 sq. in. Neglect field current.

(*a*) What is the temperature rise?

(*b*) What would be the rating of this machine, based on 40° C. rise?

5–18. During the heat run on a generator, the temperature of a field coil as measured by thermometer is 86° C. What is the approximate temperature of the hottest point inside the coil?

6–18. A certain motor is to be operated in a location where the temperature of the surrounding air is 50° C. The machine has class A insulation. What would be the maximum safe temperature rise for the windings as measured by thermometer.

7–18. The motor of Prob. 1–18 is rated on a 40-degree basis. What would be the allowable horsepower output on a 50-degree basis?

8–18. A 50 hp. motor rated on a 40-degree basis operates in a room having a temperature of 35° C. The temperature of the armature winding, as measured by thermometer, is 73° C. when the machine is carrying rated load.

(*a*) What is the approximate temperature of the hottest spot inside the winding when carrying rated load?

(*b*) What would be the approximate temperature of the hottest spot inside the winding when the motor is delivering 62.5 hp.?

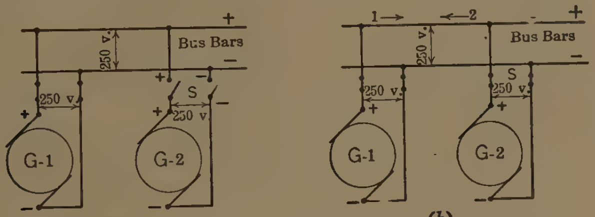
CHAPTER XIX

PARALLEL OPERATION OF D.C. GENERATORS

157. Necessity for Parallel Operation.—The reasons for the common practice of operating several generators in a power station in parallel are: (*a*) The ability to vary the number of generators in service to meet changing load requirements so that the machines may operate at approximately full load and the station efficiency may be maintained as high as possible. (*b*) Provision of excess capacity which can be used to replace a machine which requires repair or breaks down while in service. (*c*) Ability to care for increase in load by adding units to the station as load requirements grow. In some special applications, such as electrochemical work, where the load on a single circuit is large and it is desired to regulate each circuit independently, as in the case of electrolytic refining of copper, it is often found desirable to provide a separate generator for each portion of the load and to operate them independently, but, for the usual requirements for power, lighting, and railway service, parallel operation is used in order to obtain maximum flexibility of operation.

158. General Principles.—Two d.c. generators which are to be operated in parallel must be designed to produce the same terminal voltage, and terminals of like polarity must be connected together so that the voltages of the two machines will oppose each other and there will be no circulating current through the armatures of the two machines. Thus, in Fig. 129, assume that each machine generates 250 volts at its terminals and that No. 1 is connected to the bus-bars. Then, if machine No. 2 is adjusted to produce exactly 250 volts at its terminals and is connected as shown, i.e., positive to positive, there will be zero difference of potential across the switch *S* and it can be closed without any current circulating through the armatures of the two machines. Assume that the switch *S* is closed and there is no external load on the bus-bars Fig.129 *b*. Then No. 1 machine will tend to send current through the circuit between machines in the direction of arrow 1 while machine No. 2 will tend to send current in the direction of arrow 2. The voltage between terminals of either machine or the bus-bars will be 250 volts as indicated. If the e.m.f. of machine No. 1 be decreased slightly by weakening the field, then No. 2 will send a current through the two armatures in the direction of the arrow 2. On the other hand, if the e.m.f. of No. 1 were made larger than No. 2, then No. 1 would send current through the two armatures in the direction of arrow 1. The amount of current which will flow will depend upon the resistance of the two armatures and upon the difference in the voltages of the two machines. Since, as a rule, the armature resistance is low, a difference of a few per cent will usually be sufficient to allow full-load current to flow.

If there is an external load on two machines connected in parallel, the total load will divide between the two machines according to their voltage characteristics as will be explained in subsequent articles. In order that two machines shall operate successfully in parallel and shall be stable, that is, not change the proportion of load carried by each machine as long as there is no change in external load or change in field rheostat, each machine must have a drooping voltage characteristic so that it tends to " shirk " the load. If a machine had a rising characteristic, then, with increased load, its voltage would increase and this would cause it to take more load which would further increase the load until the machine would carry all the load and might even drive the other machines as motors. On the other hand, a drooping characteristic results in a decrease of voltage as the load increases so that the machine tends to take less load and, if all the machines have drooping characteristics, they all tend to shirk the load and it divides between the machine according to their individual resistances. In this case the division is stable and will not change unless the load changes or the field rheostats are adjusted. Shunt wound or separately excited generators, as a rule, have drooping characteristics while compound generators may have rising characteristics, and, therefore, special precautions are necessary when operating the latter in parallel. This is discussed in Article 160.



(a) (b)

FIG. 129.—Diagrams Showing Principle of Parallel Operation.

159. Parallel Operation of Shunt Generators.—Suppose that two shunt generators which are to be operated in parallel have voltage characteristics as in Fig. 130. Let the voltage of each machine be adjusted to 250 volts at no load and the machines paralled as described in the preceding article. (See Fig. 129.) The machines are assumed to have the same rating but the voltage of No. 2 falls off more rapidly than No. 1 as the load is increased. Now suppose that a total load of 535 amperes is connected to the bus-bars; then each machine will tend to take a portion of the load, and the voltage at the bus-bars will drop. Since the two machines are in parallel, the terminal voltage of both machines must be alike; therefore, the load will divide in such a way as to give the same terminal voltage on each machine. For the load of 535 amperes, this occurs when No. 1 takes 325 amperes and No. 2, 210 amperes and the terminal voltage is 200 volts. Similar points for any other total load can be found by drawing a horizontal line through the characteristics at the position corresponding to the given total load. Although No. 1 takes more than its share of the load, operation is perfectly stable because, if the voltage of No. 2 is decreased, due to a momentary slowing down of its prime mover, its voltage decreases, and No. 1 will take additional load. This would cause the terminal voltage of No. 1 to decrease to correspond with the change in No. 2. On the other hand, if the voltage of No. 2 is increased, it takes load from No. 1 and the voltage of each machine increases. For proper parallel operation, the voltage characteristics of the two machines should be alike so that they will deliver the same terminal voltage when they are carrying the same percentage load. For machines of unequal size, the total load should divide in proportion to their ratings.

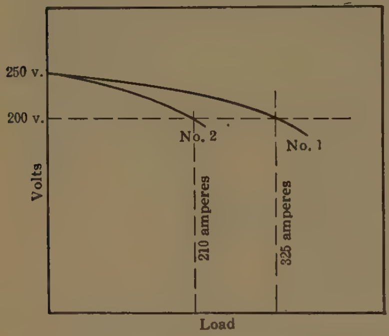


Fig. 130.—Parallel Operation of Shunt Generators.

160. Parallel Operation of Compound Generators.—As a rule compound generators are either flat or over-compounded and, therefore, if paralleled only at their line terminals, they would be unstable. In order to ensure stability under all conditions of operation, it is necessary to parallel the armatures of the machines through an equalizer connection (Fig. 131). If the equalizer connection were not provided, it would not be practicable to operate the machine in parallel, as can be seen by considering the following conditions. Suppose that the equalizer connection is open so that the current in each series field is the same as the current in the corresponding armature. If No. 1 machine should slow down slightly this would tend to throw more of the load on No. 2, but its voltage would not drop as in the case of shunt machines (see Article 159) so that it would tend to take still more of the load. The result would be that No. 2 would take all of the load and would in addition send current through No. 1 in such a way as to change it to a motor. But this current would flow through the series field in such a direction as to oppose the shunt field and, hence, would decrease the voltage of No. 1 still more. The result would, therefore, amount practically to a short-circuit on machine No. 2 which would open the machine circuit breakers and shut down the plant. When compound machines are paralleled without an equalizer connection, they are in a condition of unstable equilibrium and a slight change in voltage of one machine is sufficient to cause the trouble just described.

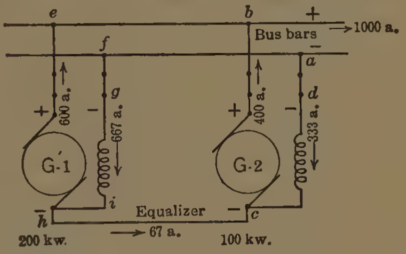


FIG. 131.—Connections for Compound Generators in Parallel.

When an equalizer connection is used (Fig. 131) the series fields are in parallel and the load current divides between the several series fields inversely as their resistance, and this division is practically independent of the current carried by the various machine armatures. Thus, referring to Fig. 131, suppose the total load is 1000 amperes and the machine ratings are 200 kw. and 100 kw., respectively. Then for proper operation No. 1 machine should carry about 667 amperes and No. 2, 333 amperes. If the series field circuits have the proper resistances, the current will divide as shown in Fig. 131. Suppose for some reason, however, that No. 1 machine slows down slightly. Then the armature of No. 1 will tend to take less than 667 amperes, but the current through the series field of No. 1 will remain the same since the division between the fields depends upon their resistance and not upon the current in the machine armature. A condition like that shown in Fig. 131 might then occur. The armature of No. 1 would supply 600 amperes and No. 2, 400 amperes, a current of 67 amperes flowing in the equalizer from No. 1 to No. 2 machines. Under these conditions, it will be seen that the series field excitation of both machines is constant as long as the system load is constant;

hence, any tendency of one machine to take more load causes a drop of voltage at the armature terminals or, in other words, these machines have drooping voltage characteristics as measured at the armature terminals, which we have already seen (Article 158), is necessary for successful parallel operation. Under normal conditions, the current in the armature and series field of a machine would be alike so that no current would flow in the equalizer. It is apparent that if the current in the several series fields is to be properly apportioned, the resistances by the different paths must be inversely as the rating of the generators. This resistance includes that of the connections *a–d* and *f–g*. The resistance of the equalizer connection *c–b* must also be low, for instance, $\frac{1}{4}$ to $\frac{1}{3}$ the resistance of the series field and lead of the largest machine. When machines of different types are paralleled, as, for example, an engine-driven generator and a turbo-generator, there will be considerable difference in the series field resistance even for machines having the same rating. The turbo-generator, having a low resistance series field, will take more than its share of the load current and in order to correct this, additional resistance must be placed in series with its series field, preferably between points *a–d* (Fig. 131). Compound machines which are to be operated in parallel must have approximately the same voltage characteristic. Compounding shunts must, therefore, be adjusted on each machine so that, when running independently, the same amount of compounding is secured. If the characteristic curves are not similar for all loads, then one machine may take more than its share of the load. Division of a given total load can be adjusted by changing the shunt field rheostat as is done with shunt machines. Commutating pole machines are likely to have a flatter voltage characteristic than non-commutating pole machines and, therefore, they will tend to take more than their share of the load. The difference, however, is usually not enough to cause trouble, and, in any event, this type of machine can carry a temporary overload more easily than the non-commutating pole machine. When commutating pole machines are operated in parallel with others, it is important that the brushes shall be properly set on the neutral because a slight forward shift of brushes in a commutating pole machine will give a rising, rather than a drooping voltage characteristic at the armature terminals, and this will cause unstable operation even with an equalizer. With commutating pole machines, the equalizer is connected to point *i* (Fig. 131) and the commutating winding is connected between points *i* and *h* so that the commutating pole winding will always carry the same current as the armature. It should be noted that the ammeter should be connected to the terminal opposite to the series field (Fig. 131) so that it will always indicate the armature current and thus give an idea of the load on the machine. When three-wire compound generators are operated in parallel, the series fields are divided, with half on each side of the system, and two equalizers are required.

PROBLEMS ON CHAPTER XIX

1–19. Two shunt-wound, 125-volt generators are arranged to be connected in parallel. The resistance of the armature circuit of each machine is 0.05 ohm, including brushes.

(*a*) How much current would circulate between the two machines if they were paralleled when the e.m.f. of one is 125 volts and of the other 130 volts? Neglect change in shunt field and assume no external load.

(*b*) What becomes of the power represented by this circulating current?

2–19. Two shunt generators, A and B, have a voltage regulation as follows:

Machine A, 125 volts full load, 135 volts no load.

Machine B, 125 volts full load, 145 volts no load.

The full-load current rating of each machine is 100 amperes. Assume that the external characteristic is a straight line.

(*a*) Plot the two external characteristics.

(*b*) Assume that machine A is carrying 100 amperes load and is adjusted to 125 volts on the bus-bars. Assume that the field rheostat of machine A is not changed, when B is thrown in parallel and the e.m.f. of B adjusted until it takes one-half the ampere load. What would be the voltage at the bus-bars?

3–19. Referring to the machines of Prob. 2–19, assume that they are in parallel and each carrying 100 amperes with 125 volts on the bus-bars. Assume that the field rheostats are not changed and that the total load drops to 150 amperes.

(*a*) Determine the current carried by each machine and the voltage at the busbars.

(*b*) How should the field rheostats be adjusted to bring the voltage back to normal (125 volts) and to produce the proper division of load?

4–19. A 1500-kw. load is to be carried by two generators rated respectively 1000 kw. and 800 kw. How much load should each carry for most satisfactory operation?

5–19. A 2000-kw., 575-volt, compound-wound generator has a no-load voltage of 525 volts and a full-load voltage of 575 volts. When the series field is not used, the voltage falls to 400 volts at full-load current. Assuming that the change in voltage due to the series winding is proportional to the load current, what would be the voltage of the machine if it were carrying full load as a compound machine and the series field coil of another similar compound generator not in operation were thrown in parallel with it?

6–19. Referring to the machines illustrated in Fig. 131, determine the sizes of rubber-covered wire required for positive, negative, and equalizer for each machine, allowing for a 25 per cent overload. Assume 250 volts at bus bars.

CHAPTER XX

SPECIAL TYPES OF D.C. GENERATORS

161. Three-wire balancers are motor-generator sets designed to provide the neutral circuit for a three-wire system. It is possible to operate a three-wire system (see Chapter XXIII) from two 125-volt machines connected in series. (See Fig. 164.) There is greater economy, however, in using a single 250-volt machine to supply the system. When this is done, some means must be provided to care for an unbalanced load. Where a balancer-set is used for this purpose the arrangement may be as shown in Fig. 132. The main generator *G* is designed to operate at 250 volts. The balancer consists of two shunt-or compound-wound machines, coupled together and connected in series between the positive and negative wires of the system. The neutral wire is connected to the junction. When the load on the system is balanced, the two machines operate as motors, and no current flows in the neutral. If the load is unbalanced (Fig. 132), the neutral current divides, part going through each of the machines. If we neglect the losses in the two machines, the current would divide equally, giving 12.5 amperes in each armature. The machine which is on the negative side (*G*) is a generator and that on the positive side (*M*) is a motor. The main generator always carries the entire load, and the balancer-set serves only to transfer power from the lightly loaded side to the heavily loaded side of the system. In the particular example given in Fig. 132 the main generator furnishes 100 amperes at 250 volts directly to the load, and it also supplies 12.5 amperes at 250 volts to the balancer-set. This current flows through the motor *M* and thence through the neutral to the load on the

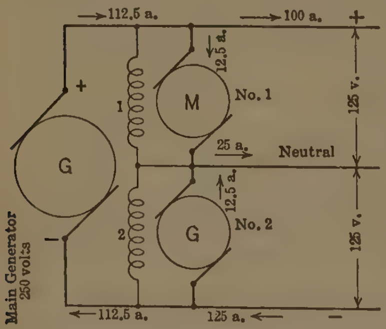


FIG. 132.—Connections of a Three-wire Balancer.

negative side. The generator *G* also furnishes 12.5 amperes which flows through the neutral and the load on the negative side. If the positive side of the system had the heavier load, machine 1 would become a generator, and machine 2 a motor. In every case the machine on the heavily loaded side would be a generator. In practice, because of the losses in the armature, the motor would carry slightly more and the generator slightly less than half the neutral current. With shunt-wound balancers connected as shown, the voltage on the two sides would not be exactly constant at all loads because of the drop in the armatures of the two machines. If the connections of the fields are interchanged, so that No. 1 field is connected to the negative side and No. 2 to the positive, better voltage regulation results. By the use of compound windings on these machines, the regulation can be still further improved. Since the unbalanced load on well-designed three-wire systems is small, usually not more than 10 per cent, it is apparent that the capacity of the balancer-set need be only a small fraction of the main generator capacity.

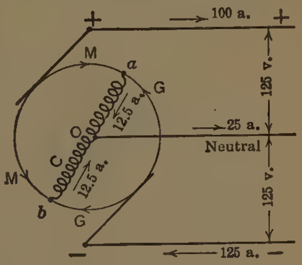


Fig. 133.—Connections of a Three-wire Generator.

162. A three-wire generator is a machine designed to provide the neutral connection for a three-wire system without the use of a balancer-set. The machine is a d.c. generator designed to operate at the voltage between the outside wires, 250 volts in the example shown in Fig. 133. A coil having high reactance[1] and low resistance is connected permanently between two diametrically opposite points *a–b* on the armature. Since the voltage induced in the armature coils is alternating, an alternating e.m.f. is applied at the terminals of this coil. Only a small current flows, however, because the coil has a high reactance. The middle point *O* of this coil is always at a potential midway between *a* and *b* so that the neutral of the system can be connected to point *O*. When there is an unbalanced load, the neutral current divides at point *O* and flows through *oa* and *ob* to the armature winding. There is very little opposition to the flow of the neutral current because this is direct current, and the coil has a low resistance. The high reactance of the coil does not cause any voltage loss when direct current flows. The action of the neutral current in the armature is like that in the balancer-set, this current flowing with the generated e.m.f. on the heavily loaded side (– in Fig. 133) and against this e.m.f. on the other, thus giving a motor action. Three-wire generators may be built with the balance coil *C* mounted on the armature, in which case connection is made between point *O* and the neutral by means of a single slip-ring. The more common arrangement, however, is to employ one or two balance coils mounted separately from the generator, in which case two or four slip-rings are required to make connection with the proper points on the armature.

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[1] The coil is wound on a laminated iron core. See Article 224 for explanation of term reactance.

163. Ward-Leonard System.—When starting a d.c. motor it is necessary to apply a reduced voltage to the armature terminals. Usually this is accomplished by means of the motor starter (Chapter XXI) which inserts a resistance in the armature circuit during the starting period. This arrangement must be used if the motor is supplied from a constant-potential source. There is, however, a power loss in the starting rheostat, and with large motors which are frequently started and stopped, this loss would result in a large expense for power. The Ward-Leonard system provides a method of starting d.c. motors without the use of a starting resistance, by applying a variable voltage to the motor armature. This voltage is produced by an adjustable-voltage generator. The system is illustrated in Fig. 134. The motor *M*, which in this case is driving a mine hoist, is a shunt-wound machine with its field supplied from a constant potential source produced by the exciter *E*. The motor armature is connected directly to a d.c. generator *G*, without any extra resistance, but with a circuit breaker *CB* to protect it against overload. The generator field is also supplied from the exciter *E* and both exciter and generator are driven by an a.c. motor, *ACM*, which is connected to the power supply. The field excitation of the generator can be varied and reversed by means of the controller *FC* and, therefore, the voltage applied to the motor armature may be adjusted and the motor

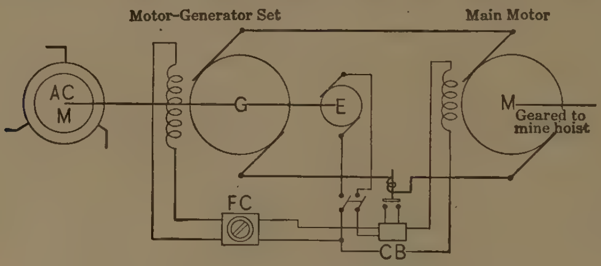

FIG. 134.—Ward-Leonard System.

reversed by means of this controller. When lowering a load, the motor may be allowed to act as a generator and feed back power to the motor-generator set, which will return power to the supply. In some cases, additional speed adjustment is secured by weakening the motor field. The generator is provided with commutating poles and a compensating winding (Article 141) in order that it may operate without sparking at the weak field strength required to produce a low voltage. Sometimes the motor has a compensating winding also. The principal advantage of the Ward-Leonard system is that the speed is adjustable through a wide range, without rheostatic losses; hence, it operates with high efficiency at all speeds. The disadvantage is that a special motor-generator set is required for each motor, and the losses in this set are high if there are considerable periods when the motor is not running.

The **Ilgner system** is a modification of the Ward-Leonard system designed to reduce the maximum power required to drive the motor. A flywheel is added to the motor-generator set, and the motor *ACM* which takes power from the supply is designed to slow down when a heavy load is thrown upon the set so that the flywheel can assist in carrying the load. The Ilgner system is required where the power taken by the motor-generator set is large as compared with the capacity of the supply system. The Ilgner-Ward-Leonard system is extensively used for large mine hoists and for rolling mills where a reversing drive is required.

164. Generators for automobiles must operate at varying speed and therefore differ in some respects from d.c. generators which are designed to operate at constant speed. (See Chapter XII.) The generator used on gasolene-driven automobiles is employed to charge a storage battery which furnishes power for lighting and for starting the engine. To avoid overcharging the battery, it is necessary to regulate the generator voltage as the speed of the car changes. Some of the more important methods used in practice are: (*a*) Regulation by varying the resistance of the field circuit of a shunt-wound generator by means of a relay which is operated by the generator voltage. As usually designed, this relay vibrates rapidly, thus opening and closing a short-circuit around a resistance in series with the shunt field winding. In some designs, the relay is operated by the current supplied by the generator and in others by a combination of current and voltage. (*b*) Regulation by the use of a differentially wound, compound generator. The series winding is so proportioned as to limit the current supplied by the generator to a safe value regardless of the speed. (*c*) Regulation by field distortion, commonly called the "third-brush" method. The third-brush system of regulation which is at present the method most commonly used, is illustrated in Fig. 135. The main brushes *A* and *C* are connected to the storage battery through the compound relay, which is designed to disconnect the battery from the generator when the speed drops so low that the generator no longer charges the battery or when the battery tends to feed back into the generator for any cause. The generator is shunt-wound, with one terminal of the field winding connected to the third brush *B* and the other to the main brush *A*. The voltage across the field, and hence the field flux, depends upon the voltage between *A* and *B*. When the generator is carrying a small armature current, the flux in the air gap is approximately uniform, but as the armature current increases, there is a distortion of this flux due to armature reaction (Article 137) and the flux is shifted in the direction of rotation, giving two pole tips, *ss*, with strong flux and two tips, *ww*, with weak flux. Since the voltage between brushes *A* and *B* depends on the flux, this voltage will decrease when the armature reaction causes a decrease of flux. The shunt field, being connected to *AB* therefore receives a reduced voltage when this flux is shifted. When the speed of the car increases there is a tendency for the armature voltage to increase, thus sending more current through the battery. This increased current, however, produces a greater armature reaction and therefore weakens the generator field. This prevents an excessive rise of generator voltage at high speed and thus avoids overcharge

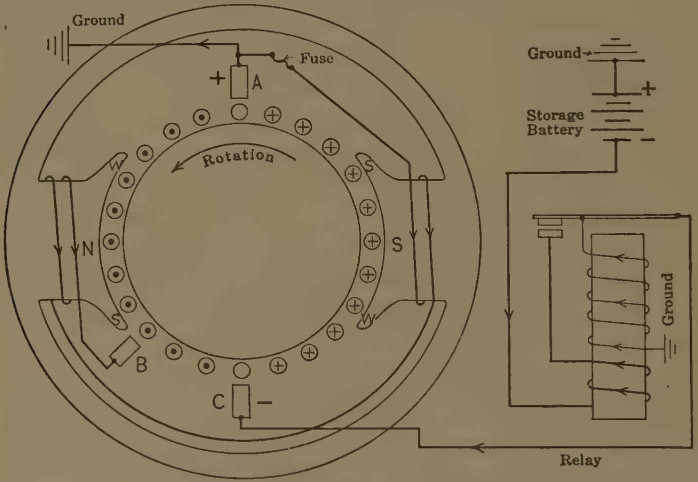

FIG. 135.—Third-brush Generator as Applied to Automobiles.

of the battery. The third brush is movable to permit adjustment of the voltage of the generator to suit varying operating requirements. If the battery does not receive sufficient current to keep it in good condition, the charging current can be increased by shifting the brush in the direction of rotation, that is, towards C (Fig. 135), since this will raise the voltage on the field circuit. Shifting in the opposite direction would reduce the charging current.

165. Arc-welding Generators.—Special types of d.c. generators have been developed to supply current for welding metals by means of an electric arc. Where several welding operators are to be supplied with current, a low-voltage compound generator is sometimes used. This generator is designed to operate at from 60 to 75 volts. Each welding circuit supplied from the generator is provided with a series resistor which reduces the voltage at the arc to the proper value, and also prevents excessive variations in the welding current as the length of the arc is varied.

Where a single welding operator is to be supplied, or where economy is important, a specially designed machine is used. This machine automatically maintains an approximately constant current with variations in the arc length. The generator made by one manufacturer gives 60 volts on open circuit and 20 volts when welding. The welding current can be adjusted for values from 200 to 75 amperes, and is held constant automatically.[2] The principle of operation is illustrated in Fig. 136 *a*. The machine has a two-pole armature, although there are four main poles. They are not, however, alternately of N and S polarity, but are arranged in pairs of like polarity. The armature, therefore, carries two fluxes Φ_m and Φ_c at right angles. The strength of flux Φ_c depends upon the excitation of poles N_c, S_c, and is not affected by the flux in N_m, S_m, except for the saturation of the portion of the circuit which carries both fluxes. The load current is taken from the brushes AC; hence, the armature reaction, due to this current, has a direction coinciding with the brush axis. The armature reaction, represented by line OF (Fig. 136 *b*), can be resolved into two components at right angles. The component OD acts in the same direction as the main flux Φ_m, while OE opposes the cross flux Φ_c. The main field circuit N_mS_m is saturated so that OD cannot affect the main flux, but the cross-field circuit N_cS_c is not saturated; hence the armature reaction weakens the cross flux. A third brush, B, is placed between two poles of the same polarity. It will be seen that the voltage between brushes A and B is constant, because it is

[2] See "A New Type of Arc Welding Generator," General Electric Review, May, 1920.

due to the main flux Φ_m which is not affected by armature reaction. A shunt-winding is used on all four poles of the machine and this winding is connected to brushes A and B; hence, the shunt excitation is constant. The voltage between brushes B and C decreases with increased armature current because this voltage is due to the flux Φ_c which is decreased by armature reaction. The poles N_c S_c have a series winding (Fig. 136 *c*) which is so connected that it opposes the shunt excitation of these poles. Hence, the series winding also tends to decrease the voltage between B and C. Since the voltage BC decreases with the load, the terminal voltage AC also decreases with load and the machine gives a practically constant current. Adjustment of the current is effected by cutting in more or less of the series turns on poles N_c and S_c. The machine is also provided with commutating poles. The complete diagram of connections is shown in Fig. 136 *c*.

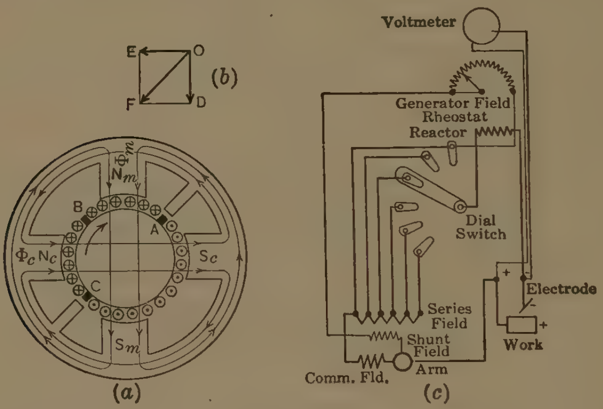

FIG. 136.—Arc-welding Generator. *General Electric Co. type.*

PROBLEMS ON CHAPTER XX

1-20. A three-wire, 110–220-volt system has a load of 50 amperes on the positive side and 35 amperes on the negative. The current in the neutral wire is 15 amperes. The neutral is supplied through a three-wire balancer set.

(*a*) Draw a diagram of connections showing main generator and balancer set and indicate the voltage and current for each machine. Neglect losses in balancer set.

(*b*) If the motor carries 62 per cent of the neutral current, calculate the current in each machine.

(*c*) Calculate the currents when the positive load is 75 amperes, the negative 100 amperes and the neutral 25 amperes. Neglect losses.

2–20. A 125–250-volt, three-wire system has a load on the positive side of 1000 amperes and on the negative side 900 amperes. The neutral current is 100 amperes. If each machine of the balancer set has an efficiency of 85 per cent, calculate the current in each machine, including the main generator.

3–20. Referring to Prob. 1–20 (*a*), calculate:

(*a*) The power supplied by the main generator.
(*b*) The power supplied to each side of the three-wire system.
(*c*) The power supplied to, and delivered by the balancer set.

4–20. Referring to Prob. 2–20, calculate:

(*a*) The power supplied by the main generator.
(*b*) The power supplied to each side of the three-wire system.
(*c*) The power supplied to, and delivered by the balancer set.

5–20. If the three-wire system described in Prob. 2–20 were supplied by a three-wire generator:

(*a*) What would be the current for positive and negative terminals of the machine?

(*b*) What would be the d.c. current in the balance coil?

CHAPTER XXI

D.C. MOTOR STARTING AND CONTROLLING DEVICES

166. Starting Requirements.—We have seen in Article 115 that the resistance of d.c. motor armatures is so low that a starting rheostat must be used for all except very small machines. Furthermore, if the speed is to be adjusted, a rheostat must be used, either in the armature or the field circuit. A description of the important types of d.c. motor starters and controllers is given in the articles following.[1]

167. Capacity of Starters.—Starters are rated according to the sizes of motors for which they are designed. A different starter is required for different voltages and sizes of motors. With d.c. motors, if the starter is too large, the current taken by the motor will be excessive; if too small, the motor may not start and the starter may be burned out. Ordinary starters are intended for occasional use only and do not have sufficient capacity to carry the starting current continuously or to start the motor at very frequent intervals. A rheostat designed to be left in the armature circuit continuously, for the purpose of adjusting the speed, is called a speed regulator.

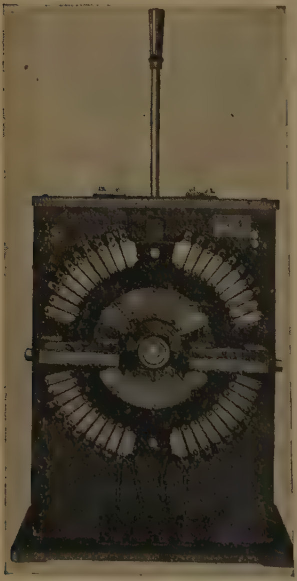

FIG. 137.—Face-plate Starter. Reversing type. *The Electric Controller & Mfg. Co.*

168. Starters for Series Motors.—For motors which are to be started only at fairly long intervals, a starter with no-voltage release, similar to that described in Article 169, is used. Where motors are to be frequently started, stopped and reversed, as in crane service, either face-plate starters (Fig. 137) or drum controllers (Fig. 140) are used. The motor is reversed and brought up to speed by movements of a single handle.

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[1] For more complete information on this subject, see "Controllers for Electric Motors," by H. D. James.

169. Starters for shunt and compound motors which have to be started infrequently are of the face-plate type (Fig. 138) for sizes up to about 50 hp. For larger motors, a number of separate switches, either hand- or electrically-operated, are used to cut out the starting resistance. The illustration (Fig. 138) shows a face-plate starter connected to a compound motor, but the same type of starter can be used for series or shunt motors by modifying the connections as indicated in the illustration. These starters are usually provided with a low-voltage release. This consists of an electromagnet which holds the rheostat arm in the running position as long as there is voltage on the motor circuit. In case of a failure of the line voltage, the magnet releases this arm and it is returned to the off position by a spiral spring. Hence the motor will not be damaged if the voltage is thrown on again. The magnet does not release until the motor has slowed down, because, for a short time, the motor acts as a generator and keeps the magnet energized. The low-voltage release magnet is either connected directly across the line (as in Fig. 138) or is placed in series with the shunt field winding. When connected across the line, the strength of the magnet is independent of the current in the motor field. If connected in series with the shunt field, the magnet might become so weak (due to weakening the shunt field) that it could not hold the arm in the running position. If the magnet is in the field circuit, however, it will release and stop the motor if the field circuit is opened; whereas, if connected across the line it would not release and the motor might run away. Usually, in such cases, there is sufficient load on the motor to prevent it from running away and the heavy current taken by the motor would be sufficient to open the circuit breaker or blow the fuses. Starters of this type are frequently equipped with an **overload release**. This consists of a coil in series with the armature which operates similarly

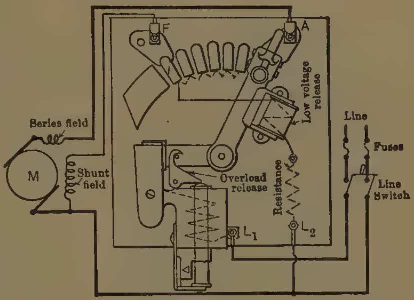


FIG. 138.—Face-plate Starter. With low-voltage and overload release. Connections shown are for a compound motor. For a shunt motor, *A* is connected directly to armature. For a series motor, connection to *F* is omitted. When arm is on the starting-point (1), the motor receives full field. As the armature resistance is cut out by moving arm to the right, this resistance is cut into field circuit. The value of this resistance is so low, however, compared with that of the field, that it has no effect upon the speed of the motor.

to a circuit breaker. Figure 138 shows a starter with overload and no-voltage release as made by one manufacturer. The overload release is only used on comparatively small motors (up to about 15 hp.). For larger motors, separate circuit breakers or fuses are used. It is now standard practice to enclose this type of starter in a metal box with the operating handle accessible from the outside. With this arrangement, the operator cannot come in contact with any part of the circuit and the starter mechanism is protected from dirt and possible mechanical injury. Where the motor is frequently started and stopped, as in machine-tool service, the drum-type controller (Article 171) is more satisfactory because it will withstand harder usage.

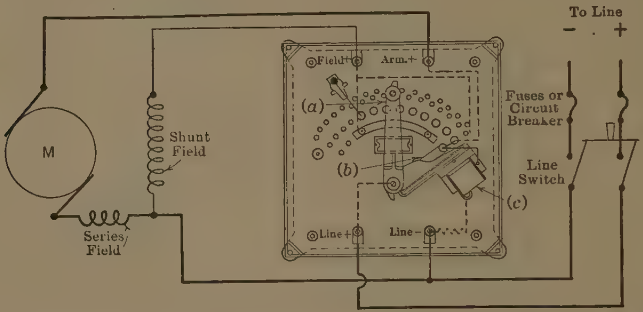


FIG. 139.—Compound Starter. For starting and adjusting speed of shunt and compound motors. The upper row of buttons is connected to the field resistance, the middle row to the armature. The curved segment below the buttons is used to short-circuit the field resistance when starting. There are two movable arms (*a*) and (*b*) pivoted to the same hub. These move together when the motor is being started. Arm (*a*) makes contact with both rows of buttons and (*b*) with the segment. When the arms have reached the running position (to the extreme right) arm (*a*) can be moved back, leaving (*b*) as shown. This inserts resistance in the field circuit. A low-voltage release coil (*c*) causes both (*a*) and (*b*) to return to the off position if the supply fails.

170. Compound starters are arranged to cut in the field resistance after the motor is up to speed (Fig. 139). Movement of the arm (*a*) towards the running position first cuts out the starting resistance with full field on the motor. After the starting resistance is all out, a movement of arm (*a*) in a reverse direction inserts resistance in the shunt field. (See also Fig. 8 *b*.)

171. Drum Controllers.—For adjustable speed, shunt or compound motors operating machine tools, the drum controller is best (Fig. 140). This contains contacts for starting the motor and also contacts for inserting field resistance, as in compound starters. These controllers are also arranged to reverse the motor by a change in the direction of motion of the handle. They are used in capacities up to about 50 hp. For larger motors the controller operates electromagnetic switches, mounted separately, which make the necessary connections for starting.

172. Automatic Starters.—The starters previously described are all hand operated, and in the hands of a careless workman may be so operated as to damage the motor by starting too quickly or to burn out the starter by running too long in the starting position. There are also cases where the motor must be started and stopped in response to changes in pressure, water level, etc. Automatic controllers are now produced to meet a large variety of requirements, but the machine-tool controller illustrated in Fig. 141 is typical of common practice. The controller shown is for a shunt or a compound motor controlled by start and stop push buttons. The speed is adjustable by means of a field rheostat. When the line switch is closed, the motor can be started by means of the "start" button which completes a circuit from the + side of the line through the "stop" button and the operating coil of the line contactor (No. 1) to the − side of the line. This closes the contactor and the motor starts with all the resistance in circuit. Contactor No. 1 is held in the closed position by the circuit through 9, 0, and contact 0 which closes when contactor No. 1 closes. Pushing the stop button opens this circuit and stops the motor. The coil operating the accelerating contactor (No. 2) is connected across the motor terminals and a portion of the starting resistance; hence, the voltage on the operating coil of the contactor increases as the counter e.m.f. of the motor increases with increase in speed. When the speed has increased sufficiently, contactor No. 2 closes and the starting resistance is short-circuited. Contactor No. 3 is a full-field relay which provides full field while the motor is accelerating. The operating coil of this relay is connected across the starting resistance and hence this relay closes as soon as the line contactor closes. When contactor No. 2 closes there is no voltage across the coil of relay No. 3 and its contacts open, thus cutting in the field rheostat. Low-voltage protection is provided on this type of starter. If the voltage drops too low the line contactor (No. 1) will open, thus opening con-

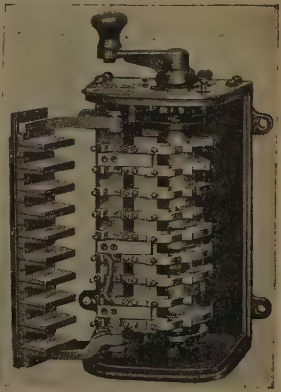
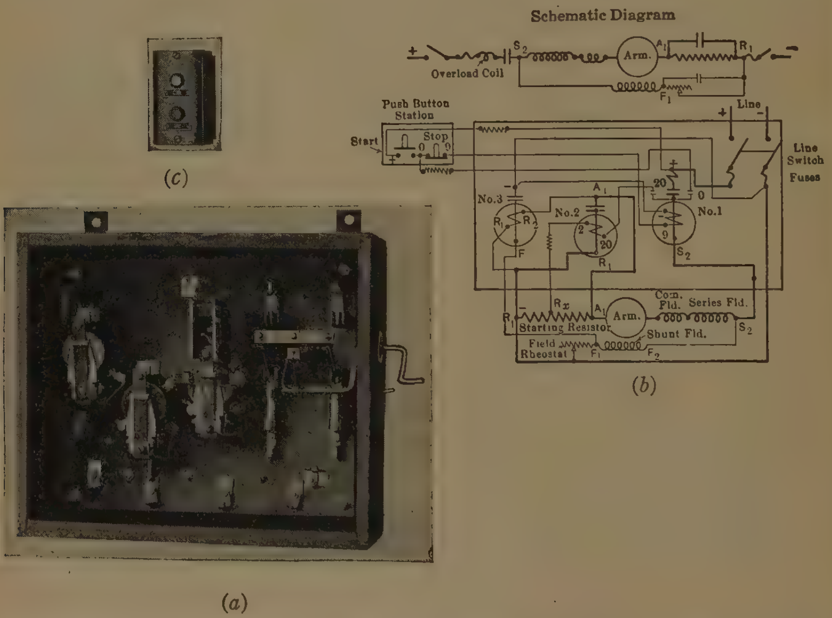


FIG. 140.—Drum Controller.
The General Electric Co.

tact 0. The motor will not start again until the start button is pushed. Automatic starters may be provided with devices to give dynamic braking (see Article 173) and to provide for reversing the motor. The type of starter illustrated is used for small motors up to about 10 hp. For large motors, there are more accelerating contactors, and these are closed as the starting current decreases to a predetermined value.

173. Dynamic Braking.—If a motor driving an elevator or other load having considerable inertia is disconnected from the line, the load will drive the motor. Under these conditions, a shunt motor will become a generator without any change in the connections. A resistance connected across the armature will absorb power from the load and slow down the machine. The same effect can be produced with series motors if the field winding is connected temporarily to the line (through a suitable resistance). This action is used either to make a quick stop or to retard a descending load. Dynamic braking **for making a quick stop** is employed for elevators, printing presses, and machine tools. The controller is arranged to connect a resistance to the armature circuit after it is disconnected from the line. The



(c)

(b)

(a)

FIG. 141.—Automatic Starter. (a) Front view with door removed; (b) Connection diagram; (c) Start and stop push-button. *The Westinghouse Elec. & Mfg. Co.*

field circuit is kept connected to the line to give a high braking effect. As the motor slows down, the voltage generated by the armature decreases, and hence the current through the resistance would decrease. To obtain the greatest braking effect, therefore, provision must be made to reduce the resistance as the machine slows down. The final stop is made by a friction brake which is controlled by an electromagnet. Dynamic braking **for retarding a descending load** is employed on cranes and ore-handling machinery. The motors are generally series wound, so the field winding is placed across the line in series with a suitable resistance. The resistance across the armature is then adjusted until the required speed is secured. Sometimes the armature is connected to the line in series with a resistance. In this case, the machine will return power to the supply.

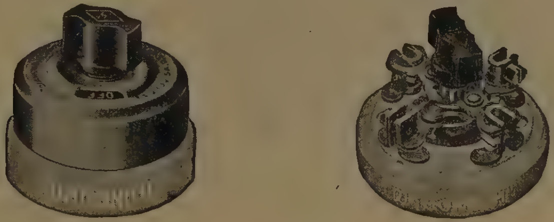

(*a*) (*b*)

FIG. 142.—Rotary Snap Switches. Double pole 10 amperes. *The Hart Mfg. Co.*

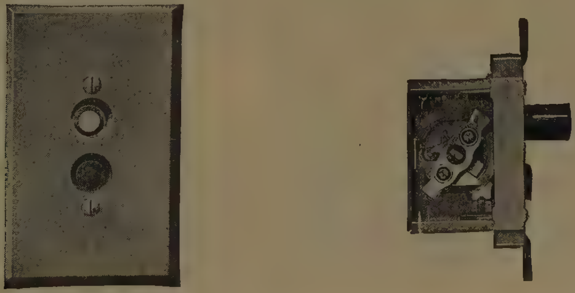

(*a*) (*b*)

FIG. 143.—Push-button Switch. (*a*) Front view showing switch plate· (*b*) Side cut away to show mechanism. *The Hart Mfg. Co.*

174. Switches.—For controlling lamps, small motors, or circuits not exceeding about 10 amperes, either rotary snap switches (Fig. 142) or push-button switches (Fig. 143) are extensively employed. These switches are made single- or double-pole to suit requirements. As a rule switches of these types are not used on circuits above 250 volts, although snap switches can be obtained for currents up to 30 amperes at 600 volts.

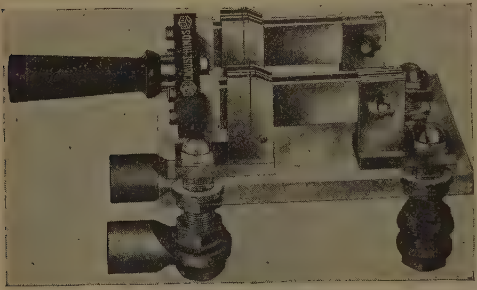
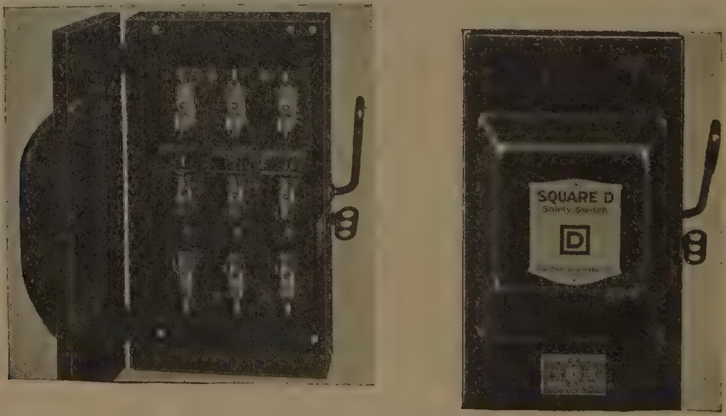


FIG. 144.—Knife Switch. 800-ampere, 250 volts, double-pole. *The Crouse-Hinds Co.*

Knife switches consist of one or more blades made of flat copper bar, which are hinged at one end and are designed to enter clips or jaws, thereby closing the electrical circuit (Fig. 144). These switches are made single-, double- or triple-pole and also double-throw; i.e., there are two clips per pole, one on either side of the hinge so that the blade can be used to close either of two circuits. Connections to the circuit are made either at the back of the supporting panel as shown in Fig. 144 or on the front. Knife switches frequently have combined with them suitable fuse holders or clips to save space on the switchboard or panel board. Except when the switches are grouped together on a switchboard in a power station or on a distributing panel board, it is customary to enclose the switch and fuses in a metal box, making what is called a safety-type switch (Fig. 145). This arrangement is used extensively for controlling motor and lighting circuits.



(*a*) (*b*)

FIG. 145.—Safety-type Switch. *The Square D Co.*

175. Fuses are devices intended to melt and open a circuit whenever the ampere load on the circuit exceeds a safe value. Plug fuses (Fig. 146) have a piece of lead-alloy fuse wire mounted in a porcelain cup with a metal cover. A screw-thread contact is provided, similar to the base on an ordinary incandescent lamp. Plug fuses are intended for use on small capacity circuits, the most common size being made in various capacities up to 30 amperes at 250 volts. Cartridge fuses (Fig. 147) contain a fusible strip or link enclosed in a fiber tube which is filled with a non-conducting powder. Both plug and cartridge fuses are classed as enclosed fuses since the fusible strip is enclosed in such a way that the flash or arc caused by the melting or "blowing" of the fuse is confined to the casing, and hence there is little danger from fire. Cartridge fuses are made in capacities from 3 to 600 amperes and for voltages of 250 and 600 volts. Fuses intended for 600-volt service are longer and will not fit the same fuse receptacles or cutouts as the 250-volt type. Fuses of different ampere capacity are also designed for different sizes of receptacles; for example, all capacities from 3 to 30 amperes fit the same cutouts, while those from 35 to 60 amperes take a different size, and so on. Figure 148 shows one type of cutout used for cartridge fuses.

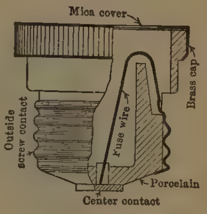
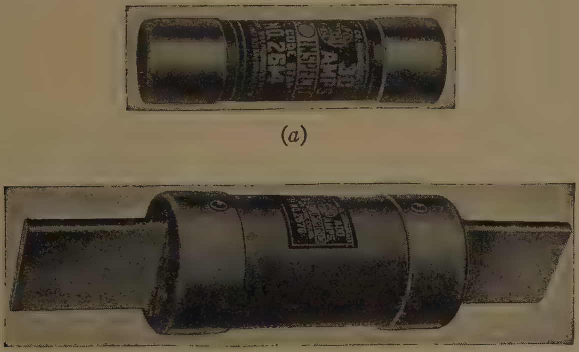


FIG. 146.—Plug Fuse.



(a)

(b)

FIG. 147.—Cartridge Fuses. 250-volt, National Electric Code Standard. (*a*) Ferrule-type contacts, 3- to 60-ampere capacity; (*b*) Knife-blade type contacts, 61- to 600-ampere capacity. *The Johns-Pratt Co.*

176. Circuit breakers are devices designed to open the circuit when an overload occurs, and, therefore, they perform the same functions as fuses. Circuit breakers must be used for the protection of circuits of large capacity, since fuses are not built in sizes above

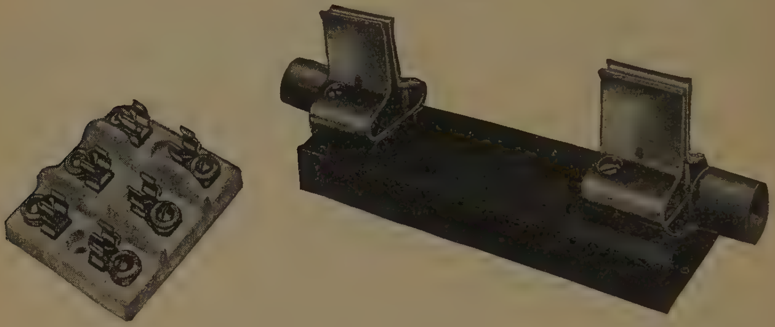

(*a*) (*b*)

FIG. 148.—Cartride Fuse Cutouts. (*a*) Ferrule contact type; (*b*) Knife-blade contact type. *The Johns-Pratt Co.*

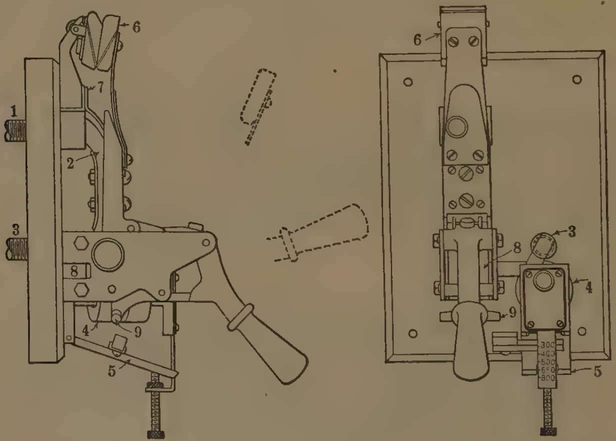


FIG. 149.—Circuit Breaker. Single-pole, 600 Volts, 400 Amperes, Plain Overload. Current enters at stud (1) and passes through laminated brush (2), lower contact block (8) and coil (4) and to stud (3). If the current exceeds the value for which the breaker is set, armature (5) is lifted by current in coil (4), thus striking arm (9) and unlatching the toggle. The breaker thus opens, taking the position shown in dotted lines. As the breaker opens, the brush leaves the contact block first, then (7) opens and finally the circuit is broken by the carbon contacts (6) so as to prevent burning of the main contacts. Breaker can be set to trip at a lower current by raising (5).

600 amperes. Circuit breakers cost more than fuses, but they are preferable for use on circuits subject to frequent overloads because the service can be restored quickly and cheaply after it has been

interrupted. Motor feeders are usually protected by circuit breakers instead of fuses. A circuit breaker of the air-break type is shown in Fig. 149. Circuit breakers can be set to open at various current values, the range being from about 80 to 160 per cent of the normal rating of the breaker. Circuit breakers are made in one-, two-, three-, and for small sizes, four-pole types. Both fuses and circuit breakers of the type shown in Fig. 149 can be used on either direct- or alternating-current circuits. For the latter service, there is also an oil-insulated type of circuit breaker which has a number of advantages. This is described in Chapter XXXVII.

CHAPTER XXII

STORAGE BATTERIES

177. A storage cell consists of positive and negative plates or elements, assembled together, and immersed in an electrolyte. Two or more cells, connected together, constitute a battery. There are two types of storage batteries in general use; the lead-acid type, which has a positive plate of lead oxide, and a negative plate of spongy lead, immersed in electrolyte of dilute sulphuric acid; and the nickel-alkaline or Edison type which has a positive plate of nickel oxide and a negative plate of iron in a finely divided form, immersed in a solution of potassium hydrate. With either type of cell, electricity is produced by chemical action involving the active material on the plates and the electrolyte. After a definite amount of electricity has been delivered by the discharge of the battery the active material of the plates is restored to its original or charged condition by passing electricity through the battery in a reverse direction. The commercial storage or secondary batteries are completely reversible and can be repeatedly charged and discharged until the plates are worn out by the dislodging of the active material. In this respect they differ from primary batteries, such as Leclanché and Edison-Lelande cells, where new plates must be used after the battery has been discharged.

LEAD BATTERIES

178. Principle of Operation.—The active material of a plate is that part which undergoes a chemical change when electricity flows through the battery. This active material is supported by a frame or grid of pure lead or lead alloy which serves the double purpose of conducting the current and carrying this active material. The construction of the plates is described more in detail in a later paragraph. When the battery is charged, the active material of the positive is peroxide of lead which is brown in color and is rather porous. The active material of the negative plate is gray in color and consists of pure lead in a spongy, porous form. The electrolyte is a solution of pure sulphuric acid in pure water, the strength of the solution varying in different types of batteries. The strength also varies with the state of charge or discharge.

When a battery is supplying energy (discharging), sulphuric acid from the electrolyte unites with the lead peroxide on the positive plate to form lead sulphate and water, while lead sulphate is also formed on the negative plate. The equation for this action is

$$\underset{\substack{\text{Lead}\\ \text{peroxide}\\ +\text{plate}}}{\mathrm{PbO_2}} + \underset{\substack{\text{Sulphuric}\\ \text{acid}}}{\overset{\text{Charge}}{\mathrm{2H_2SO_4}}} + \underset{\substack{\text{Lead}\\ -\text{plate}}}{\mathrm{Pb}} = \underset{\substack{\text{Lead}\\ \text{sulphate}\\ +\text{plate}}}{\mathrm{PbSO_4}} + \underset{\substack{\text{Lead}\\ \text{sulphate}\\ -\text{plate}}}{\overset{\text{Discharge}}{\mathrm{PbSO_4}}} + \underset{\text{Water}}{\mathrm{2H_2O}}. \quad (41)$$

Thus, while a battery is discharging, lead sulphate is formed in both plates and the amount of acid in the electrolyte is reduced; hence, the specific gravity of the electrolyte, which is heavier than water, decreases on discharge. For practical reasons, which will be explained later, the battery cannot be allowed to discharge until all the active material on the plates has been changed to lead sulphate. If current from an external source is passed through the battery in a reverse direction so that the current *enters* at the positive terminal and leaves at the negative terminal, the battery can be charged or restored to its original condition. The equation for charging is the reverse of that given above, that is, the current dissociates water in the electrolyte into hydrogen (H) and oxygen (O). The hydrogen appears at the negative plate and the oxygen at the positive. These convert the sulphate formed during discharge and produce spongy lead on the negative and lead peroxide on the positive plate. At the same time, sulphuric acid is formed, thus increasing the specific gravity of the electrolyte.

179. Construction.—The plates in common use are divided into two classes according to the method of formation of the active material. In the Planté type the active material, either sponge lead (negative) or lead peroxide (positive), is formed directly from pure metallic lead, by electro-chemical means. The Fauré or pasted type uses oxides of lead, such as red lead or litharge, in the form of a paste which is pressed into a supporting frame or grid composed of metallic lead. The paste on the plates is converted to spongy lead for negatives or changed to lead peroxide for positives by immersing the plates in electrolyte and passing current through them in the proper direction. One kind of Planté plate, called the Manchester, is made of spirally wound strips of corrugated sheet lead which are held in a lead alloy frame (Fig. 150). The negatives used with this positive have a frame or grid which is cast from an alloy of lead and antimony and is arranged with numerous small pockets or windows which are filled solidly with lead oxide paste (Fig. 151). Another common form of pasted plate is the Exide (Fig. 152). For rough service, such as is encountered with electric trucks where the battery is worked hard, the Exide-Ironclad type of positive is very effective (Fig. 153). This has the active material solidly packed into hard-rubber tubes which have

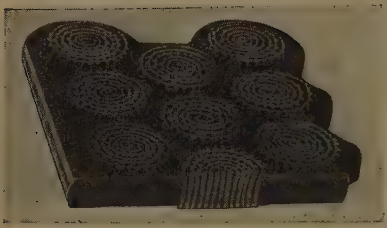


FIG. 150.

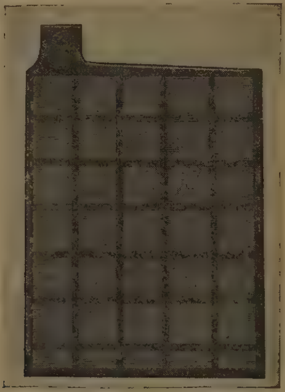


FIG. 151.

FIG. 150.—Section of Manchester Plate. *The Electric Storage Battery Co.*
FIG. 151.—Box Negative Plate. *The Electric Storage Battery Co.*

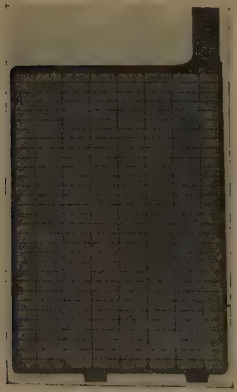
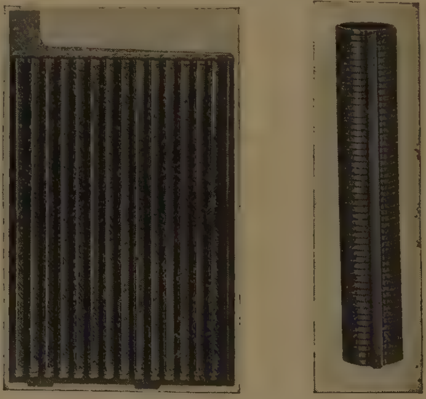


FIG. 152.



(*a*) (*b*)

FIG. 153.

FIG. 152.—Negative Plate for Ironclad Type of Battery. *The Electric Storage Battery Co.*

FIG. 153.—Positive Plate for Exide-Ironclad Type of Battery. (*a*) Complete plate; (*b*) Hard-rubber tube. *The Electric Storage Battery Co.*

very narrow slots cut in them to allow free circulation of the electrolyte, but still prevent escape of the active material. In the center of each tube is a lead core which serves to conduct the current to the frame. The tubes are assembled vertically in a lead frame, the lead cores in the tubes being welded to this frame. Pasted negatives (Exide type) are used with Exide-Ironclad positives.

It was seen in the preceding article that, on discharge, water is formed at the surface of the active material, thus reducing the density of the electrolyte and tending to stop the chemical action. Therefore, continued discharge requires that there shall be a free circulation of electrolyte through the pores of the active material. The thickness of plate which is used depends upon the service; where a high rate of discharge is required, a thin plate must be used because the electrolyte cannot penetrate a thick plate fast enough to maintain the action.

The Planté type of plate lasts longer, but is more bulky than the pasted type. For this reason, pasted plates are used for portable batteries, electric vehicles, starting and lighting service on gasoline automobiles, and similar applications. The pasted plates are made in different thicknesses to suit varied requirements. Thin pasted plates give higher discharge rates than thick plates but have a shorter life. Exide-Ironclad batteries are recommended for electric locomotive and similar service because of their high discharge ability, high electrical efficiency, rugged construction, and long life. There is usually one more negative than positive plate, so that both sides of each positive plate can be worked evenly to reduce the tendency to buckle.

For the electrolyte, it is important to use only chemically pure acid and distilled or other pure water, because impurities in either the acid or the water will shorten the life of the plates. The density of the electrolyte is higher for the vehicle type of battery because the volume of electrolyte is less in order to reduce the size and weight of the battery. When fully charged, the specific gravity for the vehicle type of cell is from 1.270 to 1.280. For stationary batteries it is about 1.210 to 1.240. On discharge, the electrolyte density of the vehicle type of battery drops from 100 to 150 points, that is, a battery having a specific gravity when charged of 1.280 would when fully discharged have a density of 1.180 to 1.130. The variation in stationary batteries, where there is more electrolyte, is not so great.

The positive and negative plates of a cell are interleaved and are kept from touching each other by separators placed between the plates. The usual form of separator is a thin sheet of specially treated wood which covers the entire surface of the plate and thus effectually prevents short-circuits caused by particles of active material or foreign substances bridging across between plates. In some cases, thin sheets of perforated hard rubber, combined with grooved wooden sheets are used.

For stationary batteries, glass jars are used for small size cells and lead-lined wood tanks for large cells. For portable and vehicle batteries, hard rubber or similar composition is used for the jars. Stationary batteries are usually covered at the top by glass plates to reduce evaporation and prevent the spreading of acid spray during charge. Portable and vehicle batteries have sealed covers with a vent for the escape of the gases formed during charging. Figure 154 shows a sectional view of a vehicle type of cell and Fig. 155 a large size stationary battery such as is used by central stations as a standby to furnish service in case of failure of the generating apparatus.

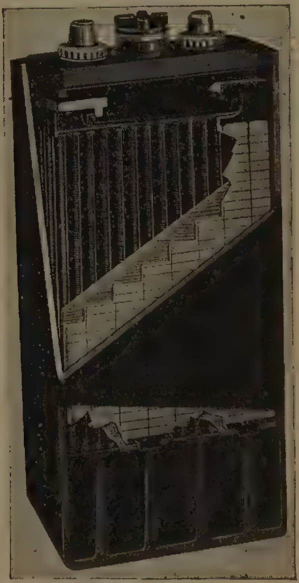


FIG. 154.

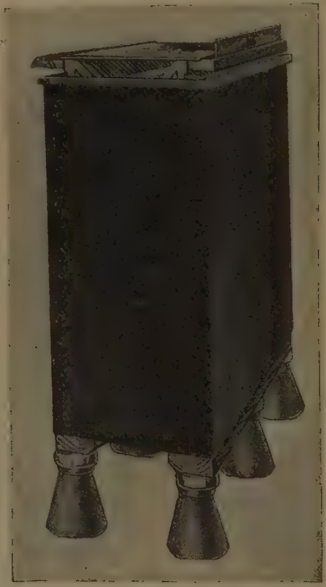


FIG. 155.

FIG. 154.—Exide-Ironclad Cell. *The Electric Storage Battery Co.*
FIG. 155.—Central-station Type of Cell. *The Electric Storage Battery Co.*

180. Discharge Characteristics.—It was shown in Article 178 that when a lead-acid battery is delivering electricity (discharging), the active material on both positive and negative plates is changed to lead sulphate. In practice, however, it is not feasible to continue the discharge until all the active material has been changed, but the discharge is stopped when the voltage of the cell reaches a certain minimum. The ampere-hour capacity and the minimum voltage for discharge rates are stated by the battery manufacturer. This capacity varies with the rate at which the discharge takes place and the minimum voltage allowed; at high rates the ampere-hour capacity is less than at low rates.

Example 1.—A certain Exide-ironclad battery has a normal rating of 476 ampere-hours, that is, it will deliver 79.5 amperes for six hours. If, however, it is discharged at the one-hour rate, it can deliver only 294 amperes for one hour or 294 ampere-hours. This is only 60 per cent of the capacity at the six-hour rate.

The decrease in capacity shown in Example 1 is due to the inability of the electrolyte to penetrate the active material rapidly enough to sustain the high discharge rate. During discharge, water is formed in the pores of the active material (see Equation 41, Article 178) and this dilutes the electrolyte in contact with the active material. The discharge capacities given in Example 1 are based on continuous discharge at the given rate. If, however, the discharge is intermittent so that there are periods of rest, during which the electrolyte has time to diffuse through the pores of the active material it is possible to obtain approximately the same ampere-hour capacity at the high as well as at the low rates. Stationary batteries of the smaller sizes usually have their capacity based on the eight-hour rate and the large central station batteries on the one-hour rate. The capacity of vehicle type batteries is usually based on a six-hour rate. The capacity of the automobile starting and lighting batteries is sometimes given in ampere-hours based on a 5-ampere discharge rate. After a battery has been used for a considerable time the ampere-hour capacity decreases, due to loss of active material which flakes off and falls to the bottom of the cell. The life of a battery depends a great deal upon the service in which it is used, but in general a battery of the Planté or the ironclad type should give 1000 cycles of charge and discharge while the pasted type of plate gives from 200 to 600 cycles. The manufacturer of the Exide-Ironclad battery claims a life of three and one-half years in truck service. Starting and lighting batteries used on passenger automobiles have a life of about two years. A lead-acid type of battery is capable of discharge at very high rates. Thus an ordinary starting and lighting battery for automobile use which has a rating of 5 amperes for twenty hours, will readily deliver 275 amperes when starting the engine.

The voltage of a lead-acid battery is approximately 2 volts per cell regardless of its ampere-hour capacity. The terminal voltage on discharge depends upon the rate, because of the drop due to internal resistance. The voltage characteristic of a battery on discharge is shown in Fig. 156. The voltage at the beginning of the discharge is about 2.06 volts and after discharge for six hours it has dropped to 1.75 volts. If the battery were discharged at the one-hour rate the initial voltage would have been 1.98 volts and the final voltage, after one hour, would have been 1.68 volts. Voltage readings are, therefore, a guide to the amount of charge remaining in the battery, but they must be taken when current is flowing, since the open-circuit voltage gives no reliable indication of the condition of the battery. A better indication than the voltage reading is the specific gravity, which decreases during discharge. The amount of decrease varies with different types of batteries, so that in any particular case it is desirable to obtain this information from the battery manufacturer.

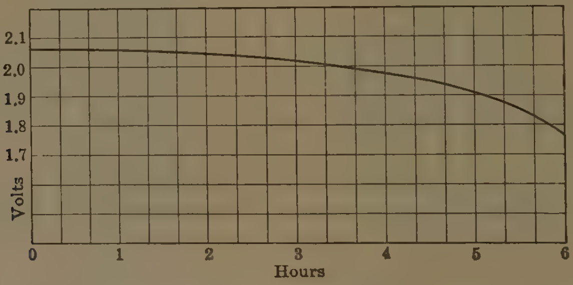


FIG. 156.—Voltage Characteristic of Lead Battery on Discharge at Six-hour Rate.
The Electric Storage Battery Co.

In general, for the portable or vehicle type of cells, there will be a drop of 100 to 150 points; that is, if the specific gravity when fully charged is 1.280, the discharge should be stopped when the specific gravity reaches 1.180 to 1.130. With electric vehicles it is customary to provide an ampere-hour meter which indicates how much electricity the battery has delivered so that the battery can be charged before the rated output has been exceeded.

Low temperatures decrease the available capacity of a storage battery, the ampere-hour capacity being reduced about 0.65 per cent per degree Fahrenheit. If a battery is well charged, it may be left inactive without danger of freezing. When discharging, the battery is heated to some extent by the I^2R loss in the battery. It is, therefore, desirable to house the battery when used during the winter for vehicles or other outdoor service because of the gain in capacity which is secured.

When a battery has been discharged, it is desirable to charge it as soon as possible and not allow it to stand idle and discharged for any considerable length of time. If it is not charged promptly, the lead sulphate, which is formed in the pores of the active material during discharge, changes to a hard, insoluble form which is very difficult to remove by charging. When this occurs the battery is said to be sulphated, and a long-continued charge at a low rate is necessary to restore the battery to normal condition.

181. Charging.—Storage batteries are commonly charged by connecting them to a constant potential d.c. source with a resistance in series to permit adjustment of the charging current. The positive terminal of the supply is always connected to the positive battery terminal so that the current during charge will pass through the battery in the opposite direction to that of discharge. A diagram of connections is shown in Fig. 157 and a typical charging unit is illustrated in Fig. 9. Alternating current cannot be used directly to charge a battery, but, by means of a rectifier (see Chapter XXXIV), alternating current can be changed to a pulsating direct current, which is suitable for charging batteries. If compound generators are used to charge storage batteries, care must be exercised to prevent the batteries from feeding back into the generator and reversing its polarity in case of a decrease in generator voltage. A resistance in series with the battery will help to prevent this but, if the battery is the principal load on the generator, it is best to use a shunt-wound machine. With compound generators, the use of a reverse-current circuit breaker is desirable. This will disconnect the battery in case it tends to discharge into the generator.

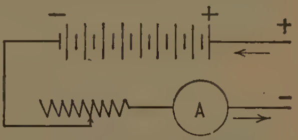


Fig. 157.—Connections for Charging a Battery.

It was shown in Article 178 that, during the process of charging, water in the electrolyte was dissociated into hydrogen and oxygen by the action of the charging current. The hydrogen appears at the negative pole and reduces the lead sulphate to spongy lead while the oxygen which appears at the positive pole converts the lead sulphate to lead peroxide. At the same time sulphuric acid is formed and the specific gravity of the electrolyte increases. At the beginning of the charging period, all the hydrogen and oxygen are used in converting the lead sulphate, but when the battery has become nearly charged most of the lead sulphate has been converted, and the current, if maintained at a constant value, is larger than is necessary for converting the lead sulphate. The excess current then produces hydrogen and oxygen gases which appear as bubbles in the electrolyte. The battery is then said to be gassing. If the current is reduced, the gassing will stop and the charge can be continued at a lower rate until gassing begins again. It is desirable, therefore, to reduce the current as the battery approaches the completion of the charge. In any event, the charging rate should be limited to a value which will not cause excessive gassing nor produce a cell temperature in excess of 110° F. Heavy gassing should be avoided as it dislodges particles of active material and thus shortens the life of the battery. In practice, charging is continued until the cell gases freely or the specific gravity rises to its maximum value which is approximately 1.210 for stationary batteries and 1.250 to 1.280 for portable and vehicle batteries. (See Article 179.) At regular intervals of a week or more, an overcharge is given at a low rate to reduce completely the lead sulphate and to bring up any cells which are somewhat lower than the others. The terminal voltage at the beginning of the charge is about 2.2 volts per cell. This voltage increases gradually until, near the end of the charge, it reaches about 2.3 volts. The voltage then rises rapidly to 2.4 to 2.6 volts. The values given are only approximate as they vary considerably with the charging rate, the age of the plates, and the temperature.

182. Applications.—Lead-acid batteries of the stationary type are used extensively by companies furnishing direct current (Edison service) for power and lighting purposes in large cities. These batteries are depended upon to maintain the service in event of complete failure of the generating or converting machinery and therefore have a large ampere-hour capacity. Another application is that for isolated plant lighting, the small farm-lighting type being designed for 32 volts and the large installations for 110 volts. Storage batteries are regularly used in large central stations for operating the oil switches, for emergency lights, and in some cases for a reserve source of generator excitation. Batteries of comparatively small capacity are extensively used for railway signal systems, both for operating of yard interlocking plants and for track signals. Telephone, fire alarm, and telegraph systems also use storage batteries extensively.

The portable type of battery is employed for gasoline-driven passenger automobiles, for starting and lighting, and for propulsion of electric vehicles. Electric locomotives, equipped with storage batteries for propulsion and of a weight of 50 tons or more are in use for switching purposes in railway yards and for industrial railways. Storage batteries are a necessary part of the equipment of all electrically lighted steam railway cars. These batteries are usually charged by axle-driven generators, which also supply current to the lamps when the car is in motion.

THE EDISON BATTERY

183. In the nickel-alkaline or, as it is generally called, the Edison battery, the chemical reaction on charge and discharge is one of oxidation. In the charged state the active material of the positive plate is nickel oxide and, of the negative plate, metallic iron in a finely divided condition. The electrolyte is a 21 per cent solution of potassium hydrate. On discharge, the active material in the positive plates is reduced, forming a lower oxide of nickel and the oxygen goes to the negative plate where it oxidizes the iron, forming iron oxide. On charge, the process is reversed, the current decomposing some of the electrolyte producing hydrogen at the negative plate, which reduces the iron oxide to metallic iron while the oxygen liberated at the positive plate changes the lower oxide of nickel to a higher oxide of nickel. The process is represented by the following equation:

$$\overset{\text{Charge}}{}\qquad\qquad\qquad\qquad\overset{\text{Discharge}}{}$$

$$\underset{\substack{\text{Negative}\\\text{plate.}\\\text{Charged}}}{3Fe} + \underset{\substack{\text{Potassium}\\\text{hydrate}}}{8KOH} + \underset{\text{Water}}{4H_2O} + \underset{\substack{\text{Positive}\\\text{plate.}\\\text{Charged}}}{6NiO_2} = \underset{\substack{\text{Negaplate}\\\text{discharged}}}{Fe_3O_4} + \underset{\substack{\text{Potassium}\\\text{hydrate}}}{8KOH} + \underset{\text{Water}}{4H_2O} + \underset{\substack{\text{Positive}\\\text{plate}\\\text{discharged}}}{2NiO_4}. \qquad (42)$$

It will be seen that the amount of water and potassium hydrate does not change during charge or discharge, therefore the specific gravity of the electrolyte does not change while the battery is in service. The action during charge and discharge is simply a transfer of oxygen from one plate to the other, the electrolyte serving as the medium by which this is accomplished.

184. Construction.—The active material of the positive plates is contained in small tubes of nickel-plated steel which is perforated with a large number of fine holes. The material, in the form of nickel oxide, is solidly packed in the tubes with alternate layers of pure flake nickel in order to secure the necessary conductivity. The active material of the negative plates is contained in flat pockets also made of perforated nickel-plated steel. The positive tubes and the negative pockets are then assembled in nickel-plated steel frames, different capacities of plates being produced by using more or less of the individual units. The plates are separated by hard-rubber rods and spacers, and are contained in a nickel-plated, sheet-steel tank with seam, bottom and cover welded in place. A vent hole with valve is provided in the cover. The valve allows escape of the gases formed in the cell, but prevents free access of air to the cell and thus prevents the electrolyte from combining with carbon dioxide in the air. An assembled view of an Edison battery is shown in Fig. 158.

185. Charge and Discharge Characteristics.—A typical set of charge and discharge voltage curves is given in Fig. 159. The batteries are rated at the ampere-hour capacity based on a five-hour discharge. The charging rate in amperes is the same as the five-hour-discharge rate but the charge is maintained for seven hours. It will be seen from the curve (Fig. 159) that the average voltage on discharge is 1.2 volts per cell. On charge, at normal rate, the voltage rises to about 1.80 or 1.85 volts per cell and a cell may be considered to be fully charged when the voltage remains constant for thirty minutes when charging at a constant current rate. Voltage readings vary with the temperature and density of the electrolyte. Usually the required amount of charge is determined from an ampere-hour

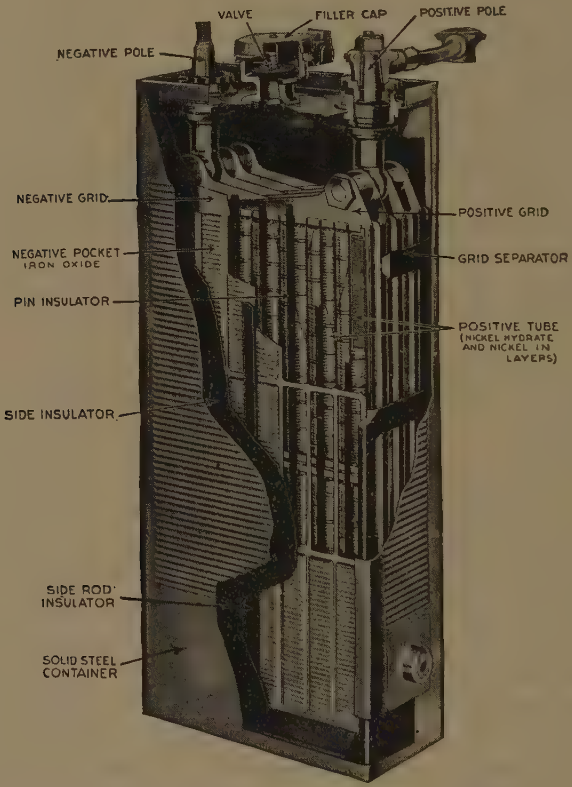


FIG. 158.—Edison Cell. *The Edison Storage Battery Co.*

meter which is so connected that it records both charge and discharge and makes proper allowance for the additional ampere-hours required on charge. Edison batteries may be charged at rates higher than normal, provided the temperature is not allowed to exceed 115° F. It is possible to charge at three times the normal rate for thirty minutes. Towards the end of the charge the cells gas freely and, since the gases given off are explosive, a naked flame should never be brought near the battery vent while the cell is being charged.

The Edison battery has a considerably higher internal resistance than the lead battery and therefore cannot give as high rates of discharge due to the danger of overheating and the voltage drop in cells. For vehicle work, a discharge 50 per cent greater than normal is allowable and rates as high as six times normal can be allowed for short periods.

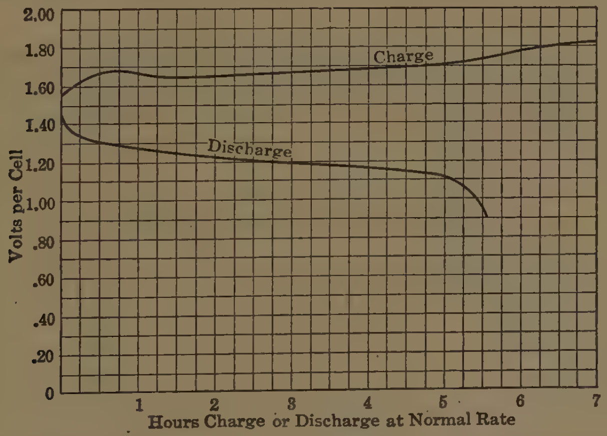


FIG. 159.—Voltage Characteristics of an Edison Cell on Charge and Discharge. *The Edison Storage Battery Co.*

Edison batteries have a reputation for long life. Edison batteries can remain idle for long periods partly or wholly discharged without damage to the plates. They can also be discharged to zero voltage and charged in reversed direction, without injury.

Since the electrolyte does not change in chemical composition during charge or discharge (see Article 183) there is no change in specific gravity as the battery discharges. The normal specific gravity of the electrolyte is 1.200. The specific gravity gradually decreases with use and should be renewed when it reaches a value of about 1.16. The interval between solution renewals averages between two and three years. Low temperature of electrolyte reduces the capacity of the battery, and for this reason, the battery compartments of electric vehicles are designed to maintain a satisfactory operating temperature even when the outside temperature is low. The more common applications of Edison batteries are in electric street-trucks, industrial trucks, railway train lighting, tractors, mine locomotives, individual miner's lamps, and radio.

186. Comparison of Edison and Lead Batteries.—The Edison battery has an advantage over the lead battery as far as weight is concerned for all types of motive power applications such as vehicles, trucks, etc. For vehicle service, a lead battery would weigh about 115 lbs. per kilowatt-hour capacity while the Edison battery would weigh 75 lbs. per kilowatt-hour. The first cost of the Edison battery is from 1.65 to 2.5 times that of a lead battery of equivalent capacity, depending upon the service.

The Edison battery has a higher internal resistance than the lead battery and is, therefore, not suitable for applications requiring high discharge rates such as occur in starting and lighting service for gasoline-driven automobiles, and in central station service as stand-by batteries.

PROBLEMS ON CHAPTER XXII

1–22. The capacity of a certain vehicle-type lead storage battery is as follows:

Six-hour rate	526 ampere-hours
Two-hour rate	384 ampere-hours
One-hour rate	315 ampere-hours
Thirty-minute rate	250 ampere-hours

(*a*) What is the current when discharging at each of the above rates?

(*b*) Assuming an average of 1.9 volts per cell, calculate the kilowatt-hour output of a battery of 42 cells when discharging at the above rates.

2–22. A battery consisting of 18 lead-type cells is used to supply incandescent lamps. What would be the voltage on the lamps when the battery is fully charged and when it is nearly discharged?

3–22. Referring to Prob. 1–22, what would be the discharge current when the battery is discharging at the one-hour rate and the thirty-minute rate at a temperature of 40° F.? Ratings given in Prob. 1–22 are at 72° F.

4–22. A battery of 42 lead-type cells is to be charged from a 125-volt supply. The current at the end of the charge is to be limited to 22 amperes with 2.6 volts per cell.

(*a*) Calculate the necessary resistance which must be inserted in series with the battery to obtain this current.

(*b*) What would be the initial charging current assuming the same resistance as in (*a*)?

5–22. A lighting system using 32-volt lamps is to be supplied from an Edison battery.

(*a*) How many cells should be used?

(*b*) What would be the voltage on the system when the batteries are fully charged?

(*c*) What would be the system voltage after discharging for 5 hours at the normal rate?

6–22. An Edison battery of 70 cells having a capacity of 150 ampere-hours at the 5-hour rate is to be charged from a 125-volt supply.

(*a*) Calculate the value of the series resistance required to limit the current initially to the 5-hour rate.

CHAPTER XXIII

SYSTEMS OF D.C. TRANSMISSION AND DISTRIBUTION

187. Methods of Power Supply.—There are two methods of distributing electricity for lighting and power supply. The **series system,** sometimes called the constant-current system as its name indicates, has all the lamps or motors connected in series in one circuit (Fig. 160). The *current* is kept constant regardless of the load, and therefore all lamps or motors must take the same current. The total voltage of the circuit is the sum of the voltages required for each piece of apparatus. The total *voltage,* therefore, increases as the load is increased by the addition of lamps. With the **multiple system,** sometimes called the constant-potential system, the *voltage* is maintained practically constant, whereas the *current* increases as load is added. The total current divides between the various lamps or motors which are connected to the system, and hence disconnecting a portion of the load has no effect upon the operation of the load which remains. The simplest method for multiple distribution is the two-wire system shown in Fig. 161. Other systems include three-wire and other multivoltage systems, three-phase and two-phase systems. All of

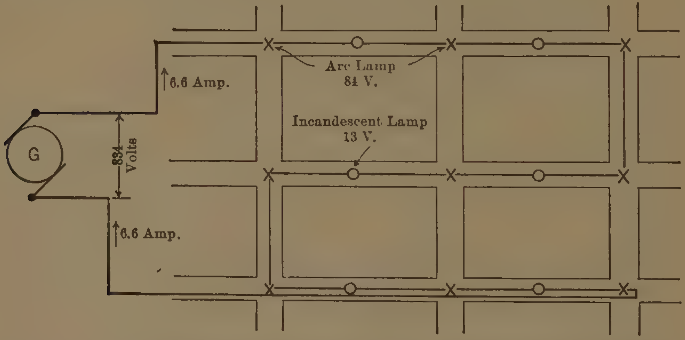


FIG. 160.—Constant-current System as Applied to Street Lighting.

NOTE.—Voltages indicated for lamps include allowance for line drop. Series circuits usually have more lamps in one circuit than are shown in diagram.

these are classed as multiple systems, since the voltage is maintained nearly constant and the lamps or motors are connected directly across the line. The **series system** is especially well adapted for street lighting, because a single wire can be run through each street and the lamps cut into the circuit at any point (Fig. 160). Also, since the current is small (usually 4 to 6.6 amperes), a small size wire may be used. The system operates at a high voltage, however, and consequently requires careful insulation and is a source of danger to any one who might come in contact with the circuit. This system is not well suited for carrying a large load on a single circuit, nor is it efficient except when fully loaded. It is not used for interior lighting nor for supplying motors,[1] because of the high voltage and the difficulties in regulating the motors with changes in load. Only the multiple system will be considered in the articles which follow.

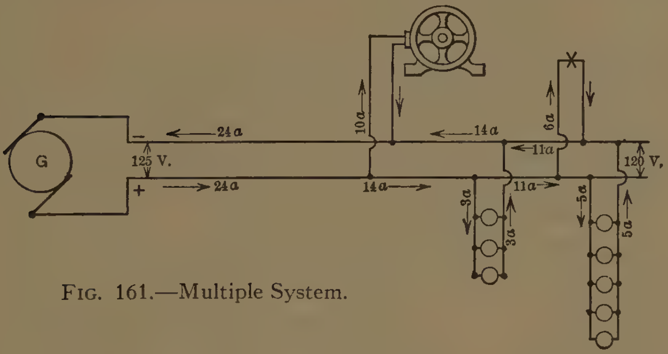


FIG. 161.—Multiple System.

188. Effect of Voltage upon Cost of Wiring.—If a given amount of power is to be transmitted, it is important to use as high a voltage as other conditions will permit. This reduces the current required and thus reduces the size of wire necessary. This is apparent from the table on following page.

The percentage voltage loss in the lines is made the same in each case, since it is the *percentage* loss and not the *actual* loss which fixes the limit in each case. In other words, the operation of lamps or motors would be satisfactory on either a 120- or a 240-volt system as long as the percentage drop were the same. The actual voltage drop in the example given would be 6 volts for the 120-volt system and 12 volts for the 240-volt system. It is apparent that, if the power transmitted is kept the same, the current required for the 240-volt system is one-half that for 120 volts. Since the allowable drop is

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[1] With the exception of a few special European installations.

EFFECT OF VOLTAGE UPON SIZE OF CONDUCTORS

Based on a feeder to transmit 100 kw. a distance of 1000 feet with 5 per cent line loss (two-wire system).

Voltage	Amperes	Line Loss, Volts	Line Loss, Per Cent	Size of Feeder* Circular Mils	Percentage of Copper,* 120-volt System = 100%
120	833	6	5	2,970,000	100
240	417	12	5	744,000	25
600	167	30	5	119,000	4
1200	83.3	60	5	29,700	1
2400	41.7	120	5	7,440	0.25

* In an actual case the feeder would be taken as the nearest standard size, which would change the percentage slightly. Correct for d.c. and nearly correct for a.c. systems.

doubled (the same percentage) the copper required is only one-quarter that needed for the 120-volt system. This means that the size of feeders for equal percentage loss varies inversely as the square of the voltage. The tabulation above takes into account voltage loss only. Other factors enter into this question, such as the greater cost of the high-voltage apparatus, the danger to users of the power, more expensive maintenance, etc., so that the voltage used must frequently be a compromise.

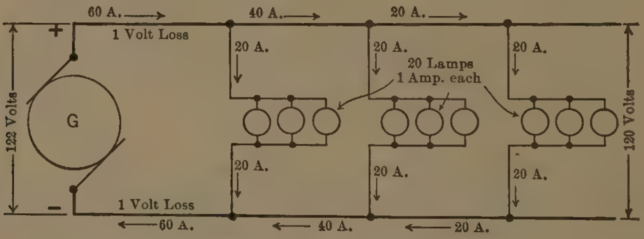


FIG. 162.—Two-wire System.

189. Systems of Wiring.—For distribution of direct current, a two-wire system having a single voltage or a three-wire system having two voltages, is generally used, although in some cases a multi-voltage system with three or more different voltages is used. The arrangement of a two-wire system is shown in Fig. 162. Although this is a very simple system it has the disadvantage that the voltage cannot be transformed readily from one voltage to another and if used for lighting, the voltage cannot exceed 250 volts, which is often too low for economical transmission. The **three-wire or Edison system** is used in order to employ a high voltage for transmitting the energy and, at the same time, to use low-voltage lamps or other devices. A series arrangement of lamps as illustrated in Fig. 163 *a* might be used, provided the two lamps in a branch circuit each require the same current. If lamp (1) is extinguished or burns out, lamp (2) is also extinguished. If the junctions between the lamps are connected together (line *B*, Fig. 163 *b*) a circuit is still maintained through lamp (2) even when (1) is extinguished. This condition is shown in Fig. 163 *c*. It is apparent that there are now four lamps on the negative side of the circuit in series with three lamps on the positive side. Since the same total current must flow in the positive and negative lines, it is apparent that the voltage of the lamps on the positive side would be above and on the negative side below 120 volts. This would cause the lamps on the side having the smaller number to burn above normal candlepower, and those on the other side below candlepower. In fact, with only a small number of lamps on one side there is always danger of burning out these lamps. The arrangement shown in Fig. 163 *b* and *c* is therefore not practical, and that shown in Fig 163 *a* is not allowed by the National Electric Code. If the wire *B* is kept at a potential midway between *A* and *C* (120 volts in the example), then lamps may be added or removed from either side and the other lamps will not be affected. Such an arrangement is called a three-wire system (Fig. 164). The middle wire, called the neutral, is kept at a voltage one-half that between the outside wires by using two generators as shown in Fig. 164, by a balancer set, a three-wire generator or by other methods which

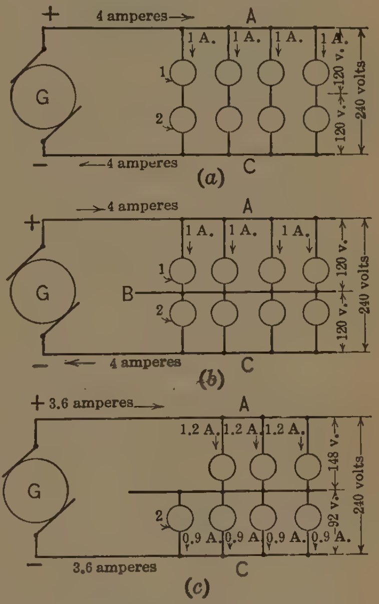
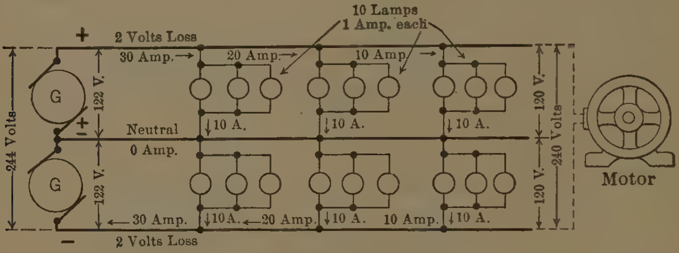
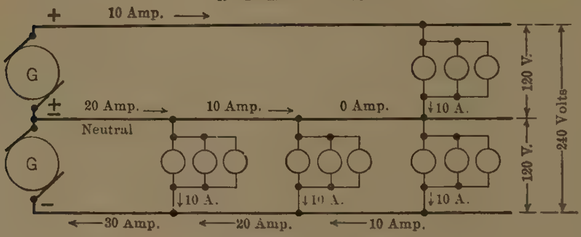


FIG. 163.—Evolution of the Three-wire System. In (*c*) it will be noted that each lamp on +side takes 1.2 amperes and on −side 0.9 ampere, although they are all rated at 1 ampere each. This would cause the lamps on the +side to burn out after a short time. Values of voltage and current given are only approximate.

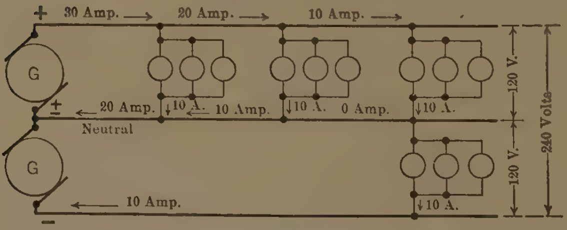
cannot be described here. (See Articles 161 and 162.) Usually the attempt is made to have equal loads on the two sides, giving a **balanced system** (Fig. 164 *a*). Under this condition, the currents in the positive and negative wires are the same and no current flows in the neutral. If the load is not the same on the two sides, the currents in the positive and negative wires are not alike, giving an unbalanced system. The neutral then carries a current which is equal to the difference between the currents in the outside wires (Figs. 164 *b* and *c*). In the example (Fig. 164 *b*) there are ten lamps on the positive side which may be considered to be in series with ten of the lamps on the negative. The current for these lamps flows directly from the positive to the negative terminal of the system. The remainder of the lamps on the negative side must be supplied through the neutral and the negative



a - BALANCED LOAD



b - UNBALANCED LOAD



c - UNBALANCED LOAD

FIG. 164.—Arrangement of the Three-wire System.

line. The direction of the current flowing in the neutral reverses when the heavier load is on the positive side (Fig. 164 *c*). The load on the positive and negative generators is the same as the load on the corresponding side of the system as is shown by the diagrams. When motors are operated from a three-wire system, they would, in general, be connected between the outside wires (as shown by the dotted lines in Fig. 164 *a*), in order to use the higher voltage. The advantage of the three-wire system is that the load is in effect transmitted at 240 volts instead of 120 volts, while at the same time it is possible to use 120-volt lamps, which have some advantages over 240-volt lamps. Consequently the size of the feeders can be greatly reduced. Three-wire systems may be either direct or alternating current, although the former is somewhat more common. A 240-120-volt Edison system is used in the business sections of many large cities for distributing electricity from central stations. The d.c. system is also used in many isolated plants, particularly for office buildings, etc. Alternating-current, three-wire systems are used to a considerable extent for distributing electricity from central stations and for lighting circuits for isolated plants.

Sometimes three-wire systems are made convertible so that they can be changed to a two-wire system by connecting together the two outside wires. Such a system requires that the neutral shall have a capacity sufficient to carry the maximum load when operated as a two-wire system and, in general, this means that the neutral must be twice the size of either outside wire, and therefore, the amount of copper is the same as for a two-wire system. Both two-wire and three-wire arrangements are used for single-phase a.c. circuits. In addition, there are three-phase and two-phase a.c. systems. These are described in Chapter XXXVI. The relative amount of copper required is given in the tabulation on the following page. The comparison in column A is based only on the ampere-carrying capacity, whereas in column B, the basis is equal percentage voltage loss, assuming the same voltage at the load in each case.

190. Choice of System for Lighting and Power Service.—The **standard voltages** for d.c. generators are 125, 250, and 600 volts; and for a.c. generators 240, 480, 600, and 2400 volts. It is apparent from the discussion in Article 188 that the system voltage should be as high as possible. For **d.c. systems** the voltage is limited to 240 volts where incandescent lamps are used. A two-wire, 240-volt system is much cheaper than a 120-volt system, but 240-volt lamps are less efficient and more expensive. The cost of maintenance of the 240-volt system would also be greater because of the higher voltage

COMPARISON OF SYSTEMS OF DISTRIBUTION

Relative amount of copper required

System	Percentage Copper Required	
	A Based on Carrying Capacity	B Based on Same Percentage Voltage Drop
Two-wire	100	100
Three-wire:		
Neutral same size as outside wires	75	37.5
Neutral half size of outside wires	63	31.2
Three-wire convertible	100	100
Three-phase:		
Three wires	87	75
Four wires	67	33.3
Two-phase:		
Four wires	100	100
Three wires	85	73

on switches and sockets. A shock from a 240-volt system is also more serious. The three-wire, 240–120-volt system costs only slightly more than a 240-volt, two-wire system, and has the great advantage that 120-volt lamps and other devices may be used. Where motors only are to be supplied, 240 or even 600 volts may be used if the feeders are very long. A voltage higher than 600 is not satisfactory for industrial purposes because of the danger from shock and the extra cost of the motor control devices. The only applications of the higher voltages are for heavy railway work, where the additional costs are justified by savings in feeders and substation apparatus. Because the voltage is limited to not more than 600 volts for industrial purposes, direct current is used only where the length of the feeders is comparatively short, as in a single building or a group of buildings located together. It is also used in the business districts of a number of large cities. Direct current must be used for charging storage batteries and for electrolytic work and it is preferable for cranes and adjustable speed motors. When **a.c. systems** are used the ease with which the voltage can be changed by means of transformers makes it possible to take full advantage of the saving resulting from the use of a high voltage. In such cases the power can be transmitted at a high voltage, requiring small-size feeders. At the points of use the voltage can then be reduced by means of transformers to a low value suitable for lamps and motors. The small loss of power in the transformers is more than offset by the saving in the feeders. Here again there are certain limitations to the voltage which may be used. As the voltage is increased the conductors must be better insulated and all switches, transformers, and other apparatus, would cost more. In a particular problem, therefore, it may be necessary to balance the saving in cost of feeders against the additional cost of the other apparatus. In general, it may be said that for industrial plants 600 volts will be sufficient even for large plants. Sometimes 2400 or 6600 volts is used, but this is not necessary unless the plant covers a large area and considerable power is to be transmitted. These voltages are rather common in steel mills. A voltage of about 2400 is extensively used by power-supply companies for their distribution circuits.

A d.c. system is satisfactory where the length of the feeders is not great or where there are decided advantages in the use of d.c. motors. Thus, office buildings supplied by isolated plants usually employ a d.c. system because of the better performance of d.c. elevator motors. For general industrial service, however, the a.c. system meets all requirements and it is therefore generally used. If there is a motor load, the three-phase, three-wire system is generally used although there are many two-phase installations.

191. Methods of Installing Circuits.—For outdoor service, circuits are run overhead or underground, depending upon the voltage used and the location of the line. Since underground circuits require the use of insulated conductors, with a protective lead sheath and a system of ducts or conduits, the cost of underground construction is very much greater than an overhead line run on either wood or steel poles. Furthermore, at present, the maximum voltage for which it has been found feasible to design underground cables is 60,000 volts. Underground systems are, therefore, limited in their application to localities where overhead wires would be objectionable, as in the more congested sections of cities.[2] Uninsulated or bare wire is generally used for overhead lines, except for circuits in the yards of industrial plants or for distribution circuits from central stations, where a triple-braid weatherproof covering is used. (See Article 192.)

For interior wiring, insulated conductors are used except in special cases, such as leads to electric furnaces, bus-bars for switchboards and similar applications. The kind of insulation used depends upon the manner in which the work is installed. All interior wiring should be so installed that it will be protected from mechanical injury and will be safe as regards fire hazard and danger from shock. Wherever wiring is installed in a building upon which fire insurance is issued, it is necessary to conform to the rules of the **National Electric Code,** which is issued by the National Board of Fire Underwriters and is in effect throughout the United States and Canada. This code gives definite rules for the installation of all kinds of wiring and also specifies the kinds of material, wire, conduit, fuses, etc., which must be used. The code also applies to outdoor lines wherever they are installed close to buildings, as in factory yards or on roofs.

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[2] Further discussion of such systems is given in Chapter XXXVI.

The common methods of installing interior wiring are: (1) rigid conduit, (2) armored cable, (3) moulding, (4) concealed on insulators, and (5) exposed on insulators. The relative cost of these systems is in the order given, the rigid conduit system being the most expensive. In the rigid conduit system, the wires, which must be rubber-insulated (see Article 192), are placed in soft-steel pipe or conduit which forms a continuous wire-way between points where fixtures or other fittings are attached. At these points the conduits terminate in sheet-steel or cast-iron " outlet boxes " which permit access to the wires for connecting the fixtures. The conduit is run either exposed or concealed in floors or walls of the building. Figure 165 shows a **rigid conduit**

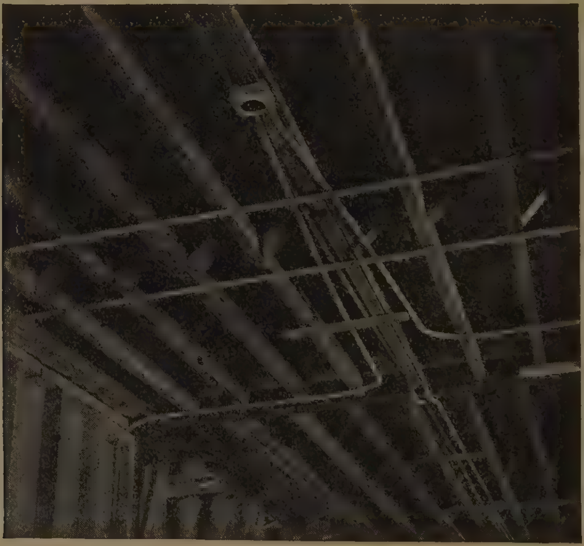


Fig. 165.—Example of a Rigid Conduit System, Showing Outlet Box Located on Floor Joist.

system, in the ceiling of a partly finished residence. In the **armored cable system,** the two conductors of a circuit are rubber-insulated and assembled together inside a protective sheath or armor composed of steel strip wound spirally in such a manner that the completed cable is sufficiently flexible so that it may be bent easily and be installed readily inside floors or walls of finished buildings. Armored cable is used extensively in wiring old buildings or for extensions to existing installations (Fig. 166). The protection to the conductors is nearly as good as that for the rigid conduit system. Armored cable is used for small capacity circuits for lights or small motors while rigid conduit is used for all sizes of circuits, conduit as large as 6 inches inside diameter being available. For small capacity circuits which are to be run exposed, **wood or metal moulding** (*raceways*) are sometimes used. In this system, rubber-insulated wire is placed in grooves in a wood strip or in a thin sheet-steel trough. Figure 167 shows types of raceways used. For cheap, concealed wiring in frame buildings, the **knob and tube system** is used. Rubber-insulated wire is run con-

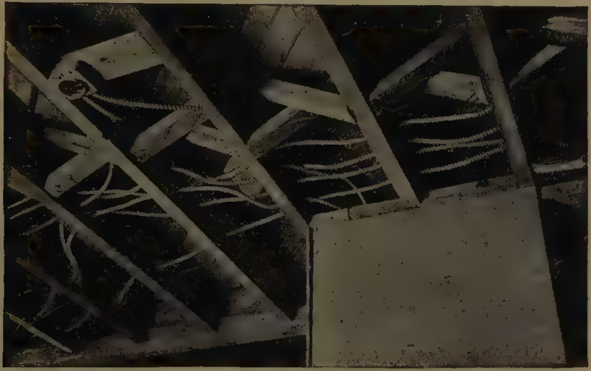
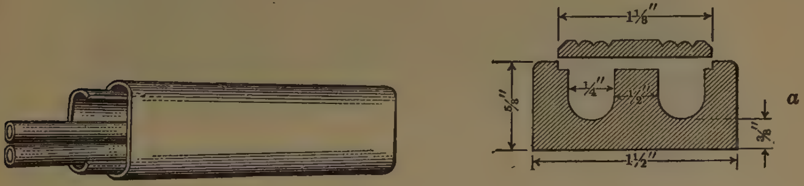


FIG. 166.—Example of an Armored Cable Installation, Showing Ceiling Outlet and Armored Cable (BX) Leading to Panel Box.



(*a*) (*b*)

FIG. 167.—Examples of Raceways for Wiring. (*a*) Metal Moulding; (*b*) Wooden Moulding.

cealed between floor joists or partition studding and is supported by porcelain insulators. Where the wires pass through any of the timbers, porcelain tubes are slipped over the wires. Where there is no objection on the score of appearance, wires are run open or exposed. They are supported on cleats or insulators as shown in Fig. 168. Either rubber or slow-burning weatherproof insulation is allowed on the conductors, but the former is better. This type of wiring is much used in factories where the circuits can be run on or near the ceiling of the room where they are not subject to mechanical injury.

192. Insulated Wires.—The term **magnet wire** is applied to insulated conductors used for field coils and, in some cases, armature coils of dynamos. This wire is of several types, one type having one or more layers of fine silk or cotton threads wrapped around it. Another type, called **enamel wire,** has a thin insulating coating of enamel baked on the copper. Sometimes the wire is enameled and then covered with cotton or silk wrapping, as first described. Enamel wires have the advantage that the thickness of insulation is less than for either silk or cotton insulation and, therefore, more turns can be wound in a given space. **Rubber-covered wire,** which is so commonly used for various systems of wiring (see Article 191), consists

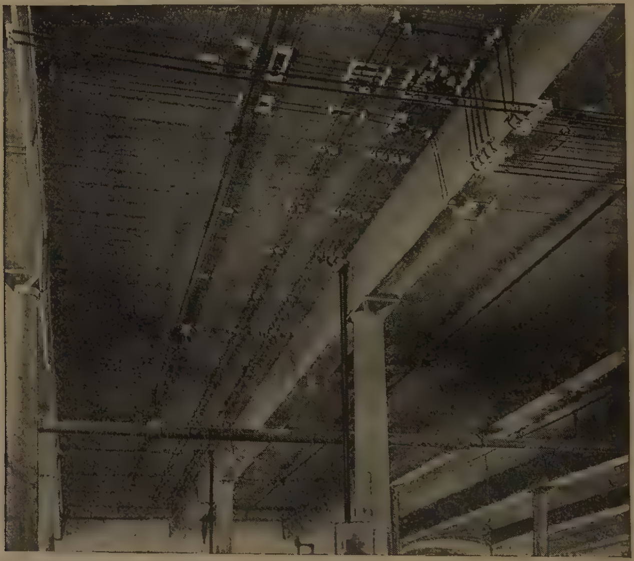


Fig. 168.—Example of Open Wiring. *The Factory Mutual Insurance Companies.*

of a tinned copper conductor upon which is moulded a seamless covering of soft, vulcanized rubber compound. This gives an insulation which is impervious to moisture and is capable of withstanding severe operating conditions. The thickness of rubber covering depends upon the voltage for which the wire is designed and the size of conductor. For ordinary service, so-called 600-volt insulation is used. This has a thickness varying from $\frac{3}{64}$ inch for No. 14 to $\frac{1}{8}$ inch for 2,000,000 cir. mils. The rubber insulation is covered by one or more layers of cotton thread, braided on, to protect the wire against mechanical injury. **Weatherproof wire** has the copper conductor covered with three braids of fibrous yarn. During the process of manufacture, the braids are saturated with a waterproof compound. This wire is used chiefly for outdoor service. **Slow-burning weatherproof insulation** is not so inflammable as weatherproof insulation. It consists of an inner weatherproof braid and an outer braided covering which has been treated to make it flameproof. This wire is used for exposed interior wiring. **Varnished-cloth insulation** consists of spirally wrapped layers of cotton tape which has been treated with an insulating varnish. In the process of wrapping, the layers are coated with a thick insulating compound to fill the spaces left in wrapping. Wire of this type is covered by braids like rubber-covered conductors. Varnished-cloth insulated conductors cost less than rubber-insulated and are as satisfactory except where they would be permanently exposed to considerable moisture.

193. The current-carrying capacity of conductors depends upon the kind of insulation and the location of the conductor. Insulated conductors in dynamos are rated generally on the allowable watts per square inch of radiating surface of the winding, but, for cotton-insulated conductors in field coils, approximately 1500 cir. mils per ampere should be allowed and for armatures 500 cir. mils per ampere. The carrying capacity of conductors used for interior wiring is specified by the National Electric Code and is given in Tables 1 and 2, in Appendix. Rubber insulation cannot withstand as high a temperature without deterioration as other classes of insulation; therefore, the safe carrying capacity is less. All interior wiring circuits must have protective devices such as fuses or circuit breakers, so arranged that the circuit will be opened if the current exceeds the values given in Tables 1 and 2. The Electric Code rates the carrying capacity of a wire at the same value whether it is in conduit or is run exposed. For outdoor overhead lines the same table may be used, but, generally, the size of these circuits is fixed by voltage drop rather than by carrying capacity. (See Article 195.)

194. Parts of a Circuit.—The connection between the point of supply and the individual lamps or motors is made by means of feeders, mains and branches. The **feeder** (Fig. 169) is the part of the circuit between the switchboard and the first distributing center *A*. The feeder is usually the longest part of the circuit. The **mains** (sometimes called sub-feeders) connect the first distributing point, *A*, with the other points, *B*, *C*, *D*, etc. Sometimes there are no mains. The **branches** supply individual motors or groups of lamps. In general, as the mains are smaller than the feeder, fuses are required at the junction point *A* to protect these mains. The branches must also be fused to protect the lamps or motors. When the electricity is supplied by a central station, a **service or service mains** are run in from the street circuits to the switchboard in the building.

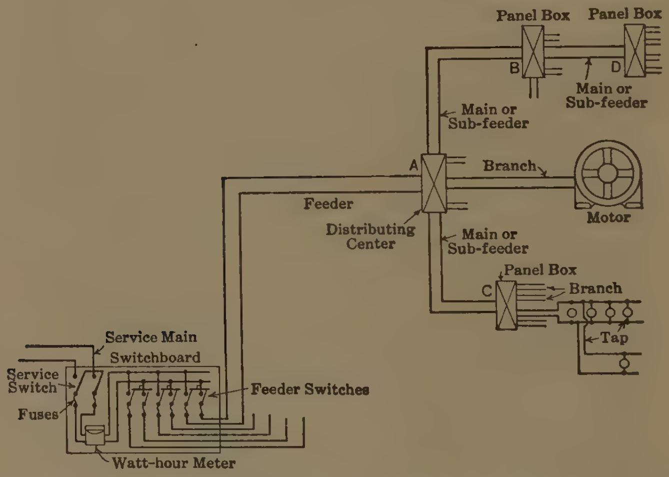


FIG. 169.—Parts of a Circuit. Fuses and switches for branch circuits and mains not shown. Usually motors are run on separate feeders and not on a lighting feeder as shown.

195. Calculating a Two-wire System.—The first step in determining the size of a circuit is to estimate the load. With lighting circuits this can be taken as the total connected load; but, with motors, the actual load on the system will, in general, be less than the sum of the full-load ratings of all the motors. This follows from the fact that all the motors would not be carrying full load at the same instant. For this reason, the actual load on a feeder or main must be determined by the use of a demand factor. Table 3, in Appendix, gives these factors for average conditions. The full-load current for d.c.

motors, can be obtained from Table 4, in Appendix. The size of conductor for the branch circuit for a single motor can also be determined from this table. If one of the motors on a feeder is considerably larger than the others, starting as well as running conditions should be considered when determining the size of circuit. For shunt or compound motors, the starting current can be taken as 1.25 times the full-load current and, for series motors, 1.5 times full-load current.

Example 1.—Shunt motors, 120-volt system.

Running conditions:

1–5 hp. motor		40 amperes
1–3		24
1–2.5		20
2–1		17.6
1–3	(spare)	24
Total		125.6 amperes

From Table 3, the demand factor is found to be about 0.75. Hence the maximum running current is

$$125.6 \times 0.75 = 94.3 \text{ amperes.}$$

The circuit must therefore be No. 2 if rubber insulation is used.

Example 2.—If the panel which supplies the motors in Example 1 also had connected to it a 50-hp. motor we would have to consider the starting conditions as follows:

Running load, six motors	= 94.3 amperes
Starting load, 50-hp. motor, 356×1.25	=443
	537.3 amperes

This would require a 750,000 cir. mils cable.

As a rule, some allowance must be made for future increase in load, the amount being determined by the requirements of each case.

A circuit must not only be large enough to carry the load without overheating as determined by the previous paragraph, but also must be of sufficient size so that the voltage drop will not be excessive. In general, a short or a high-voltage feeder will have its size determined by carrying capacity. A long or a low-voltage feeder usually has to be increased in size over the minimum required for carrying capacity in order that the voltage drop shall not be excessive. The total allowable drop should not exceed about 5 per cent for lighting circuits, and 10 per cent for power circuits; this should include the entire drop from point of supply to the individual light or motor. The drop should be apportioned about as follows:

	Branches, Per Cent	Mains, Per Cent	Feeders, Per Cent	Total, Per Cent
Lighting circuits	1.5	1.0	2.5	5
Power circuits	2.0	2.0	6.0	10

The drop in a two-wire circuit can be calculated from the formula

$$\text{Volts drop } E = \frac{2 \times K \times L \times I}{\text{Circular mils}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

where K = ohms per circular mil foot, which can be taken as 10.7 to allow for the rise in temperature of the conductor;
L = length of the line in feet;
I = current in amperes.

Example 3.—Referring to Example 1, it was found that a No. 2 rubber-covered wire was required to carry the maximum load of 94.3 amperes. If the length of the feeder is 150 feet, the drop, according to Equation (43), is

$$E = \frac{2 \times 10.7 \times 150 \times 94.3}{66{,}400} = 4.6 \text{ volts.}$$

This is a drop of 4.2 per cent and, since 6 per cent is allowable, this size of wire would be sufficient.

Example 4.—If the feeder of Examples 1 and 3 were 300 feet long, the drop for a No. 2 wire would be 8.4 per cent which is excessive. If we limit the drop to 6 per cent or 7.2 volts, the size of wire can be found from Equation (43).

$$\text{Circular mils} = \frac{2 \times 10.7 \times 300 \times 94.3}{7.2} = 84{,}200.$$

A No. 1 wire has 83,700 cir. mils and hence should be used instead of a No. 2 in order to keep the drop to a reasonable figure.

196. Calculation of a Three-wire System.—The total load is determined by the same method as for two-wire systems, lights and small motors being distributed on the two sides in order to give practically a balanced system. (See Article 189.) Large motors would be connected across the two outside wires. As far as carrying capacity is concerned, the size of the two outside wires can be determined as soon as the load on each is known. The neutral would, as a rule, be the same size as either outside wire since it must be capable of carrying the entire load of either side of the system if the other were disconnected. In some instances, where the size of the outside wires must be increased in order to reduce the voltage drop the size of neutral need not be increased as long as its ampere-carrying capacity is sufficient.

A three-wire system is usually so designed that it operates with only a small neutral current. The drop in the neutral can, therefore, be neglected in calculating the voltage loss.

Example 1.—A three-wire lighting feeder has a lighting load of 165 amperes on each side of the system, the length is 130 feet, and the voltage 120 to neutral. The minimum size of conductor for rubber insulation is, according to Table in Appendix, No. 000 which can carry 175 amperes. The drop on an outside wire is given by a modification of Equation (43).

$$E=\frac{10.7\times L\times I}{\text{Circular mils}}.$$

This is the drop on a single wire L feet long, whereas Equation (43) gives the drop on two wires each L feet long. Substituting we have

$$E=\frac{10.7\times 130\times 165}{168{,}000}=1.37 \text{ volts.}$$

If necessary to calculate the drop with an unbalanced load, the neutral drop must be considered.

Example 2.—Referring to Example 1, calculate the drop with a load of 165 amperes on the positive and 135 amperes on the negative sides. Using No. 000 for each wire the drop will be:

For the positive wire 1.37 volts as in Example 1.

For the negative wire

$$E_n=\frac{10.7\times 130\times 135}{168{,}000}=1.12 \text{ volts.}$$

The current in the neutral is 30 amperes which comes from the positive side, since this has the larger load. The drop is

$$E=\frac{10.7\times 130\times 30}{168{,}000}=0.25 \text{ volt.}$$

Since the neutral current comes from the positive, the total drop on the positive side is the sum of the drop on neutral and positive or

$$E_p=1.37+0.25=1.62 \text{ volts.}$$

The total drop on the negative side is the difference of the drop on negative and neutral wires,

$$E_n=1.12-0.25=0.87 \text{ volt.}$$

In general, it may be stated that the drop in the neutral wire adds to the drop on the heavily loaded side and subtracts on that lightly loaded. Hence, when the load is unbalanced and the drop on the neutral is appreciable, there is an unbalance of voltages on the system, the side having the heavier load having a lower voltage than the other. As a rule, the unbalance is small; hence, the effect of neutral drop is negligible.

PROBLEMS ON CHAPTER XXIII

1–23. A group of 50 arc lamps used for street lighting are arranged on a series system. Each lamp requires a voltage of 68 volts and a constant current of 4 amperes. The drop in the line wires is 200 volts.

(*a*) What is the required current and voltage which must be supplied from the power house when all lamps are operating?

(*b*) What would be the total power supplied to the circuit?

(*c*) If ten lamps were extinguished, what would be the current and voltage supplied by the power house?

2–23. A factory has a load of 50 hp. in motors and is lighted by 1000 tungsten lamps rated at 40 watts each. The load is supplied on a two-wire multiple system, giving 110 volts at the lights and motors.

If 5 per cent of the total delivered power is lost in the feeders, what would be the current and voltage at the power house?

3–23. In a series-lighting system, what would happen if the circuit were short-circuited near the generator terminals?

4–23. A certain trolley road would require 30,000 pounds of copper wire if 600 volts were used. What would be the saving if 1200 volts were used? Assume copper at 25 cents per pound.

5–23. A feeder for a 2-wire system has loads as follows: Point *A*, 100 feet from generator, 65 amperes; point *B*, 135 feet from generator, 125 amperes; point *C*, 150 feet from generator, 35 amperes; point *D*, 165 feet from generator, 45 amperes. Draw a diagram, similar to Fig. 162, showing the current in each part of the feeder, and in each branch circuit, at the four load points. Show also direction in which the current flows.

6–23. A three-wire, 110–220-volt system has a load of 50 amperes at 110 volts on the positive side and 33 amperes at 110 volts on the negative side.

(*a*) Calculate the current in each wire and indicate by a diagram the direction of flow of this current.

(*b*) Calculate the power on each side of the system.

7–23. A three-wire system has three load points, *A*, *B*, and *C*, point *A* being nearest the generator. The loads at each point are:

A. 55 amperes positive side, 65 amperes negative side.
B. 43 amperes positive side, 40 amperes negative side.
C. 25 amperes positive side, 20 amperes negative side.

Determine the amount of current and direction of flow in each part of the circuit.

8–23. A load of 500 tungsten lamps each rated at 40 watts, requires 110 volts for proper operation.

(*a*) Calculate the total current in each wire to supply these lights if they are connected on an ordinary two-wire multiple system.

(*b*) What is the minimum size of rubber-insulated wire which should be used for this system?

(*c*) If the lamps specified were supplied on a balanced three-wire system at their rated voltage, calculate the current in each of the three wires of the system.

(*d*) What would be the current in each wire of the three-wire system of (*c*) if all the lamps on the positive side of the system were not operating?

(*e*) What is the minimum size of rubber-insulated wire which should be used for the system as specified in (*c*) and (*d*)?

9–23. What is meant by the terms, "feeder," "main," and "branch," when speaking of an electric circuit?

10–23. A No. 00 feeder carrying a load of 90 amperes supplies two panel boards, one of which is fed through No. 4 mains. What fuse protection is required? Assume rubber insulation.

11–23. A panel board supplies the following 230-volt shunt motors: Three 5-hp., two 7.5-hp., two 15-hp., one spare 7.5-hp. circuit. These motors are used for individual drives. Find the maximum running current, including the spare.

12–23. A panel board supplies the following 230-volt shunt motors used for individual drive: One 3-hp. (future), one 5-hp., one 7.5-hp., one 20-hp. Find the maximum running current and the heaviest load caused by starting one motor with others running.

13–23. A feeder has the following connected load of shunt motors: Two 10-hp., one 15-hp., two 20-hp., one 25-hp. The motors are 230-volt, on group drive. Find the maximum running current in the feeder.

14–23. A feeder supplies three panel boards with the following loads: *Panel A*, one 3-hp., one 5-hp., one 7.5-hp., one 20-hp.; *Panel B*, three 5-hp., two 7.5 hp. two 15-hp.; *Panel C*, four 5-hp., one 7.5 hp., one 20-hp. The motors are used for individual drive and are 230-volt shunt machines. Calculate the feeder load.

Note—Allow 15 per cent for extensions and select demand factor on the basis of the total number of motors specified.

15–23. What size of rubber-insulated wire and fuses should be used for a 65-hp. 230-volt d.c. shunt motor?

16–23. Find size of main and fuses required for panel in Prob. 11–23. Slow-burning weatherproof wire to be used.

17–23. Find size of feeder and fuses required for panel in Prob. 12–23. Slow-burning weatherproof wire to be used.

18–23. Find size of feeder and fuses required to supply motors of Prob. 13–23. Weatherproof wire to be used.

19–23. Find size of feeder required for Prob. 14–23, if weatherproof wire is to be used.

20–23. Calculate the volts lost on a 2-wire d.c. No. 0000 feeder carrying 300 amperes a distance of 275 feet.

21–23. Calculate the volts lost on a series arc lamp circuit composed of No. 6 wire, if the current is 5 amperes and the *total length of wire* is eight miles.

22–23. What size of cable would be required to transmit 150 amperes 500 feet with a loss of 5 volts?

23–23. (*a*) What size of 2-wire feeder is required to carry a load of 150 amperes with 5 volts drop, if the distance is 300 feet?

(*b*) What size would give the same drop if the distance is 50 feet?

(*c*) Would the size found in (*b*) carry the load without overheating? If not, what size should be used, if rubber-insulated?

24–23. A branch circuit feeding a 30-hp., 115-volt d.c. motor is 125 feet long. Size of branch as per Table **4**.

(*a*) What is the voltage loss when motor is carrying full load?

(*b*) What is the loss at 50 per cent overload?

25–23. A load consisting of motors totalling 200 hp. is operated at 220 volts. The motors are 1000 feet away from the power house. Assume a temperature of 25° C. for wires.

(*a*) Assuming that the drop in the feeder supplying the motors is 5 per cent of the voltage at the motors, determine the size of wire required.

(*b*) How much current could the wire determined in (*a*) carry if rated according to the National Electric Code Rules for rubber wire?

(*c*) If the motors were supplied at 550 volts and the same percentage of voltage was lost in the line, calculate the required size of wire.

26–23. If the distance between the lamps in Prob. 8–23 and the power house is 500 feet, and the assumed temperature of the wires is 25° C.:

(*a*) Calculate in Prob. 8–23 (*a*) the size of wire required if a *total* drop of 5 volts is allowed for the feeder.

(*b*) Calculate, in Prob. 8–23, the size of wire required, including neutral, if a drop of 5 volts on *each side* of the three-wire system is allowed.

(*c*) How does the total amount of copper compare in the two cases?

27–23. With wire sizes calculated in Prob. 26–23 (*b*) determine:

(*a*) The voltage at the lamps on positive and negative sides, if the load on the negative side is one-half the amount used in Prob. 26–23.

(*b*) The voltage at the lamps on positive and negative sides if there is no load on the negative side. Positive load in (*a*) and (*b*) same as in Prob. 26–23 (*b*). Voltage at switchboard assumed to remain constant and to be same as in Probs. 8–23 and 26–23.

PART III

ALTERNATING CURRENTS

CHAPTER XXIV

ALTERNATING CURRENTS AND VOLTAGES

197. Definitions.—An alternating current or voltage not only reverses its direction regularly,[1] but also varies in magnitude at different instants. The changes which occur in an alternating current or voltage may be represented by curves of which Fig. 170 is typical. Interpreting this curve, we can say that one terminal of the generator producing this alternating voltage remains positive with respect to the other terminal for a time represented by the distance 1–3 which is $^1/_{120}$th of a second. The polarity then becomes negative for the time 3–5, which is also $^1/_{120}$th of a second. At the instant represented by point 1, the voltage is zero, and it increases gradually until, at point 2, which shows conditions $^1/_{240}$th of a second later, the voltage reaches its maximum value of E_m volts. It then decreases at the same rate for $^1/_{240}$th of a second and becomes zero at point 3. Between points 3 and 5, the process is repeated, except that the polarity is reversed although the voltage reaches the same maximum value as before. The entire change, represented by the distance 1–5, takes place in $^1/_{60}$th of a second for this particular example. The complete set of values between 1 and 5 is called a

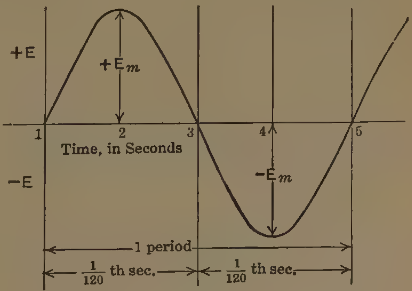


FIG. 170.—Example of an Alternating Voltage.

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[1] See definition in Chapter I.

cycle, and the time in seconds required for the voltage to pass through one cycle is called a *period*. The number of cycles per second is called the *frequency* and is represented by the letter f. In Fig. 170, the period is $^1/_{60}$th of a second, and the frequency is 60 cycles. These definitions also apply to alternating currents.

198. Some Differences between A.C. and D.C. Circuits.—A certain a.c. arc lamp requires a current of 6 amperes with a terminal voltage of 110 volts. The potential difference across the arc is 72 volts and across the regulating mechanism, which is in series with the arc, the potential difference is 50 volts. According to the laws of the d.c. circuit it would be expected that the sum of the voltages in the lamp would equal the terminal voltage, 110 volts, whereas the sum is actually 122 volts. In many series a.c. circuits the total voltage is less than the arithmetical sum of the voltages measured across the various parts of the circuit.

An alternator supplies 62 amperes to one feeder and 55 amperes to another, according to readings of ammeters connected in these feeders. The generator ammeter reads only 108 amperes, although the sum of the readings of the two feeder ammeters is 117 amperes. Frequently in parallel a.c. circuits the arithmetical sum of the currents in the branch circuits is not the same as the total current as measured by an ammeter in the supply circuit.

In an a.c. circuit carrying a current of I amperes at a potential difference of E volts the power in the circuit, in general, *is not* equal to $E \times I$ but is considerably less than this. The product $E \times I$ is called the apparent power and is expressed in volt-amperes. The true power which may be determined by a wattmeter[2] or by other means is expressed in watts. The apparent power can be expressed in kilovolt-amperes (kv.-a.), and the true power in kilowatts (kw.) by dividing the volt-amperes or watts by 1000. The horsepower for an a.c. circuit is found by dividing the true power expressed in watts by 746. The ratio $\frac{\text{true power}}{\text{apparent power}}$ is called the power factor of the circuit. It may have any value from one to zero and is frequently expressed in per cent. Stated in another way, the power factor may be called the constant by which the apparent power is multiplied to obtain the true power. (See Article 225.)

Example 1.—A certain transformer had one winding connected to a 2500-volt a.c. supply, while the other winding was without load. The current in the circuit was 1.82 amperes and the power 1520 watts as measured by a wattmeter. The apparent power is

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[2] This is an instrument for measuring the true power in an a.c. circuit and is described in Article 229.

$$1.82\times 2500 = 4550 \text{ volt-amperes,}$$

or

$$= 4.55 \text{ kv.-a.}$$

The true power is 1520 watts or 1.52 kw.

The power factor is

$$\frac{1520}{4550} = 0.334 \quad \text{or} \quad 33.4 \text{ per cent.}$$

If an alternating current of E volts is impressed on a circuit, and a current of I amperes flows, the ratio $E \div I$ is not, in general, the resistance of the circuit but is a quantity called the impedance of the circuit, which is represented by the symbol Z and is expressed in ohms. That is

$$Z = \frac{E}{I} \text{ ohms.} \qquad (44)$$

In many cases the impedance is much greater than the resistance of the circuit, due to the effects of self induction or electrostatic capacity.

Example 2.—Referring to the transformer of Example 1, it was found that to force 1.82 amperes, direct current, through the windings would require 0.895 volt. Hence the resistance is

$$R = \frac{0.895}{1.82} = 0.492 \text{ ohm.}$$

To force 1.82 amperes, alternating current, through the same winding requires 2500 volts.

Hence the impedance is

$$Z = \frac{2500}{1.82} = 1375 \text{ ohms.}$$

The great difference between the alternating and direct voltages required to force the same current through the winding is accounted for by the self-induction of the winding. This will be explained in detail in a later article.

199. Advantages of A.C. Systems.—It was shown in Article 188 that the quantity of copper required for the transmission of a given amount of power varies inversely as the square of the voltage. High voltages are, therefore, required for the economical transmission of power over long distances. On the other hand, motors, incandescent lamps, and other power-consuming devices require comparatively low voltages and, therefore, a transformation of voltage is necessary before the power can be utilized. This change can be made easily in an a.c. system by means of static transformers while voltage transformation on a d.c. system requires the use of a motor-generator set.

Static transformers are very simple in design, have no moving parts, and cost only about one-fourth as much as a motor-generator set. Transformers can be placed out of doors and require very little attention, while motor-generator sets must be protected by a building and require an attendant. Furthermore, the loss of power in the transformer is much less than in the motor-generator set. Alternating-current generators are much simpler than d.c. generators and, since they require no commutator, can be built for higher voltages and are more substantial. They are built with stationary armatures and are, therefore, especially well adapted for steam-turbine drive.

The a.c. system usually requires more generator capacity than would be necessary for a d.c. system of the same kilowatt rating because of the increased current due to a low power factor. The a.c. system also causes disturbing effects in neighboring telephone or telegraph lines. Furthermore, a.c. motors are not so satisfactory as d.c. motors for some power applications, such as cranes and adjustable speed motors. In spite of these disadvantages, however, the flexibility of the a.c. system in the matter of easy conversion to the voltages required for different kinds of load so far outweighs the disadvantages stated that nearly all the electrical power produced in this country is generated and transmitted as alternating current and most of it is used without conversion to direct current. However, direct current is used for electric railways and for general lighting and power supply in the business districts of a number of large cities which use the so-called Edison system.

200. Principal Parts of an A.C. Generator.—The only source of alternating current for general power supply is the a.c. generator or alternator. This machine, like the d.c. generator, has groups of conductors which are cut by a magnetic field, but it has no commutator. The magnetic flux of an alternator is produced by a field winding which is supplied from a d.c. source. This is usually a small d.c. generator called an exciter. The armature winding of the alternator consists of a large number of conductors which are grouped together to form coils and are placed in slots in a laminated steel core. For a single-phase alternator, all the armature conductors are connected together in a single circuit with two terminals; in a two-phase alternator, the conductors are divided into two groups, thus making two distinct circuits; in a three-phase machine, there are three groups of coils and three circuits. Usually the three circuits of the three-phase machine are connected together in such a way that there are only three machine terminals.[3]

[3] See Chapter XXXI for further description of alternator windings.

201. Elementary Principle of the A.C. Generator.—Imagine two conductors, *a* and *b*, connected in series and placed on diametrically opposite sides of the armature core of a bipolar generator (Fig. 171). The field is excited from a d.c. source. As these conductors are rotated in a counter-clockwise direction, an e.m.f. is induced in each conductor, and since they are in series, the e.m.f. at the slip rings will be the sum of the e.m.f.'s in the two conductors. When the conductors are in the position shown by diagrams 1 or 3, they are cutting no flux and, hence, the e.m.f. is zero. While conductor *a* is cutting the flux of the N pole (diagram 2), the direction of e.m.f. is such as to make slip-ring and brush *d* positive with respect to *c*, and it will remain positive as long as conductor *a* is under the N pole. When conductor *a* is under the S pole, the e.m.f. is reversed and slip-ring and brush *d* are negative. It is possible to measure the actual e.m.f. induced when the conductors are in different positions during one revolution, and if these values are plotted, a curve similar to that of Fig. 171 will be obtained. This represents the e.m.f. between brushes for one complete revolution of the armature. The intersections of the lines 1, 2, 3, and 4, with the curve, give the value of the e.m.f. at the instant when the conductors are in the position shown in corresponding diagrams, 1, 2, 3, or 4. The same result is obtained if the two conductors are stationary and the field revolves, as in Fig. 174. A commercial alternator has a large number of armature coils which are distributed over the surface of the armature, and are connected together in series so that their voltages add. The e.m.f. produced by these coils would have a wave form like that shown in Fig. 174, but one complete revolution would still be required to produce one cycle if this alternator had a bipolar field.

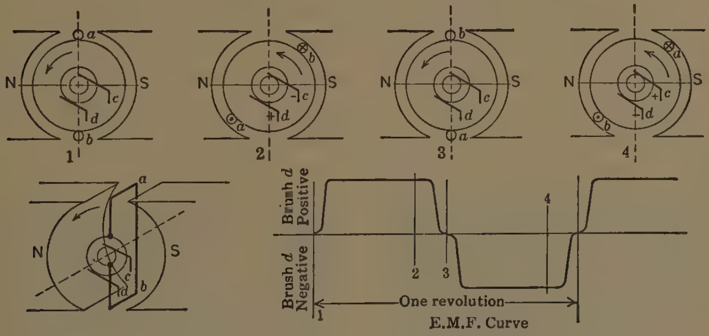

FIG. 171.—Production of an Alternating E.M.F.

202. Hydraulic Analogy.—A d.c. circuit acts, in many respects, like a hydraulic system consisting of a centrifugal pump, arranged for circulating water in a length of pipe (Fig. 1 *d*). While the gate valve is closed, a difference of pressure exists at the valve, the pressure being higher (or positive) on the left side of the valve. In the electric circuit the d.c. generator maintains a difference of potential (or electric pressure) across the switch, and the left-hand terminal remains positive at a definite and steady value. If the gate valve were opened, water would flow through the pipe circuit in the direction of the arrow, the quantity depending upon the amount of resistance in the system. If the switch in the electric circuit were closed, a current would flow from the positive to the negative terminal, the amount of current depending upon the electrical resistance in the circuit.

An a.c. system acts like a hydraulic system having a plunger pump without valves (Fig. 2). Considering a flow of water from discharge opening 1 as positive, it can be seen that when the piston is moving to the left the flow of water will be out at 1 and in at 2, or the flow will be in a positive direction through the system. When the piston is moving to the right, the flow reverses and is in or negative at 1. Hence, a complete cycle of the hydraulic system is accomplished in two strokes of the piston, or one complete revolution of the crank shaft which drives the piston. The flow of water in the pipe reverses or is alternating. Similarly, in the electric circuit, terminal 1 is first positive and then negative for each cycle, and the current produced by this e.m.f. is also alternating in the direction of flow.

203. Wave Form.—The shape of an e.m.f. or current wave, when plotted as in Fig. 170, is called the wave form of e.m.f. or current. The wave form of e.m.f. produced by an alternator is of great importance in electrical engineering. It has been found by experience that, for the purpose of general power supply, the sine wave[4] is the best. The present practice is, therefore, to design alternators to produce as closely as possible a sine-wave form. If the length of air gap in the alternator is uniform, as in Fig. 171, and the conductors are concentrated in a few slots, the wave form is flat-topped as shown. By increasing the air gap at the pole tips, thus decreasing the field strength at these points and by suitably arranging the armature conductors, the wave form, at no load, can be made to approach closely a sine wave. The wave form of the alternator, when carrying a load, may differ considerably from its shape at no load, due to the distorting effect that is produced by electrostatic capacity in the form of underground cables or long overhead transmission lines. Voltage or current waves which have the shape shown in Fig. 170 are called sinusoidal waves; the other forms are called non-sinusoidal and are said to contain "harmonics."[5]

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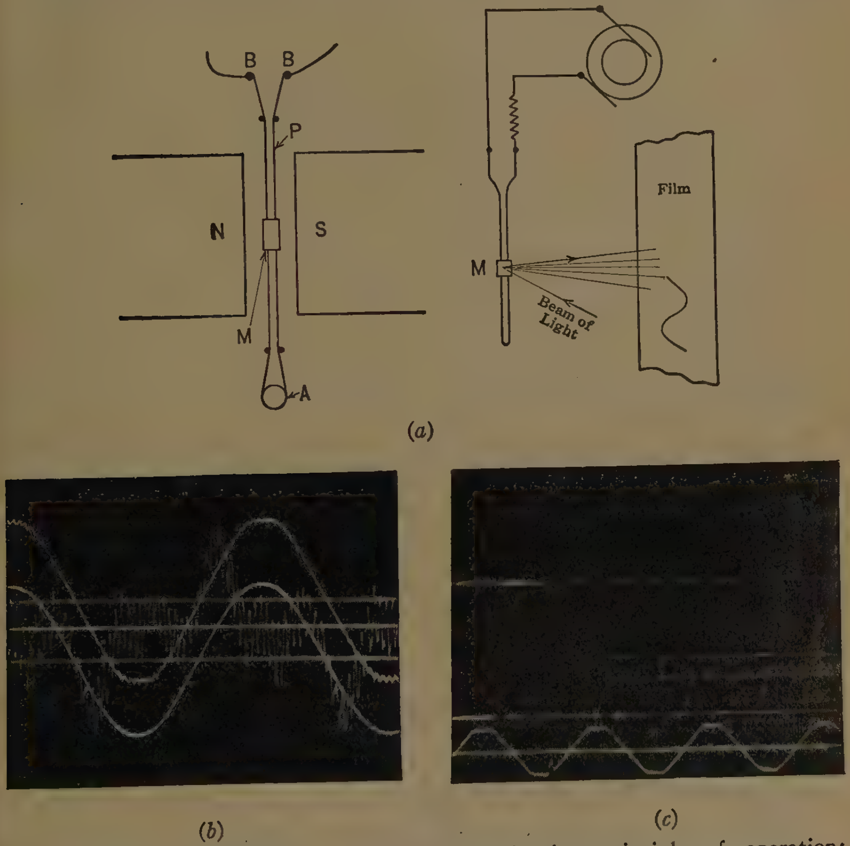
[4] The properties of a sine wave are discussed in the next chapter.

204. The oscillograph is a special type of galvanometer having a moving element, called a vibrator, which is so designed that it will respond to extremely rapid changes in a voltage or a current. It was shown in Fig. 170 that a 60-cycle alternating voltage changes from zero to its maximum value in $^1/_{240}$th of a second. Evidently no indicating instrument of the ordinary type, using a moving coil with a pointer, could respond to such a rapid change in voltage. The



(*a*)

(*b*)

(*c*)

FIG. 172.—The Oscillograph. (*a*) Diagrams showing principle of operation; (*b*) Examples of alternating waves; (*c*) Record of circuit-breaker operation.

[5] Non-sinusoidal waves are discussed in Article 218.

moving element of the oscillograph, however, because of its very small mass, will not only follow the changes taking place in a 60-cycle voltage, but will also show variations which occur in a very small fraction of a cycle. It is possible, therefore, by the use of an oscillograph to produce a curve similar to that in Fig. 170 which will indicate accurately the voltage or current value for every instant during a cycle even when this change is extremely rapid. The instrument (Fig. 172 *a*) operates on the principle of the ordinary Weston d.c. instruments, although the field *NS* is produced by a d.c. winding instead of a permanent magnet since an intense field is required. Between the poles of this magnet is placed a single-turn coil consisting of a loop of fine phosphor bronze ribbon, *P*, held at the center by a pulley, *A*, and having the two ends clamped to insulated supports, *B*, *B*. A very small mirror, *M*, is attached to the loop, and a definite tension is applied to the loop by means of a spring. When in operation, current passes through the loop by way of the supports *B*, *B*, and when this occurs the two sides of the loop tend to move in opposite directions in the magnetic field, thus twisting the mirror about a vertical axis. If the current reverses, the mirror moves in the opposite direction. When an alternating current is passed through the loop, the mirror vibrates with the same frequency as that of the current and the deflection of the mirror will be proportional to the strength of the current. The mirror reflects a spot of light from an arc or an incandescent lamp upon a photographic film which is attached to a drum. With the drum stationary, the spot of light would draw a straight line parallel to the axis of the drum. When in use, the drum is rotated at a high speed, and then the spot of light draws, on the film, a curve which indicates the wave form for one or more cycles, as is required. A curve of this kind is called an "oscillogram." The loop *P* can carry only a small current, therefore, when the vibrator is used as an ammeter, it must be provided with a shunt and, when used as a voltmeter, it must have a high resistance in series with the loop. Oscillographs are usually provided with three independent elements or vibrators, each with its loop and mirror, so that three curves can be drawn simultaneously on the same photographic film.

The oscillograph is indispensable in electrical engineering practice and is used to study unknown wave shapes and to compare them with a sine wave. Because the moving system is very light, it can respond to rapid changes in current strength and, therefore, will indicate momentary and transient effects which may take place in an extremely short interval of time. The commercial forms of the oscillograph are capable of indicating changes which occur in as short an interval as $^1/_{1200}$th of a second. For this reason, the oscillograph is frequently used in studying the effect of short circuits and the operation of circuit breakers and fuses. Fig. 172 *b* and *c* shows several oscillograms.

205. Frequency.—It was shown in Article 201 and Fig. 171 that the e.m.f. of a two-pole alternator completes one cycle in one revolution of the machine. With a four-pole machine, such as is shown in Fig. 173, the e.m.f. of a conductor passes through one cycle when the conductor has moved from position 1 to position 5, or one-half a revolution. Hence, there are two cycles per revolution. It is apparent, therefore, that the e.m.f. passes through one cycle when the conductor has been cut by the flux of one pair of poles. For an alternator having P poles, the cycles per revolution equals $P \div 2$ and the frequency or cycles per second would be

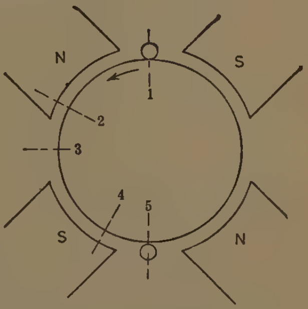


FIG. 173.—Frequency Produced by a Four-pole Machine.

$$f=\frac{P}{2}\times\frac{\text{Revolutions per minute}}{60}=\frac{P\times\text{revolutions per minute}}{120}. \qquad (45)$$

Example 1.—What frequency is produced by a twelve-pole machine operating at 250 r.p.m.?

$$f=\frac{12\times250}{120}=25 \text{ cycles.}$$

In the United States, frequencies of 60 and 25 cycles are commonly used, but there are some installations using 40 cycles. In European countries, frequencies of 50 and 25 cycles are generally used. Frequencies ranging from 12,000 to 3,000,000 cycles per second are used in radio communication.

The frequency of a power-supply system is usually maintained constant, and therefore the speed of the alternators supplying the power is definitely fixed and depends upon the number of poles. With d.c. generators, on the contrary, there is no fixed relation between the number of poles of the generators and the particular system to which they are connected. The possible speeds for alternators are given in the table on the next page.

Number of Poles	Speed in Revolutions per Minute		
	25 Cycles	40 Cycles	60 Cycles
2	1500	2400	3600
4	750	1200	1800
6	500	800	1200
8	375	600	900
10	300	480	720
P	$3000 \div P$	$4800 \div P$	$7200 \div P$

The speed of many other kinds of a.c. machines, such as induction and synchronous motors and synchronous converters, is also fixed by the number of poles and the frequency of the supply.

A frequency of 60 cycles is in common use by central stations which supply general lighting and power service in cities and by isolated plants which serve individual factories. Where synchronous converters are used, as in railway service, a frequency of 25 cycles is common. At the present time, however, improvements in design make it possible to secure very satisfactory results from 60-cycle converters and they are, therefore, generally used in new installations, instead of 25-cycle machines. Generators, synchronous motors, induction motors, and transformers, designed for 60 cycles are cheaper and lighter in weight than those designed for 25 cycles. Very slow-speed induction motors, such as are used for rolling mill drives and large air compressors are, however, more satisfactory when built for 25 cycles because they operate at a higher power factor. Incandescent lighting at 25 cycles is not so satisfactory as 60 cycles, because of the noticeable flickering of the lamps. A frequency of 25 cycles gives a lower voltage loss in transmission lines, but in spite of this fact, by far the greater proportion of the high voltage, long-distance power-transmission systems in the United States are now operated at 60 cycles.

PROBLEMS ON CHAPTER XXIV

1–24. A certain alternating-current generator supplies 100 amperes at a voltage of 2300.

(*a*) What is the *apparent power* in kv.-a. supplied by the generator?

(*b*) If the power factor of the circuit were 1.0, what would be the true power in kw.?

(*c*) If the power factor of the circuit were 0.8, what would be the true power in kw.?

2–24. In Prob. 1–24 if we neglect the losses in the a.c. generator:

(*a*) Calculate the horsepower required to drive the generator under conditions as specified in Prob. 1–24, (*b*).

(*b*) Calculate the horsepower required to drive the generator under conditions as specified in Prob. 1–24, (*c*).

3–24. A generator is carrying a load of 5000 kw. at a power factor of 0.70. The generator voltage is 11,000. Calculate the generator current.

4–24. A single-phase alternator supplies a full-load current of 100 amperes at 2300 volts. What is the rated kv.-a. output of the machine?

5–24. The rated output of a single-phase alternator is 5000 kv.-a. at 11,000 volts. What is the full-load current?

6–24. A certain coil when applied to a 250-volt a.c. circuit allows 1.5 amperes to flow. What is the impedance of the coil?

7–24. A single-phase transmission line has an impedance of 22.5 ohms. What would be the fall of potential in the line when carrying a current of 105 amperes?

8–24. An a.c. generator has sixteen poles and operates at 450 r.p.m. What is the frequency of the voltage supplied by this generator?

9–24. What is the highest speed in r.p.m. for which an alternating-current generator can be built to give a voltage having a frequency of 25 cycles per second?

CHAPTER XXV

SINUSOIDAL CURRENTS AND VOLTAGES

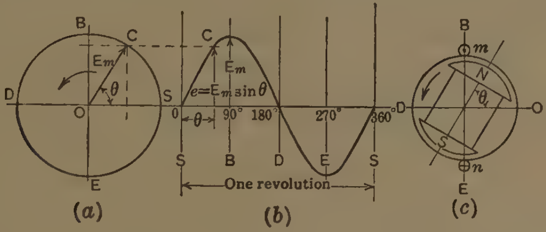
206. The Sine Curve.—In Fig. 174 *a* is shown a circle of radius E_m. The projection of E_m upon the vertical axis *BE* will be $E_m \sin \theta$, where θ is the angle *COS*, or the angular distance that the line E_m has rotated, measured from the zero point *S*. When the line E_m is at the point *S* its vertical projection is zero, and as the line revolves in a counter-clockwise direction this projection increases until $\theta = 90°$, when it is equal to E_m. As the angle θ changes from 90° to 180°, the vertical projection decreases at the same rate that it previously increased, until when $\theta = 180°$, this projection is again zero. As the angle changes from 180° to 360°, the same process is repeated, except that the vertical projections are now below the line *DS* and are therefore called negative.

The changes which occur in the length of the vertical projection of E_m as it rotates at a uniform velocity can be represented by plotting the values in rectangular coordinates, as shown in Fig. 174 *b*. The horizontal scale gives the value of the angle θ and the vertical scale the length of the vertical projection of E_m. This is called a sine curve. The ordinates of a sine curve need not necessarily be determined by plotting the projection of a revolving line E_m but may be determined by substituting in the equation

$$e = E_m \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

In this equation, E_m is the maximum ordinate of the curve and e is the ordinate corresponding to the angle θ, measured from the zero point of the curve.

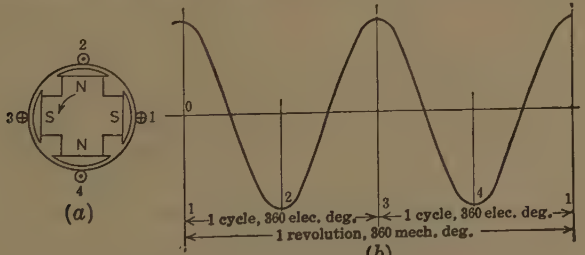
It was shown in Article 201 that when a two-pole alternator makes one complete revolution the induced e.m.f. makes one complete reversal or passes through one cycle. If the alternator produces a sine wave e.m.f., then the curve in Fig. 174 *b* would represent the actual e.m.f. between the alternator terminals at different positions of the field magnet (Fig. 174 *c*) and an oscillograph, if connected to the alternator terminals, would draw a curve like Fig. 174 *b*. When the N pole is at the point O, the e.m.f at the terminals m, n is zero. When the field reaches position B, this e.m.f. is a maximum, equals E_m, and terminal m is positive with respect to n. At position D, the e.m.f. is again zero. At position E, it becomes $-E_m$, and m is negative with respect to n. The actual e.m.f. between m and n, at any instant, can be found by substituting in Equation (46), provided E_m is taken as the maximum value of the e.m.f. between the terminals mn, and θ is measured in degrees from the reference point O. A sine wave of e.m.f. which acts upon a closed circuit will produce a current of the same frequency. In general, this current is approximately sinusoidal, but it may be distorted in some instances, due to the effect of electrostatic capacity or to other causes.



(a) (b) (c)

FIG. 174.—Production of a Sine Curve.

207. Electrical and Mechanical Degrees.—In a two-pole machine, such as is shown in Fig. 174 *c*, the field must make one complete revolution to complete one cycle. In a four-pole machine (Fig. 175 *a*) a cycle is completed when the field moves from point 1 to point 3, or one-half revolution. The angle through which the field moves in generating one cycle of e.m.f. is expressed as 360 *electrical degrees.* In the four-pole machine, this is one-half a revolution, or 180 mechanical degrees, whereas, in a two-pole machine, 360 electrical degrees corresponds to one revolution, or 360 mechanical degrees. In any alternator, the field (or armature) must move an angular distance



(a) (b)

FIG. 175.—Electrical and Mechanical Degrees.

equal to the angle subtended by two consecutive poles, in order to complete one cycle. Hence, a four-pole machine would rotate at one-half the speed of a two-pole to produce the same frequency. A curve such as is shown in Fig. 175 *b* can be used to represent the e.m.f. of a machine of any number of poles, and it is not necessary to associate an e.m.f. with the particular machine which produced it in order to solve problems in a.c. circuits.

208. Effective Value of Current and Voltage.—If a sine wave of e.m.f. is impressed upon a circuit having a resistance of R ohms, the alternating current which flows will also have a sine-wave form and can be represented by the equation

$$i = I_m \sin \theta. \qquad \qquad (47)$$

This curve is illustrated in Fig. 176. Since the current evidently varies from zero to a value of I_m, the power converted into heat varies from zero to $I_m{}^2R$. At the particular point in the cycle represented by the angle θ the power converted into heat equals i^2R. Variations in i^2, as the angle θ changes, are shown by the shaded curve. The power converted into heat at any instant during the cycle is, therefore, equal to the product of the ordinate of this current-squared curve and the resistance R; that is, $i_1{}^2R$, $i_2{}^2R$, $i_3{}^2R$, etc. Suppose that the value of i is calculated for a large number of equally spaced points on the current curve, for example, for each degree, and let these values be i_1, i_2, i_3, etc. The average rate at which power is converted into heat in the resistance is then the average of $i_1{}^2R$, $i_2{}^2R$, $i_3{}^2R$, etc., which equals average $(i_1{}^2 + i_2{}^2 + i_3{}^2 + \text{etc.})R$. Let I represent the value of a steady direct current which, when passed through the same resistance R, will convert electric power into heat at the same average rate as the alternating current $i = I_m \sin \theta$. Then the steady rate at which power is converted into heat when direct current is flowing is I^2R; and by definition, this is equal to the average rate at which power is converted into heat when the alternating current is flowing. Hence,

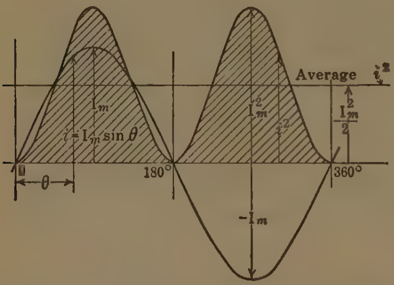


FIG. 176.—Effective Value of an Alternating Current.

$$I^2R = \text{average } (i_1{}^2 + i_2{}^2 + i_3{}^2 + \text{etc.})R,$$

or

$$I = \sqrt{\text{average } (i_1{}^2 + i_2{}^2 + i_3{}^2 + \text{etc.})}.$$

When the values i_1, i_2, i_3, etc., are the instantaneous values for a complete cycle, the expression $\sqrt{\text{average } (i_1^2+i_2^2+i_3^2+\text{etc.})}$ is the effective value of the current wave and is represented by the symbol $I_{\text{eff.}}$ Hence, the effective value of the alternating current is equal numerically to the steady value of the direct current which will produce the same average heating effect in the resistance R. Since the effective value can be determined by calculating the square root of the mean of the squares of the instantaneous values of current during the entire cycle, the effective value is sometimes spoken of as the root-mean-square value of current (abbreviated, r.m.s. value). The effective value is also called, by some writers, the virtual value.

In practice, the effective value of any wave form can be calculated by determining the instantaneous values at a number of equally spaced points on the curve, as, for example, every 5 degrees. These instantaneous values are then squared and averaged. The square root of this average is the effective value of the wave. It is apparent that the larger the number of points selected, the more closely will the result approach the exact value which is found when an infinite number of instantaneous values are used.

When the current curve is a sine wave, the effective value can be calculated mathematically without the necessity of determining the instantaneous values in the manner described in the previous paragraph. For a sine wave of current which reaches a maximum value of I_m, this is:

$$I_{\text{eff.}} = I_m \div \sqrt{2} = 0.707 I_m. \quad \ldots\ldots \quad (48)$$

The effective value of an alternating voltage is the square root of the mean of the squares of the instantaneous values of the voltage during a cycle. For a sine wave, this is

$$E_{\text{eff.}} = E_m \div \sqrt{2} = 0.707 E_m. \quad \ldots\ldots \quad (49)$$

Indicating voltmeters and ammeters, such as are commonly used for a.c. measurement, read effective values of voltage or current, regardless of the shape of the wave. The maximum value of a sine wave can be found by measuring the effective value by means of an ammeter or voltmeter and substituting in Equations 48 or 49. That is,

$$\text{Maximum value of voltage, } E_m = E_{\text{eff.}}\sqrt{2} = 1.41 E_{\text{eff.}}$$

$$\text{Maximum value of current, } I_m = I_{\text{eff.}}\sqrt{2} = 1.41 I_{\text{eff.}}$$

Example 1.—If a direct voltage of 100 volts is impressed on a circuit, having a resistance of 10 ohms (Fig. 177), a current of $100 \div 10 = 10$ amperes direct current would flow. If an alternating voltage of 100 volts effective is impressed upon the same resistance of 10 ohms, the actual voltage impressed upon this resistance will vary during each half cycle from zero volts to a maximum value of $E_m = 100 \times 1.41 = 141$ volts. The current, as read by an ordinary indicating meter, would be 10 amperes (effective), but the actual current would vary from zero to a maximum of $I_m = 10 \times 1.41 = 14.1$ amperes.

In the work which follows, the effective values of voltage or current are to be understood, unless there is a definite statement to the contrary. Hereafter, the effective values will be expressed by E or I, without subscripts.

209. Average Value of Current and Voltage.—As the shape of the positive and negative loops of a sine wave is the same, the average value of current or voltage for a complete cycle is zero. The average value of one loop or a half cycle has, however, a definite value which is numerically equal to the average height of the loop. For a sine curve, this is found to be

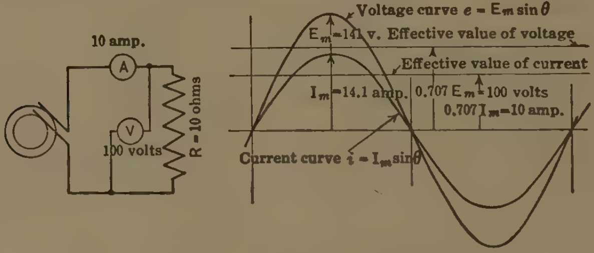
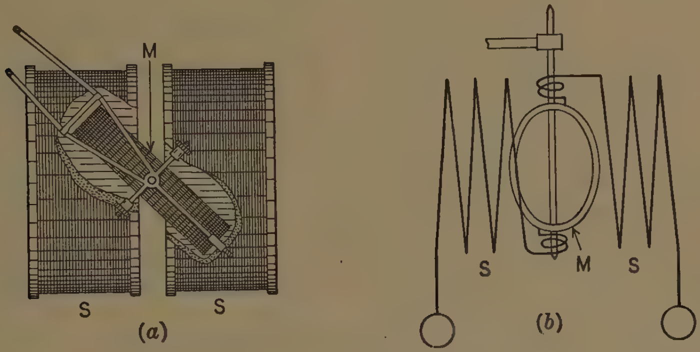

10 amp.
A
V
100 volts
R = 10 ohms
Voltage curve e = E_m sin θ
E_m = 141 v. Effective value of voltage
Effective value of current
I_m = 14.1 amp.
0.707 E_m = 100 volts
0.707 I_m = 10 amp.
Current curve i = I_m sin θ

FIG. 177.—Effective Value of Current and Voltage.

$$\text{Average value of voltage, } E_{\text{av.}} = \frac{2}{\pi} E_m = 0.637 E_m. \quad . \quad . \quad (50)$$

$$\text{Average value of current, } I_{\text{av.}} = \frac{2}{\pi} I_m = 0.637 I_m. \quad . \quad . \quad (51)$$

210. Voltmeters and Ammeters for A.C. Circuits.—With a permanent-magnet type of d.c. voltmeter, the pointer deflects either to the right or to the left, depending upon the polarity of the voltage applied to its terminals. If this meter were connected to an a.c. supply, the pointer would tend to move first in one direction and then in the other, as the polarity of the supply was reversed each cycle. Since, in the commercial a.c. circuits, these reversals are very rapid, the pointer could not move fast enough because of its inertia and would remain at zero. If, however, the stationary field of an instrument is produced by a solenoid connected in series with a moving coil, then the fields of the stationary and moving coil will reverse at the same time and the force, tending to turn the coil, will always be in the same direction. An instrument of this kind is called an electrodynamometer and is very commonly used in making a.c. measurements. The arrangement of one style of electrodynamometer is shown in Fig. 178. The stationary coils S are in series with the movable coil M to which the pointer is attached. Connection is made with the moving coil through two springs, which also resist the turning force exerted by the current. The force tending to move the coil M is proportional to the product of the field strength of the stationary and moving coils. The field strength of each of these is proportional to the current i, which flows through both, since they are in series. Hence, the force tending to turn the coil at any instant is proportional to the square of the current flowing at that instant. As this current varies from zero to a maximum I_m every half cycle, the force also varies from zero to a maximum each half cycle. Because of the inertia of the coil, however, it cannot move fast enough to follow these rapid fluctuations and, accordingly, it moves to a point corresponding to the average value of the deflecting force. This is a measure of the average i^2 and, hence, is proportional to the effective value of the current. By using a high resistance in series with the coils the instrument can be used to measure alternating voltages. An electrodynamometer type of instrument, if calibrated by direct current, will read effective values of alternating current regardless of the wave form. Electrodynamometers are used as a.c. voltmeters and wattmeters. (See Article 229.) They can be used as ammeters only when a small current is to be measured because of the small carrying capacity of the moving coil.



(a) (b)

FIG. 178.—Principle of the Electrodynamometer.

211. Vectors.—Certain quantities, such as areas, masses, and temperatures, can be represented and compared graphically by lines of varying length, referred to an arbitrary scale. Lines used in this way are called scalars. Two or more scalar quantities of the same kind, namely, several temperatures, may be combined by addition or subtraction according to the rules of algebra. Quantities which require, for their complete representation, a statement not only of magnitude but also of direction, such as forces in mechanics, are represented graphically by vectors. When so represented, the magnitude of the quantity is indicated by the length of the line according to a convenient scale, and the direction in which the force acts is shown by an arrow head placed at one end of the line, which is drawn in the direction in which the force acts, referred to an arbitrary axis of reference. Thus, in Fig. 179 *a* a certain mass is acted upon by a force of 15 pounds downward, 5 pounds to the right, and 10 pounds to the left. The vector diagram representing these forces is shown in Fig. 179 *b*. Both the 5-pound and the 10-pound forces act in a horizontal direction so that, until arrow heads are placed upon the horizontal lines representing these forces, the vector is not completely defined.

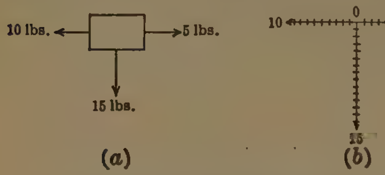


FIG. 179.—Representation of Forces.

212. Vector Representation of Alternating Currents and Voltages.—It was shown in Article 206 that the changes which occur in the value of an alternating current or voltage during a cycle can be represented by a curve plotted to rectangular coordinates. This is the kind of curve which would be drawn by the oscillograph. With a sine-wave form, it is not necessary to draw this curve since the successive values of the current or voltage can be represented by a vector, as shown in Fig. 180 *a*. The vector I_m has a length equal to the maximum ordinate of the sine curve (Fig. 180 *a*) and is supposed to be rotating at a uniform velocity in a counter-clockwise direction. At the instant marked *x* on the curve the vector has moved an angular distance of θ degrees measured from the point of reference *S*. The value of the current at that instant is $i = I_m \sin \theta$. Hence, the position

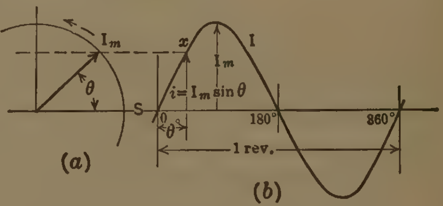


FIG. 180.—Properties of the Sine Curve.

of the vector I_m indicates the distance which it has moved or the portion of the cycle which has been completed. The angle θ is always measured in electrical degrees and, hence, one cycle is completed when the vector has made one complete revolution, starting from S. It is important to note that, in all vector diagrams, the vectors are assumed to rotate in a *counter-clockwise direction.*

The position of a vector, representing an alternating e.m.f., indicates also the position of the revolving field (or armature) of the alternator, measured from the point of zero voltage. For example, the two-pole alternator in Fig. 181 produces a sine wave of e.m.f. having a maximum value of 100 volts. The vertical projection of a vector representing this maximum value of 100 volts is the instantaneous e.m.f. at the terminals of the alternator at different points during one revolution. Thus, when the field is horizontal, the e.m.f. is zero, since the conductors are not cutting any flux. At the instant when the field is 30 degrees from the zero point S, the value of the e.m.f. is represented to scale by the vertical projection of the vector when it is 30 degrees from point S as shown in diagram, Fig. 181 *a*. At that instant, the terminal e.m.f. is, therefore, $e_1 = 100 \sin 30° = 50$ volts. Similarly in diagrams *b*, *c*, and *d*, the instantaneous e.m.f. is 70.7 volts, −86.6 volts, and −70.7 volts, respectively.

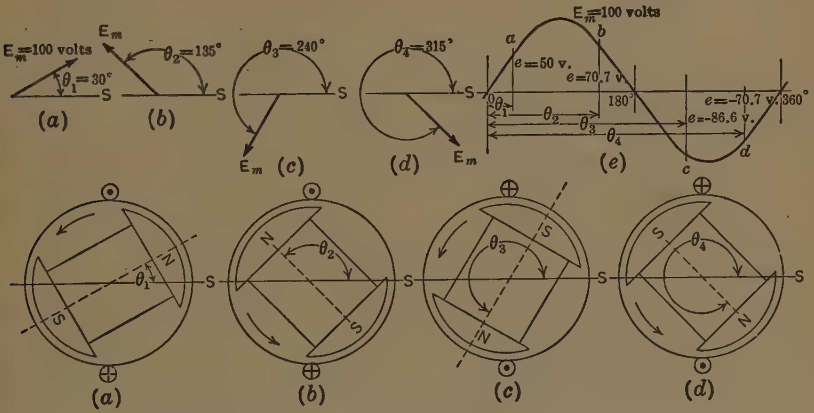
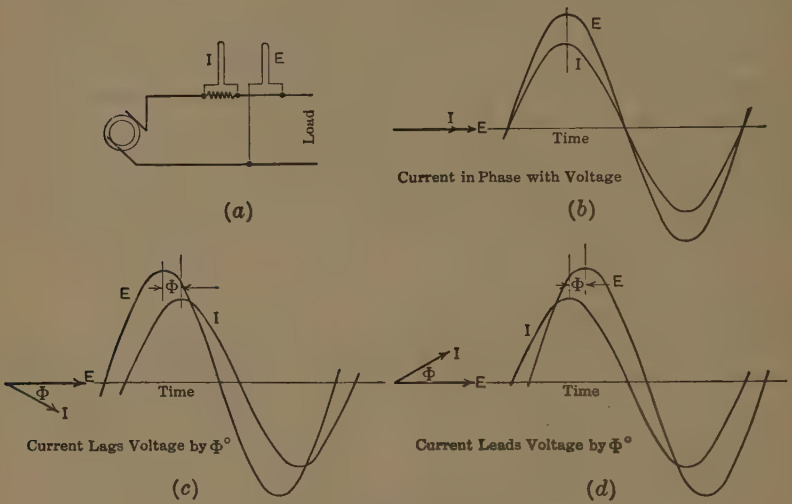


FIG. 181.—Representation of a Voltage by Vectors.

213. Phase Relations in A.C. Circuits.—If two oscillograph elements are so connected to an a.c. circuit that one draws the voltage curve and the other the current curve, it will be found that the maximum values of the voltage and current curves do not necessarily coincide but may be displaced according to the kind of load connected to the circuit. When both current and voltage reach their positive maximum values at the same instant, as in Fig. 182 *b*, the current and voltage are said to be in phase; when the current reaches its positive maximum at a later time than the voltage, as in *c*, the current is said to lag behind the voltage and the angular distance is called the phase displacement. Similarly, in *d* the current leads by Φ degrees. When comparing sine curves it should be remembered that elapsed time is measured from left to right so that the current curve in diagram *d* reaches its maximum before the voltage, and hence the current is ahead or leads the voltage. The current I, produced by the voltage E (Fig. 182) must evidently have the same frequency, hence the angle Φ will remain constant, as there is no change in the character of the load. When the phase angle remains constant in this



(*a*)

Current in Phase with Voltage

(*b*)

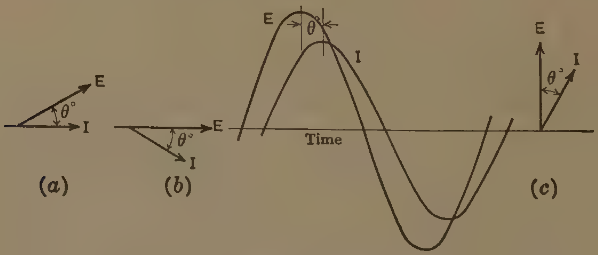
Current Lags Voltage by Φ°

(*c*)

Current Leads Voltage by Φ°

(*d*)

FIG. 182.—Phase Relations.



(*a*) (*b*) (*c*)

FIG. 183.—Representation of Phase Position by Vectors.

way, the conditions existing can be represented by vectors. Thus in Fig. 183 the current and voltage can be represented, respectively, by a vector I and E drawn from the same center in such a way that I is located θ degrees behind E. The position in which the two vectors are drawn is unimportant so long as they are separated by the angle θ. Three positions in which they might be drawn are shown but, whatever the position, they represent the same sine curves which are separated by the angle θ. The phase angle between two sine waves does not change with variations in the effective values of the waves, as will be apparent by a study of the curves shown in Fig. 183.

When several quantities are to be considered, it is more convenient to employ vectors than to draw the sine curves representing these quantities. The vector diagram in Fig. 184 *a* indicates that voltage E_A leads E_B by an angle of 30 degrees and that E_C lags E_B by 60 degrees, or it could be said that E_B is 60 degrees ahead of E_C and 30 degrees behind E_A. Either of these statements is sufficient to enable the sine curves for these three voltages to be plotted in their correct phase position. Similarly, it can be said that, for the currents represented in Fig. 184 *b*, I_2 is 45 degrees behind I_1, and I_3 is 90 degrees behind I_1. Special terms are used to designate certain phase angles as is shown in Fig. 185.

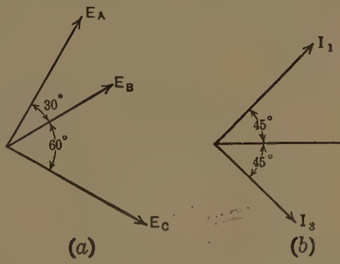
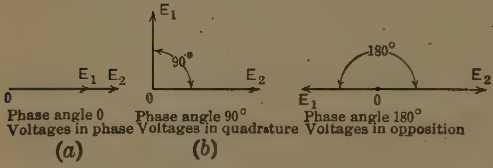


FIG. 184.—Phase Relations of Currents and Voltages.

214. Combining Alternating Voltages.—When two d.c. generators, A and B (Fig. 186 *a*) are connected in series, the total voltage E_C is the numerical sum of the voltages E_A and E_B. Suppose that two bipolar alternators are rigidly coupled together, as shown in Fig. 186 *b* and *c*, in such a way that the N pole of machine B is Φ degrees behind the N pole of machine A measured in the direction of rotation, which is counter-clockwise. The voltages of the two machines will then have the same frequency and can be represented by the curves shown in Fig. 186 *d*. Terminal 1 will have a maximum voltage in the positive direction when the N pole is directly in line with 1, while terminal 3 will not have a maximum voltage in a positive direction until the fields have rotated Φ degrees further. Hence, the voltage of machine B lags behind the voltage of A by Φ degrees. The voltage between terminals 1 and 4 at any instant is equal to the algebraic sum of the voltages e_A and e_B which are produced by the machines at that instant. By plotting the sum of these voltages taken at different instants during a cycle, the curve E_R is obtained. This represents the voltage between terminals 1 and 4 at different points in a cycle. The sine curves E_A and E_B can be represented by two vectors which have lengths equal to the maximum values of E_A and E_B according to any convenient scale and are so drawn that there is an angle of Φ degrees between them (Fig. 186 *e*). The diagonal of the parallelogram formed by the vectors E_A and E_B is the maximum value of the curve E_R in Fig. 186 *d*, and this curve could be constructed by plotting the vertical projections of E_R in the manner explained in a previous paragraph. That E_R is the maximum value of the sine curve of the combined voltage of the two alternators can be seen from the fact that the vertical projection of E_R at the instant shown



Phase angle 0 Voltages in phase (*a*)
Phase angle 90° Voltages in quadrature (*b*)
Phase angle 180° Voltages in opposition

FIG. 185.—Special Designations of Phase Angles.

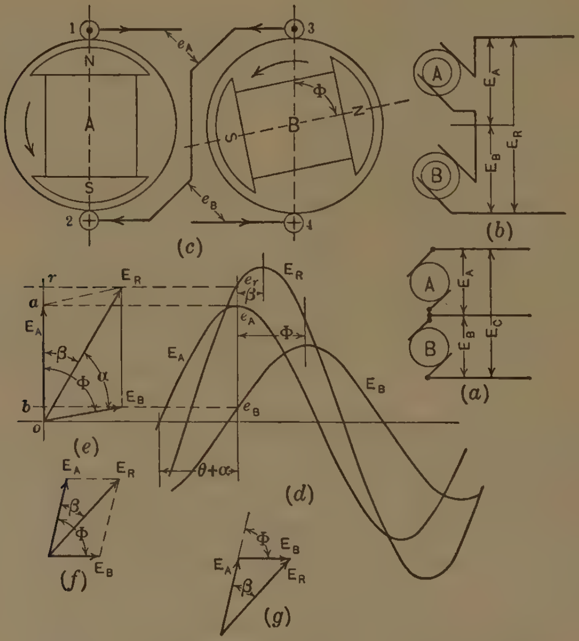
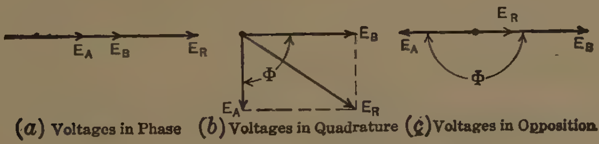


FIG. 186.—Combining Two Alternating Voltages.

in *d* is the line *or* which is equal to *oa*+*ob*. This holds true for any other point in the cycle, since the three vectors revolve at the same speed and therefore the angle Φ is constant. The angle β is the phase angle between the voltage of machine *A* and the total or resultant voltage between terminals 1–4, while α is the phase angle between machine *B* and this resultant voltage.

The explanation given in the previous paragraph shows that it is not necessary to draw the sine curves of a.c. voltages in a series circuit in order to find the total voltage as in Fig. 186 *d*, but that this resultant voltage E_R can be found by combining two vectors drawn to the proper scale and having an angle between them equal to the phase angle of the two voltages. In the vector diagram shown in Fig. 186 *e* the vectors represent the maximum values of the voltages. Since, however, the effective values are 0.707 times the maximum values, the same kind of diagram (Fig. 186 *f*) drawn to a different scale would represent the effective values of the voltages of the two alternators and the combined voltage. The maximum and effective values of a wave and the elapsed time between the maximum values of two waves are of importance when solving problems, and all of these can be shown by vector diagrams. It is necessary to draw the actual wave form, by an oscillograph or other means, only when instantaneous values must be considered. An arrangement of vectors as in Fig. 186 *e* or *f* is called a clock diagram. In Fig. 186 *g* is shown another vector diagram where the back end of vector E_B is placed against the point of E_A, and E_R is found by drawing a line to close the triangle. The corresponding angles are also shown. It is apparent that either construction leads to the same result. Sometimes the arrangement shown in Fig. 186 *g* is more convenient but usually it is better to employ the clock diagram with all the vectors radiating from one center, as this facilitates the determination of the phase angles.



(*a*) Voltages in Phase (*b*) Voltages in Quadrature (*c*) Voltages in Opposition

FIG. 187.—Resultant of Two Voltages.

It is apparent that the magnitude of the resultant E_R will depend upon the value of the angle Φ. The resultant will be a maximum when the two voltages are in phase and $\Phi = 0°$ (Fig. 187 *a*), in which case $E_R = E_A + E_B$. When $\Phi = 90°$, the voltages are in quadrature and

$E_R=\sqrt{E_A^2+E_B^2}$. When $\Phi=180°$, the voltages are in opposition and $E_R=E_B-E_A$.

215. Resolution of a Vector into Components.—This is the reverse of the process described in the previous article, where the method of finding the resultant of two vectors was explained. In general, any vector can be resolved into two components which may be at right angles or at any angle less than 180 and more than zero degrees. For example, the vector E (Fig. 188 *a*) may be resolved into two vectors E_1 and E_2 at right angles (Fig. 188 *b*); or into two vectors E_3 and E_4, making an acute angle θ with each other (Fig. 188 *c*); or into two vectors E_5 and E_6 (Fig. 188 *d*), making an obtuse angle Φ with each other. In each of the three cases, the pairs of vectors shown in Fig. 188 *b*, *c*, or *d* are the exact equivalent of the single vector E. There are, in fact, an infinite number of pairs of vectors or components into which E could be resolved. In most cases in practice, the vector is resolved into two components at right angles, as in Fig. 188 *b*.

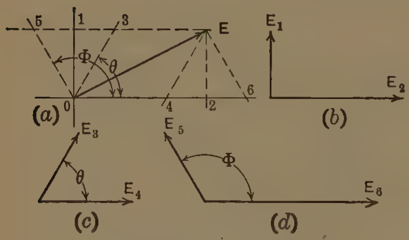


FIG. 188.—Resolution of a Voltage into Components.

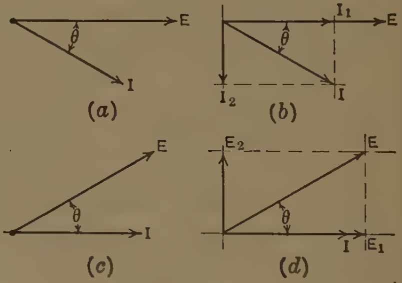


FIG. 189.—Resolution of a Vector into Components at Right Angles.

A common application of this idea is shown in Fig. 189 *a* where the current I lags θ degrees behind the voltage E. The current can be resolved into two components, as shown in Fig. 189 *b*. The component I_1 in phase with the voltage is called the *active* or in-phase component of the current. The component I_2 at right angles to the voltage is called the *reactive* or quadrature component of the current. The voltage E also can be resolved into two components, as shown in Fig. 189 *d*. The component E_1 is the active component of the voltage and E_2 the reactive component. According to the discussion of the previous article the two components of current or of voltage (I_1 and I_2 or E_1 and E_2) are the exact equivalent of the actual current I or voltage E in the circuit, and these components could be substituted in a vector diagram for the single vector I or E. An examination of Fig. 189 will show that the components can be calculated as follows:

Active component of E is $E_1 = E \cos \theta$.
Reactive component of E is $E_2 = E \sin \theta$.
Active component of I is $I_1 = I \cos \theta$.
Reactive component of I is $I_2 = I \sin \theta$.

216. Voltage and Current Relations in Series Circuits.—A series circuit is one in which the same current flows in all parts of the circuit.

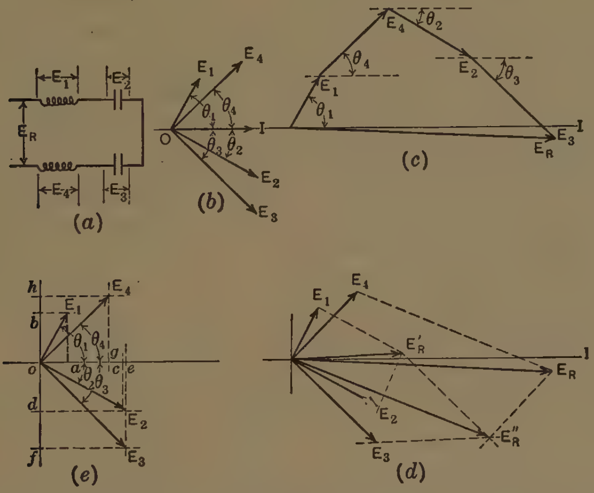


FIG. 190.—Methods of Combining Voltages in a Series Circuit.

When such a circuit is carrying an alternating current the terminal voltage may be less than the numerical sum of the voltages measured across the various parts. For example, in Fig. 190 *a* the voltage E_R is very much less than the sum of E_1, E_2, E_3, and E_4. The voltage E_R is, in fact, the *vector sum* of these four voltages. To find this sum, a vector diagram should be drawn and the value of the resultant voltage E_R found graphically or by computation. In a series circuit, where there is a single current I, it is convenient to use the current vector as a line of reference and to draw it horizontally to the right of the center O (Fig. 190 *b*). The voltages are then located at the proper angle, lagging or leading, with respect to the current vector I. The position of the voltage vectors must be determined by the conditions of the problem, and if the phase angles are not given, they must be computed. The effective values of current and voltage are used in these diagrams.

When solving a vector diagram graphically (Fig. 190 *d*) the vectors are combined two at a time. Thus, E_1 and E_2 when combined give a resultant E_R'. This resultant, combined with E_3 gives E_R'' and E_R'' combined with E_4 gives E_R, the total or terminal voltage. Another graphical method, employing what is called a polygon of vectors, is shown in Fig. 190 *c*. The voltage vectors are arranged to form the sides of a polygon with the proper angles between them as determined by the phase position indicated in the problem. The line E_R which completes the polygon is the total voltage. This method, which is known as the topographical method, gives a simpler diagram than the arrangement shown in Fig. 190 *d* when there are more than two vectors to be combined. Graphical methods are, however, rather inaccurate, especially when the phase angles are small. For this reason, mathematical methods are preferable, but when they are used, a vector diagram should also be drawn. This diagram should show the vectors approximately to scale so that possible errors in the calculated results can be detected. By drawing such a diagram, a more definite idea of the physical conditions of the problem can be secured.

It was shown in Article 215 that any vector can be resolved into two components at right angles to each other. If the vectors E_1, E_2, E_3, and E_4 (Fig. 191) are resolved into their components along horizontal and vertical axes, the sum of the horizontal components will be the horizontal component of the total or resultant voltage E_R, and the sum of the vertical components will be the vertical component of E_R. From these two components E_R can be calculated. Referring to Fig. 191 *b* the components are:

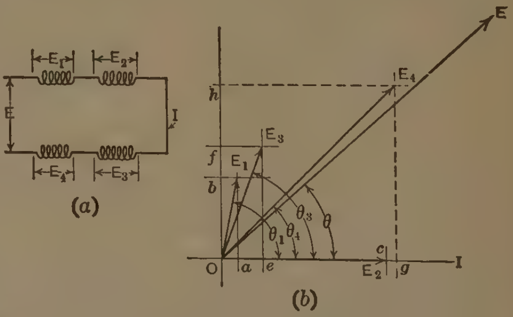


FIG. 191.—Resultant of Voltages in a Series Circuit.

Horizontal Components, H	Vertical Components, V
For E_1, $oa = E_1 \cos \theta_1$.	$ob = E_1 \sin \theta_1$.
For E_2, $oc = E_2 \cos \theta_2$.	$od = E_2 \sin \theta_2$.
For E_3, $oe = E_3 \cos \theta_3$.	$of = E_3 \sin \theta_3$.
For E_4, $og = E_4 \cos \theta_4$.	$oh = E_4 \sin \theta_4$.

The total horizontal component is

$$\begin{aligned} H &= oa+oc+oe+og \\ &= E_1 \cos \theta_1 + E_2 \cos \theta_2 + E_3 \cos \theta_3 + E_4 \cos \theta_4. \end{aligned}$$

The total vertical component is

$$\begin{aligned} V &= ob+od+of+oh \\ &= E_1 \sin \theta_1 + E_2 \sin \theta_2 + E_3 \sin \theta_3 + E_4 \sin \theta_4. \end{aligned}$$

The resultant voltage is the vector sum of the components, or

$$\begin{aligned} E_R &= E_1 \oplus E_2 \oplus E_3 \oplus E_4 \\ &= \sqrt{H^2+V^2}. \end{aligned}$$

The above method can be applied to any number of vectors with any value of the phase angles θ_1, θ_2, etc.

Example 1.—In a series circuit consisting of four parts (Fig. 191 *a*), the voltages are: $E_1 = 100$ volts, leading 80°; $E_2 = 200$ volts, in phase; $E_3 = 150$ volts, leading 70°; and $E_4 = 300$ volts, leading 45°. Find the total voltage E impressed on the circuit. The vector diagram is shown in Fig. 191 *b*. Resolving the voltages into horizontal and vertical components:

H	V
For E_1, $oa = 100 \cos 80° = 17.4$	$ob = 100 \sin 80° = 98.5$
For E_2, $oc = 200 \cos 0° = 200$	$od = 200 \sin 0° = 0$
For E_3, $oe = 150 \cos 70° = 51.3$	$of = 150 \sin 70° = 141$
For E_4, $og = 300 \cos 45° = 212.1$	$oh = 300 \sin 45° = 212.1$
$H = 480.8$	$V = 451.6$

$$E = \sqrt{480.8^2 + 451.6^2} = 659.3 \text{ volts.}$$

When computations are to be made by means of a slide rule, the resultant E can be found most easily by first computing the tangent of the angle θ between the resultant voltage E and the current I:

$$\text{Tan } \theta = \frac{V}{H} = \frac{451.6}{480.8} = 0.925.$$

Then

$$\theta = 42.8° \quad \text{and} \quad \cos \theta = 0.7337.$$

Hence

$$E = \frac{H}{\cos \theta°} = \frac{480.8}{0.7337} = 659 \text{ volts.}$$

The total voltage E leads the current by 42.8°.

The vectors do not always lie in the same quadrant as in Fig. 191, and, therefore, their components may have to be subtracted instead of added.

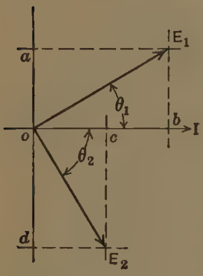


Fig. 192.—Signs of Components.

With two vectors (Fig. 192), the voltage E_1 leads the current by θ_1 degrees and E_2 lags by θ_2 degrees. The H components are oc and ob, and both are in the same direction; hence, they should be added, i.e.: $H=oc+ob$. The vertical component of E_1 is oa and of E_2 is od, and these are in opposite directions; therefore, they must be subtracted. It is customary to call oa positive and od negative, hence, $V=oa-od$. The position of a vector with respect to the reference line (which in series circuits is taken as the current) will determine the signs for the two components. This is shown in Fig. 193, where the signs are given for vectors located in the four quadrants.

Example 2.—A series circuit (Fig. 194) with a current I, consists of four parts having voltages as follows: $E_1=100$ volts, leading 45°; $E_2=200$ volts, lagging 85°; $E_3=150$ volts, leading 70°; and $E_4=300$ volts, lagging 80°. Find the total voltage E which is impressed on the circuit. The H and V components are:

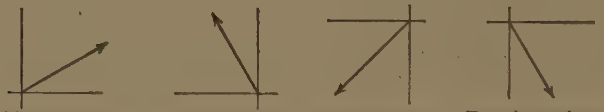

First quadrant $\begin{smallmatrix}+H\\+V\end{smallmatrix}$ (*a*)
Second quadrant $\begin{smallmatrix}-H\\+V\end{smallmatrix}$ (*b*)
Third quadrant $\begin{smallmatrix}-H\\-V\end{smallmatrix}$ (*c*)
Fourth quadrant $\begin{smallmatrix}+H\\-V\end{smallmatrix}$ (*d*)

Fig. 193.—Signs of Components of Vectors in Four Quadrants.

H	V
For E_1, $oa=100\cos 45°=\ \ 70.7$	$ob=\ \ 100\sin 45°=\ \ 70.7$
For E_2, $oc=200\cos 85°=\ \ 17.4$	$od=-200\sin 85°=-199.2$
For E_3, $oe=150\cos 70°=\ \ 51.3$	$of=\ \ 150\sin 70°=\ \ 141$
For E_4, $og=300\cos 80°=\ \ 52.1$	$oh=-300\sin 80°=-295$
$H=191.5$	$V=-282.5$

Since V is negative and H positive, the resultant voltage E will lie in the fourth quadrant according to Fig. 193 *d*.

$$\text{Tan }\theta=\frac{282.5}{191}=1.47, \quad \theta=55.8°.$$

$$\text{Cos }\theta=\cos 55.8°=0.562.$$

$$E=\frac{191.5}{0.562}=341 \text{ volts.}$$

The resultant voltage E, therefore, lags 55.8° behind the current I.

217. Voltage and Current Relations in Parallel Circuits.—In a parallel circuit the voltage is the same across every branch, and the total current is the vector sum of the currents in the individual branch circuits. In general, the total or resultant current is less than the numerical sum of the current in the different branches. In drawing a vector diagram for a parallel circuit the single voltage vector is used as a line of reference and the current vectors are located at the proper angle with the voltage. Thus, for the circuit shown in Fig. 195 *a* the currents in the three branches are I_1, I_2, and I_3, and the total current I_R is the vector sum of I_1, I_2, and I_3, or

$$I_R = I_1 \oplus I_2 \oplus I_3.$$

The vector diagram is shown in Fig. 195 *b*. The total current I_R can be found by resolving the branch currents into horizontal and vertical components, as was described in the previous article. The horizontal component of the resultant current is

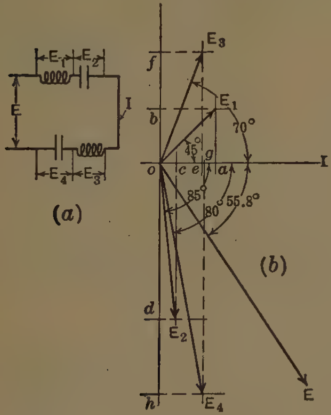


FIG. 194.—Vector Diagram for a Series Circuit.

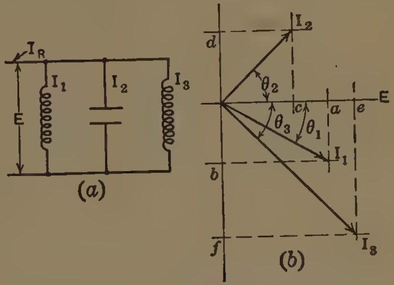


FIG. 195.—Resultant Current in a Parallel Circuit

$$H = oa + oc + oe$$
$$= I_1 \cos \theta_1 + I_2 \cos \theta_2 + I_3 \cos \theta_3.$$

The vertical component of the resultant current is

$$V = -ob + od - of$$
$$= -I_1 \sin \theta_1 + I_2 \sin \theta_2 - I_3 \sin \theta_3$$

and the total current is

$$I_R = \sqrt{H^2 + V^2}.$$

The angle θ between the total current and the voltage can be found from

$$\operatorname{Tan} \theta = \frac{V}{H}.$$

In the numerical solution of problems involving parallel circuits, the methods illustrated in Examples 1 and 2 of Article 216 are to be followed.

218. Non-sinusoidal Waves.—The methods of representing voltages and currents which have been given in this chapter are based upon a sinusoidal wave form. The methods of calculating a.c. circuits which are given in the next chapter are also based upon sinusoidal waves. Even if the actual wave is not sinusoidal, as frequently occurs in practice, these methods may be used because the usual measuring instruments read effective values regardless of wave form and these effective values can be used as if they represented sine waves. Therefore, in the more common problems involving the performance of a.c. machinery or of loaded transmission lines, we may proceed as if the quantities were actually sinusoidal.[1]

PROBLEMS ON CHAPTER XXV

Note: A vector diagram should be drawn for each problem.

1–25. (*a*) An alternator supplies a sine wave of e.m.f. having a maximum value of 3250 volts. Calculate the instantaneous values of the e.m.f. for a complete cycle, for every 15 electrical degrees of rotation of the armature.

(*b*) Plot the sine curve from the values obtained in (*a*).

2–25. (*a*) If the alternator in Prob. 1–25 supplies a sine wave of current having a maximum value of 1410 amperes, calculate the instantaneous values of the current for one complete cycle for every 15 electrical degrees.

(*b*) Plot the sine curve from the values obtained in (*a*).

3–25. An alternator has sixteen poles.

(*a*) What fraction of a revolution must the field move to complete one cycle? Express this in mechanical degrees.

(*b*) At what speed must the machine run to produce a frequency of 60 cycles?

4–25. (*a*) Calculate the effective value of the e.m.f. produced by the alternator of Prob. 1–25, using the instantaneous values determined in this problem.

(*b*) Compare the result thus obtained with the value determined from Equation 49.

(*c*) Calculate the effective value of current from Prob. 2–25 using Equation 48.

5–25. (*a*) Calculate the average value of the e.m.f. produced by the alternator of Prob. 1–25, using the instantaneous values determined in this problem.

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[1] The more exact method used in solving problems involving non-sinusoidal waves are given in Pender's, "Electricity and Magnetism for Engineers," Part II, Chapter XVIII.

(*b*) Compare the result thus obtained with the value determined from Equation 50.

(*c*) Calculate the average value of current from Prob. 2–25 using Equation 51.

6–25. A certain transmission line is operated at 220,000 volts (effective) between wires. What is the maximum difference of potential between wires?

7–25. An incandescent lamp takes 2.6 amperes (effective). What is the maximum value of current in the lamp filament?

8–25. (*a*) If the alternator in Prob. 1–25 supplies a current having a maximum value of 1410 amperes which is in phase with the e.m.f., represent by vectors these two quantities.

(*b*) If the current were 30 degrees behind the e.m.f., represent this condition by vectors.

(*c*) If the current were 30 degrees ahead of the e.m.f., represent this condition by vectors.

9–25. Two alternating current generators are connected in series, No. 1 generates an effective e.m.f. of 2300 volts, and No. 2 an effective e.m.f. of 1100 volts.

(*a*) If the phase angle between the e.m.f.'s were 70 degrees, what would be the combined e.m.f.?

(*b*) If the phase angle were 90 degrees, what would be the combined e.m.f.?

(*c*) If the phase angle were 165 degrees, what would be the combined e.m.f.?

10–25. An alternator has a terminal potential of 2300 volts and a current of 125 amperes, which lags 15 degrees behind the voltage.

(*a*) Calculate the active component of the current.

(*b*) Calculate the reactive component of the current.

(*c*) Calculate the active component of the voltage.

(*d*) Calculate the reactive component of the voltage.

11–25. A series circuit taking 0.1 ampere is made up of three parts. Part A has a terminal voltage of 100 volts, and is in phase with the current. Part B has a terminal voltage of 200 volts which is 90 degrees behind the current. Part C has a terminal voltage of 250 volts and is 90 degrees ahead of the current.

(*a*) What is the total voltage required for the entire circuit?

(*b*) What is the phase angle between the current and the resultant voltage?

12–25. A series circuit is made up of four different pieces of apparatus and takes a current of 10 amperes. Part A has a voltage at its terminals of 55 volts in phase with the current. Part B has a voltage of 75 volts and is 60 degrees ahead of the current. Part C has a voltage of 100 volts and is 85 degrees ahead of the current. Part D has a voltage of 20 volts which is 30 degrees ahead of the current.

(*a*) Calculate the total impressed voltage for the entire circuit.

(*b*) What will be the position of this total voltage vector with respect to the current?

13–25. An a.c. arc lamp takes 6 amperes from a 110-volt supply circuit. The voltage across the arc is 72 volts, and this voltage is 25 degrees behind the line voltage.

(*a*) Calculate the necessary fall of potential of the reactance which must be put in series with the arc to give the above voltage of 72 volts.

(*b*) What is the phase angle between the voltage of the reactance and the line voltage?

14–25. A transformer running without load takes a current of 10 amperes which lags 70 degrees behind the impressed voltage of 2300 volts.

(*a*) What are the active and reactive components of this current?

15–25. In Prob. 11–25:

(*a*) Calculate the active and reactive components of voltage for each of the four parts.

(*b*) Calculate same for the line voltage.

16–25. Two generators are operated at the same frequency and their terminal voltages adjusted to 2300 volts each. Terminals of the two machines of like polarity are connected together so that the e.m.f. of *B* is 180 degrees behind *A*. The voltage of *A* is held constant and the e.m.f. of *B* is then changed to 2000 volts, the phase angle remaining the same. A current of 50 amperes then flows through the two machines.

(*a*) What is the value of the resultant voltage which causes this current to flow?

(*b*) If the current is 90 degrees behind the resultant voltage, what is the power for each machine?

17–25. The generators of Prob. 16–25 are adjusted for the same e.m.f., 2300 volts, and then the voltage of *B* is made to lag 185 degrees behind *A*.

(*a*) What is the resultant voltage which forces current through *A* and *B*?

(*b*) If the current which flows is 34 amperes, 90 degrees behind the resultant voltage found in (*a*), calculate the power in machines *A* and *B*.

18–25. The generators in Prob. 16–25 are adjusted until the e.m.f.'s are each 2300 volts and then the voltage of *B* is made to lag 175 degrees behind *A*.

(*a*) What is the resultant voltage which forces current through the circuit?

(*b*) If the current is 34 amperes lagging 90 degrees behind the resultant voltage, calculate the power in machines *A* and *B*.

19–25. A load of lamps requires 100 amperes and a load of single-phase motors requires 40 amperes. The current for the motors is 40 degrees behind the current for the lamps.

What is the combined current?

20–25. A circuit supplied from 110-volt mains is made up of three parts, all in parallel across the mains. Part A takes 10 amperes which is in phase with the line voltage. Part B takes 10 amperes which lags 45 degrees behind the line voltage. Part C takes 10 amperes which is 45 degrees ahead of the line voltage.

(*a*) Calculate the total current taken from the supply mains.

(*b*) What is the position of the vector representing the total current with respect to the line voltage?

21–25. If in Prob. 20–25, the angle of lag for *B* were 90 degrees and the angle of lead for *C* were 90 degrees instead of 45 degrees:

(*a*) Calculate the total current taken from the supply mains.

(*b*) What would be the position of the vector representing the total current?

22–25. If in Prob. 20–25, the angle of lag for *B* were 30 degrees, and the angle of lead for *C* were 90 degrees:

(*a*) Calculate the total current taken from the mains.

(*b*) What would be the position of the vector representing this current?

23–25. A certain power station has three single-phase feeders, all supplied at 2300 volts. The ammeter on feeder *A* reads 100 amperes; on feeder *B* 50 amperes and on feeder *C*, 150 amperes. Power factor is lagging.

Wattmeters on the feeders read: $A = 182$ kw.; $B = 98$ kw.; $C = 275$ kw.

(*a*) What is the power factor of each feeder?

(*b*) What is the total current taken by the feeders?

(*c*) What is the total power supplied to the feeders?

(*d*) What is the total kv.-a.?

(*e*) What is the power factor of the entire station?

24–25. A 50.-kv-a. transformer running without load on the secondary takes 2.5 amperes when operated at a normal voltage of 1100 volts on the primary. The power taken by the transformer under these conditions is 720 watts.

(*a*) Calculate the power factor.

(*b*) What is the angle of lag between the current and impressed voltage?

25-25. A certain station operates at 2400 volts, single-phase, and supplies loads as follows:

(*a*) Feeder No. 1 carries incandescent lamps. The ammeter in this feeder reads 50 amperes and the wattmeter 120 kw. What is the apparent power and the power factor?

(*b*) Feeder No. 2 carries series arc lighting. The ammeter reads 16 amperes and the wattmeter 25 kw. What is the apparent power and power factor?

(*c*) Feeder No. 3 carries lamps and motors. The ammeter reads 50 amperes and the wattmeter 100 kw. What is the apparent power and power factor?

26–25. An alternator is to be purchased to supply the loads specified in Prob. 23–25.

(*a*) What would be the cost at the rate of $17.00 per kv.-a.?

(*b*) If the power factor could be made unity with the same kilowatt loads, what would be the cost?

27–25. A certain power plant having a capacity of 10,000 kv.-a. cost $80.00 per kv.-a.

(*a*) If the load is supplied at unity power factor, what is the power plant investment per kw. of load?

(*b*) If the load is supplied at a power factor of 70 per cent what is the power plant investment per kw. of load?

CHAPTER XXVI

ALTERNATING-CURRENT CIRCUITS

219. Resistance.—The resistance of a circuit is given by Equation (3), Article 11. Let this resistance be R_c ohms. If a direct current of I amperes flows in the conductor, the power loss (Article 8) is $P=I^2R_c$ watts. The resistance R_c is called the conductor resistance or ohmic resistance. When an alternating current of I amperes flows in a circuit, the power supplied to the circuit may be considerably greater than would be found from this equation, as will be seen by the following:

Example 1.—The conductor resistance of the 2500-volt winding of the transformer described in Example 1, Article 198, is measured by the fall of potential method, using direct current and is found to be $R_c=0.492$ ohm. When 1.82 amperes direct current is passed through the coil, the power input is found to be $P_{dc}=1.82^2\times0.492=1.63$ watts. When an alternating current of 1.82 amperes (effective), at 25 cycles, is passed through the coil, the power input is found by measurement to be 1520 watts. The power lost in the circuit is evidently much greater when it carries an alternating current and the transformer winding acts as if its resistance were greatly increased.

If I is the alternating current in the circuit and P the power input, the resistance is

$$R=\frac{P}{I^2}. \qquad (52)$$

The quantity R is known as the *effective resistance* of the circuit.

This equation applies to any a.c. circuit where the power P is all transformed into heat. In Example 1 above, the effective resistance is

$$R=\frac{1520}{1.82^2}=459 \text{ ohms.}$$

Of the total power supplied to this winding, 1.63 watts is lost in heat in the copper due to the conductor resistance $R_c=0.492$ ohm and the remainder, or about 1518 watts, is used in heating the iron core, due to hysteresis and eddy current losses caused by the alternating flux. If the wire forming this coil were unwound from the core, the power input with an alternating current of 1.82 amperes flowing would be only 1.63 watts. That is, the conductor resistance and effective resistance would be equal. In general, any coil wound on an iron core would have an effective resistance higher than the conductor resistance of the coil. In a.c. calculations, the effective resistance should be used. This can be found by measuring the input to the circuit with a wattmeter and solving for R by means of Equation (52).

220. A.C. in Resistance Circuits.—If an alternating current flows in a non-inductive circuit[1] having a resistance of R ohms (Fig. 196 *a*) the only opposition to the flow of the current is produced by this resistance and the reaction to the flow of current at any instant is a voltage $e' = iR$ where i is the current at the particular instant considered. This opposing voltage will be a sine curve if the current is a sine curve and can be represented by curve E' in Fig. 196 *b*. To overcome the reaction expressed by E' there must be a voltage E impressed on the circuit in such a manner that at each instant the value e shall be equal and opposite to e'. This is the curve E. When the current is zero, E is zero and, when the current is a maximum, E must also be a maximum. Hence, the *impressed voltage* E and the current I are in phase and can be represented by the vector diagram (Fig. 196 *c*). In problems involving circuits having resistance it is customary to show only the current and the *impressed* voltage E, required to overcome the reaction due to the resistance and to omit vector E'. Ohm's Law can be applied to non-inductive, a.c. circuits in the same way that it is used in d.c. circuits. That is, if an effective

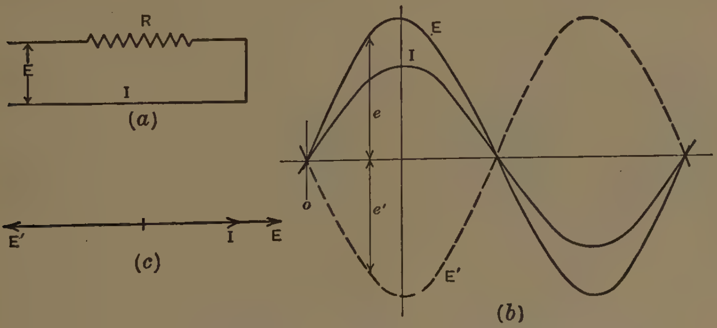

FIG. 196.—Alternating Current in a Resistance Circuit

voltage of E volts is impressed on a non-inductive circuit having a resistance of R ohms, the effective current is

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[1] A circuit is non-inductive when the magnetic field surrounding the conductor composing the circuit is so small that the counter e.m.f. produced by this field is negligible as compared with the e.m.f. required to overcome the resistance. An incandescent lamp or a water rheostat is non-inductive.

$$I=\frac{E}{R}.$$

Where there are several non-inductive resistances in an a.c. circuit they can be combined by the laws for series or parallel circuits.

221. Power in a Resistance Circuit.—It has been shown in the previous article that the voltage E and current I for a non-inductive resistance are in phase. If these are drawn as in Fig. 197 then at point x the instantaneous values are e and i. The product $e\times i$ is the rate at which power is being supplied to the circuit at that instant. At points a, c, and e the power is zero; at points b and d the power is E_mI_m and is maximum. At other points the value is between these limits. If the values of the product $e\times i$ are plotted, the curve P is obtained. This curve is positive at all times; that is, all the energy delivered to the resistance R is consumed by the resistance and none of it is returned to the supply. The power input is not steady but varies from zero to a maximum twice during each cycle. The average power input is the average height of the curve P. This can be found graphically by determining the area of one loop of the curve P and dividing by the length of the base, or it can be found mathematically. It is

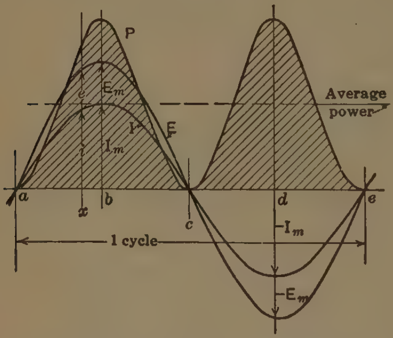

FIG. 197.—Power in a Resistance Circuit. Power curve is drawn to a smaller scale.

$$P=EI \text{ watts}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

where E and I are effective values. In measurements of an actual circuit the wattmeter reads the average power P. Also, it should be noted that since $E=IR$, the average power is $P=I^2R$ watts. Therefore, in a non-inductive circuit, the current I and voltage E may be measured by ordinary a.c. instruments, and the product of the readings will be the average power supplied to the circuit.

According to Article 198 the power factor equals the true power divided by the apparent power. For a resistance circuit the true power is, according to Equation (53), $P=EI$ watts, and the apparent power is EI volt-amperes. Therefore

$$\text{Power factor}=\frac{P}{EI}=\frac{EI}{EI}=1.$$

The current and voltage are in phase; therefore, the phase angle $\theta=o$. Since $\cos\theta=\cos 0°=1$, the power factor is equal numerically to the cosine of the angle between E and I.

222. Inductance.—In Article 59 it was shown that whenever the current in an electric circuit changes, a counter e.m.f. is produced because of the magnetic field surrounding the circuit. With d.c. circuits, this counter e.m.f. is generally of importance only when the circuit is opened or closed, but with a.c. circuits, where the current is continually changing in value even when the load is constant, the effect of self-induction is of much greater importance.

Example 1.—The transformer coil described in Example 1, Article 198, would require a steady direct voltage of $0.492\times1.82=0.896$ volt to force 1.82 amperes direct current through the winding. In order to force 1.82 amperes (effective) alternating current through this winding, 2500 volts is required. If the wire forming this coil were unwound from the iron core of the transformer, and were arranged in the form of a loop with the two sides close together, the direct voltage required to force 1.82 amperes through this wire would still be 0.896 volt. If an alternating voltage were applied to this loop, approximately 0.896 volt, instead of 2500 volts, would be required to force 1.82 amperes through the wire. This marked difference is evidently due to the arrangement of the wire. When the wire is wound into a coil and placed on the iron transformer core, an intense magnetic field is produced when current flows through the coil. This causes a very high counter e.m.f. of self-induction, which, in this case, is approximately 2500 volts. When the wire is formed into a loop, with the two sides close together, there is only a very weak magnetic field linking with the loop, and therefore, the counter e.m.f. of self-induction is practically negligible. In this case, only the voltage to overcome the resistance of the wire, 0.896 volt, is required to force the current through the wire.

A circuit like this transformer coil, in which an appreciable voltage is induced when the current changes, is said to have inductance or self-induction; a circuit like the loop is said to be practically non-inductive.

We have seen in Article 60 that the e.m.f. of self-induction depends upon the rate of change of flux linking with the circuit. If a coil having N turns is so designed that each turn is cut by the same flux, then the e.m.f. induced in each turn will be one volt when the flux is changing at the rate of 10^8 lines per second. The total voltage induced will be N times the voltage of one turn or

$$e_L=N\frac{d\Phi}{dt}10^{-8}\text{ volts}, \qquad (54)$$

or

$$e = N\,(\text{rate of change of flux})\;10^{-8}\ \text{volts.}$$

Where the flux linking with the coil is produced by the current in the coil it is frequently convenient to express the induced e.m.f. in terms of the current. It was shown in Chapter VIII that the induced e.m.f. depends upon the rate of change of this current. This is expressed by the equation

$$e_L = L\frac{di}{dt}\ \text{volts,}$$

or

$$e_L = L\ (\text{rate of change of current}). \quad . \quad . \quad . \quad . \quad . \qquad (55)$$

The quantity L is a constant called the coefficient of self-induction The unit of self-induction is called the henry. Where the coil does not have an iron core, the flux is proportional to the current and in many cases in practice, even with an iron core, the magnetic circuit is not saturated so that the flux is approximately proportional to the current. In such cases, the coefficient of self-induction is given by the equation

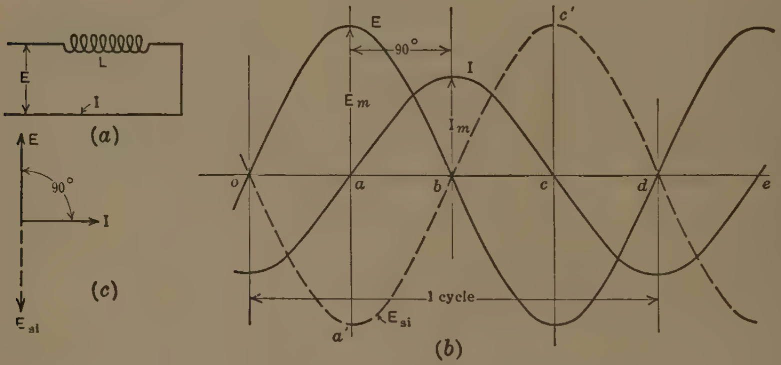

FIG. 198.—Alternating Current in an Inductive Circuit.

$$L = \frac{N\Phi}{I}10^{-8}\ \text{henries.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (56)$$

It should be noted that, where the flux is proportional to the current, the coefficient of self-induction L is as much a constant of the circuit as is its resistance. In practice, it is more common to speak of the inductance of a circuit than of its coefficient of self-induction.

223. A.C. in an Inductive Circuit.—If an alternating current I is passed through a coil (Fig. 198 a) which has negligible resistance the only opposition to the flow of current will be caused by the counter e.m.f. of self-induction, produced by the flux linking with the coil. Referring to Fig. 198 *b*, at the points *b* and *d*, the current is at the maximum value and is not changing; hence, the self-induced e.m.f. is zero. At points *a*, *c*, and *e* the current curve has the greatest slope or the rate of change of current is the highest; therefore, the induced e.m.f. is a maximum. During the interval $a-b$ the current is flowing in a positive direction and is increasing. According to Lenz's Law the induced e.m.f. must be in such a direction as to oppose this change of current and to tend to send current in the opposite or negative direction. A voltage curve drawn above the horizontal axis represents a positive voltage, that is, one which would send current through the circuit in a positive direction. Therefore, the e.m.f. of self induction at the point *a* must be in a negative direction and is represented by point a' on the dotted curve. Between points *b* and *c* the current is decreasing, and, therefore, the induced e.m.f. must tend to increase the current; that is, it must be positive. This locates the point c' and the section of the dotted curve between *b* and *c*. Between points *c* and *d* the current is increasing in a negative direction, and the induced e.m.f. must still be positive. If the inductance L is constant and the current curve is a sine wave the e.m.f. of self-induction will also be a sine wave and can be represented by the dotted curve E_{si} in Fig. 198 *b*. To overcome this e.m.f. of self-induction and force the current through the coil the voltage impressed upon the coil must, at each instant, be equal and opposite to the e.m.f. of self induction. The curve E, shown in Fig. 198 *b*, will fulfill this condition. The vector diagram is shown in Fig. 198 *c*. When dealing with inductive circuits it is customary to omit the voltage vector E_{si} and to show only the current I and the impressed voltage E. It is apparent, therefore, that the current and impressed voltage for an inductive circuit of negligible resistance[2] are 90 degrees out of phase. Furthermore, the current lags behind this impressed voltage so that an inductive circuit is sometimes called a "lagging circuit." In a later article, it will be shown that when the angle of lag for an inductive circuit is less than 90 degrees the resistance of the circuit cannot be neglected.

224. Calculation of Inductive Voltage in an A.C. Circuit.—During the interval of time, $a-b$ (Fig. 198 *b*), or one-quarter of a cycle, the current changes from zero to I_m, and during this same interval of time the e.m.f. of self-induction changes from its maximum value to zero. Since the impressed voltage is at each instant equal and opposed to E_{si}, the impressed voltage will also change from a maximum to zero in the same time $a-b$. If the frequency is f cycles per second, the time during which this change takes place, represented by the distance $a-b$, is $\frac{1}{4f}$ second. Since the self-induced e.m.f. depends upon the rate at which the current changes, the average e.m.f. of self-induction during the interval $a-b$ will be determined by the average rate at which the current changes, and this is the change in current I_m, divided by the time, $\frac{1}{4f}$. According to Equation (55) the e.m.f. of self-induction is equal to L times the rate of change of current; therefore, the average voltage during the interval $a-b$ is

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[2] A circuit has negligible resistance when the fall of potential due to resistance is so small as to be negligible in comparison with other voltages in the circuit.

$$E_{\text{av.}} = L\left(I_m \div \frac{1}{4f}\right)$$

$$= 4fLI_m.$$

But

$$E_m = \frac{\pi}{2} E_{\text{av.}}$$

Therefore

$$E_m = \left(\frac{\pi}{2}\right) \times (4fLI_m)$$

$$= 2\pi fLI_m,$$

and

$$E_{\text{eff.}} = 2\pi fLI_{\text{eff.}} \quad \text{. . . . .} \quad (57)$$

Equation (57) will hereafter be written without subscripts, since it is understood that effective values are used unless otherwise stated. In Article 223 it was shown that the impressed voltage must be equal and opposite to the e.m.f. of self-induction; therefore, Equation (57) gives the numerical value of the impressed voltage required to force a current I at a frequency of f cycles through a circuit having an inductance of L henries and a negligible resistance. It is customary to represent the expression $2\pi fL$ by the single symbol X. Equation (57) then becomes

$$E = XI, \quad \text{. . . . .} \quad (58)$$

which is similar in form to the equation of a d.c. circuit, $E=RI$. The quantity X is called the *inductive reactance* of the circuit and is expressed in ohms. For a coil without iron in the magnetic circuit, where L is constant, the reactance is evidently proportional to the frequency.

An examination of Equation (57) will show that for a coil with a constant value of inductance L the induced e.m.f. increases as the current increases. Also, for a constant impressed voltage E the current which will flow will be inversely proportional to the frequency.

Example 1.—A coil having an inductance of 0.1 henry has 110 volts at 60 cycles impressed upon it. The resistance of the coil is negligible. Find the current which will flow.

$$X = 2\pi fL = 377 \times 0.1 = 37.7 \text{ ohms},$$

$$I = \frac{E}{X} = \frac{110}{37.7} = 2.92 \text{ amperes}.$$

Example 2.—If the frequency is 25 cycles and the same voltage is impressed as in Example 1, find the current

$$X = 2\pi fL = 157 \times 0.1 = 15.7 \text{ ohms},$$

$$I = \frac{E}{X} = \frac{110}{15.7} = 7.01 \text{ amperes}.$$

It is seen that, as the frequency decreases, the current must increase in order to increase the flux and maintain the same rate of cutting of the conductor by the flux.

It is difficult, in many cases, to calculate the inductance of a coil from its dimensions and the number of turns even if L is a constant. The inductance can, however, be measured by passing a known current at a definite frequency through the coil and noting the voltage at its terminals. If the current is so small as to make the resistance drop negligible, the inductance can be calculated from Equation (57).

Example 3.—When a current of 1.5 amperes is passed through a certain coil, the voltage at the terminals of the coil is found to be 250 volts. The frequency is 60 cycles. Calculate the value of the inductance L. Transposing Equation (57):

$$L = \frac{E}{2\pi fI} = \frac{250}{377 \times 1.5} = 0.442 \text{ henry}.$$

225. Power in an Inductive Circuit.—It was shown in Article 223 that in an inductive circuit having negligible resistance, the current lags 90 degrees behind the impressed voltage (Fig. 198 *c*). A wattmeter connected in the circuit (Fig. 199 *a*) will read zero, indicating that the average power input to the coil is zero. Figure 199 *b* shows the sine curves representing the current and voltage, the effective values of which are I and E as determined by the instrument readings. The power in the circuit at any instant is equal to the product of the voltage e and current i at that instant.

An examination of the curves shows that at point a, $i = o$ and

$e=E_m$; therefore, the power is zero. At point b, $e=o$ and $i=I_m$; hence, the power is again zero. Therefore, during one cycle, represented by the distance o–d, the power is zero four times. Between points a and b, both e and i positive; therefore, the power is positive. Between points b and c, the e is negative and i positive; therefore, the power is negative. By plotting the product $e \times i$ for one cycle the shaded curve P is obtained. This shows the power in the circuit at each instant. When this curve is positive, the coil is receiving energy. The power in the coil at any instant varies during the time intervals a–b and c–d, from zero to a maximum of plus P_m watts. When the power curve is negative, the coil is returning energy to the supply. During these periods, represented by o–a and b–c, the power varies from zero to a maximum of minus P_m watts. The area of each loop of the power curve represents the total amount of energy delivered or received during the time corresponding to o–a, a–b, etc. There are four of these loops in one cycle, therefore the power in a single-phase circuit is not constant, but is pulsating at twice the frequency of the supply. The average power supplied to the coil is the average of the four power loops in one cycle. For the circuit of Fig. 199, the average power is zero, since the area of the two negative loops is exactly equal to the area of the two positive loops. During the time represented by the distance a–b, current is flowing in the coil in a positive direction, and there is being stored up in the magnetic field surrounding the coil an amount of energy represented by the area of the power loop between points a and b. This loop is positive because the coil is *receiving* energy from the supply. During the interval b–c the current is still flowing in a positive direction but is decreasing. The energy stored in the field is now acting to oppose this decrease in current; in other words, the coil is *delivering* instead

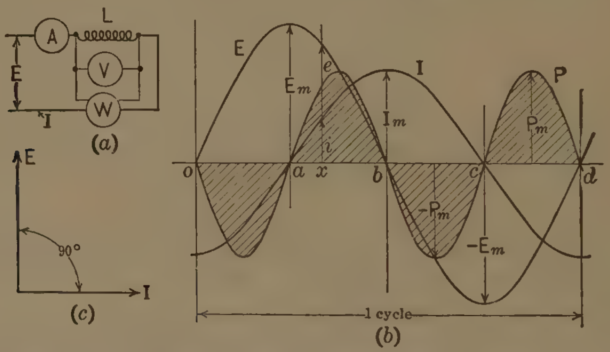


FIG. 199.—Power in an Inductive Circuit. Power curve is drawn to a smaller scale.

of receiving energy, therefore the power loop is negative. This continues for the entire period *b–c*, until at point *c* all the energy stored in the magnetic field of the coil during interval *a–b* has been returned to the electric circuit. The net delivery of energy to the coil during the half cycle is, therefore, zero and the wattmeter will read zero since it indicates the average power supplied to the coil. It follows from this that the power factor of an inductive circuit having negligible resistance is zero.[3] Since the phase angle θ, between E and I, is 90 degrees, $\cos\theta = \cos 90° = o$ and, therefore, the power factor is numerically equal to $\cos\theta$. The power equation is

$$P = EI \cos\theta = EI \cos 90° = o.$$

The power relations in an inductive circuit can be compared to the action of the flywheel of a reciprocating steam engine. During the first part of each stroke the rate at which energy is being delivered by the action of the steam is in excess of the requirements of the load, and this excess is stored in the flywheel. Later in the same stroke the rate of delivery of energy by the steam is less than the average requirements, and this deficiency is supplied by the flywheel, which slows down slightly and delivers a portion of its stored energy to the shaft. There is, therefore, a continual interchange of energy between flywheel and shaft, but the average power input to the flywheel during one revolution is zero since no energy is actually consumed by the flywheel.

226. Applications of Inductances.—When a large a.c. system is short-circuited, the current which flows is so great that difficulty is experienced in designing circuit breakers which will successfully open the circuit. Furthermore, the large short-circuit current causes severe strains in the windings of the machines and in other parts of the circuit, due to the strong magnetic field produced. If an alternator is short-circuited through a path of low resistance, the current may momentarily be as much as twenty times the full-load current. However, if the current must pass through a circuit containing an inductance, this will introduce a high counter e.m.f. which will oppose a sudden increase of current when a short-circuit occurs. By this means the current is limited to a value which can be successfully interrupted by the circuit breakers. An inductance designed for this purpose is called a **current-limiting reactor.** The type of reactor, shown in Fig. 200, consists of large stranded cable wound upon insulating supports and carefully braced to withstand the forces caused by the strong magnetic field produced when large currents are flowing. The coil has no iron core and, under normal-load conditions, produces

[3] See definition of power factor, Article 198.

a counter e.m.f. of only a few per cent of the machine e.m.f., and, therefore, does not appreciably affect the voltage of the system.

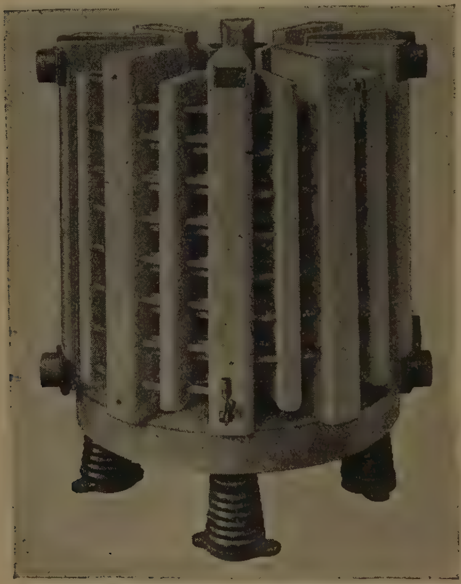


FIG. 200.—Current-limiting Reactor. *The General Electric Co.*

When a lightning discharge occurs in the vicinity of an overhead transmission line it receives an electric charge which tends to cause a current at very high potential and high frequency to flow from the line to ground through the insulation of the system. To prevent this, an auxiliary path to ground is provided as shown in Fig. 201 *a*. A gap, *G*, is inserted in this path so that the line will not be permanently grounded. Between the line and the transformer shown in the illustration an inductance, *C*, is inserted. This offers a very high counter e.m.f. when the electric charge tends to flow through the transformer windings; consequently, the voltage at *G* rises sufficiently so that there is a discharge across this gap and the transformer windings are protected. The coil *C* is called a choke coil and, in practice, consists of a few turns of

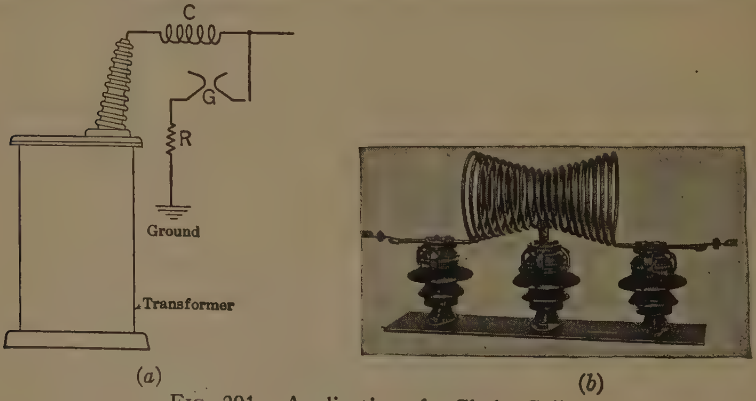


FIG. 201.—Application of a Choke Coil.

wire without an iron core, supported on suitable insulators. One of these choke coils is inserted in each line wire. The coil offers a negligible opposition to the normal-load current of the line. A choke coil suitable for a 25,000-volt line is shown in Fig. 201 *b*.

227. Resistance and Inductance in Series.—If an alternating current, I, is passed through a circuit consisting of a resistance and inductance in series (Fig. 202 *a*), the impressed voltage E will be equal to the vector sum of the voltages E_R and E_L measured, respectively, across the terminals of the resistance and reactance. (See Article 216.) The resistance voltage E_R is in phase with the current and is $E_R = IR$. (Article 220.) The reactance voltage E_L is 90 degrees ahead of the current and is $E_L = 2\pi fLI = XI$. (Article 224.) These voltages are represented by the sine curves in Fig. 202 *b*. The vector diagram is shown in Fig. 202 *c*. It is apparent that

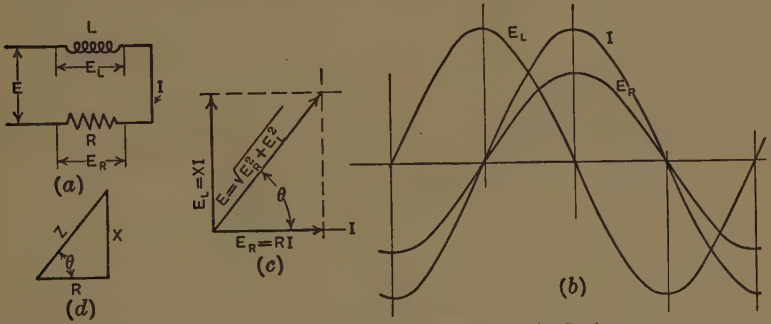


FIG. 202.—Resistance and Inductance in Series.

$$E = \sqrt{E_R^2 + E_L^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

According to Article 198, the impedance of the circuit shown in Fig. 202 *a*, is

$$Z = \frac{E}{I} \quad \text{and} \quad E = ZI.$$

By substituting the values of E, E_R, and E_L, Equation (59) becomes

$$ZI = \sqrt{(IR)^2 + (2\pi fLI)^2},$$

or

$$Z = \sqrt{(R)^2 + (2\pi fL)^2} = \sqrt{R^2 + X^2}. \quad . \quad . \quad . \quad . \quad . \quad (60)$$

The relations between Z, R, and X are shown by the right-angle triangle in Fig. 202 *d*. The angle θ is the phase angle between the current and the total voltage E.

In practice, an inductance seldom has a negligible resistance so that R and L may exist in a single piece of apparatus. The voltage relations in this case are shown in Fig. 203. The impressed voltage E can be resolved into two right-angled components. One, the active component E_R, is in phase with the current and is required to overcome the resistance of the coil. The other, the reactive component E_L, is 90 degrees ahead of the current and is required to oppose the counter e.m.f. of self-inductance of the coil. As before,

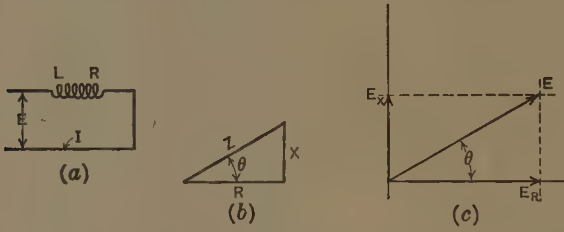


FIG. 203.—Diagram for a Coil Having Resistance and Reactance.

$$E=\sqrt{E_R{}^2+E_L{}^2} \quad \text{and} \quad Z=\sqrt{R^2+X^2}.$$

Where there are several resistances and inductances in series the voltage of each part of the circuit can be resolved into active and reactive components, which can be combined by the method described in Article 214. It can be shown, by reasoning similar to that given in the first paragraph of this article, that for any series circuit having resistance and inductance the total impedance is

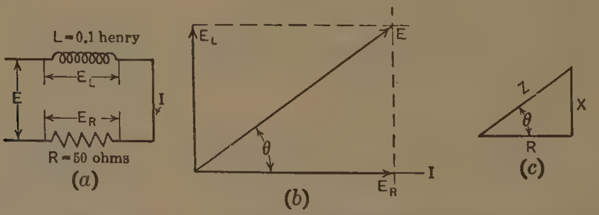


FIG. 204.—Vector Diagram for a Series Circuit with Resistance and Inductance.

$$Z=\sqrt{(R_1+R_2+R_3+\text{etc.})^2+(X_1+X_2+X_3+\text{etc.})^2}. \quad . \quad (61)$$

Example 1.—An alternating current of 5 amperes flows in a circuit consisting of a non-inductive resistance of 50 ohms in series with an inductance of 0.1 henry (Fig. 204). The frequency is 60 cycles.

(*a*) Calculate the voltage across each part of the circuit.
(*b*) What is the impressed voltage?
(*c*) What is the impedance?
(*d*) What is the phase angle between the current and the impressed voltage?

(*a*)
$$X=2\pi\times60\times0.1=377\times0.1=37.7 \text{ ohms.}$$
$$R=50 \text{ ohms.}$$
$$E_L=37.7\times5=188.5 \text{ volts.}$$
$$E_R=50\times5=250 \text{ volts.}$$

(*b*) The impressed voltage is
$$E=\sqrt{250^2+188.5^2}=313 \text{ volts.}$$

It should be noted that this is considerably less than the numerical sum of E_R and E_L which is 438.5 volts.

(*c*) The impedance is
$$Z=\frac{E}{I}=\frac{313}{5}=62.6 \text{ ohms.}$$

The impedance can also be found from Equation (60):
$$Z=\sqrt{50^2\times37.7^2}=62.6 \text{ ohms.}$$

(*d*)
$$\text{Cos } \theta=\frac{250}{313}=0.798.$$

Also
$$\text{Cos } \theta=\frac{50}{62.6}=0.798.$$
$$\theta=37°.$$

Example 2.—A certain transformer, when connected to a 2500-volt, 25-cycle supply takes 1.82 amperes in the primary winding, when the secondary is without load. The power input is 1520 watts. (See Example 1, Article 198.)

(*a*) What is the reactance of the primary winding?
(*b*) What is the inductance of the primary winding?

(*a*)
$$Z=\frac{2500}{1.82}=1375 \text{ ohms.}$$

By Article 219, the effective resistance is
$$R=\frac{1520}{1.82^2}=459 \text{ ohms.}$$

Then,
$$X=\sqrt{Z^2-R^2}=\sqrt{1375^2-459^2}$$
$$=1294 \text{ ohms.}$$

Since $x=2\pi fL$,
$$L=\frac{1294}{2\pi\times25}=\frac{1294}{157}=8.25 \text{ henries inductance.}$$

Example. 3—A series circuit consists of four parts having respectively the following constants:

Part A, resistance = 10 ohms, inductance = 0.1 henry
Part B, resistance = 0 ohms, inductance = 0.05 henry
Part C, resistance = 20 ohms, inductance = 0 henry
Part D, resistance = 2 ohms, inductance = 0.15 henry

(*a*) Calculate the total impedance of the circuit at 60 cycles.

(*b*) What voltage must be impressed on the circuit to cause 5 amperes to flow?

According to Equation (60), Article 227, the impedance is

$$Z=\sqrt{(10+20+2)^2+(\,(0.1+0.05+0.15)\,377)^2}=\sqrt{32^2+113.1^2}$$
$$=117.5 \text{ ohms.}$$

The total voltage required for the circuit is

$$E=5\times117.5=587.5 \text{ volts.}$$

The total voltage might have been found by calculating the voltage across each part and adding these voltages vectorially, as was done in Example 1. It is apparent, however, that this would be more laborious.

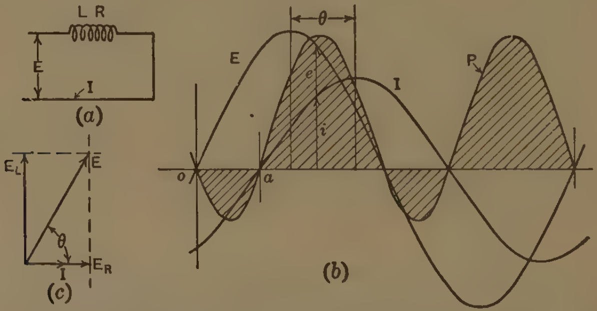


FIG. 205.—Power in a Circuit with Resistance and Inductance. Power curve is drawn to a smaller scale.

228. Power in a Circuit with Inductance and Resistance.—In a circuit like Fig. 205 *a* the current lags behind the impressed voltage E, as was shown in Article 227. At any instant, the rate at which power is being expended on the circuit is equal to the product of the voltage and current at that instant or $p=e\times i$. By plotting this product at different points in the cycle the power curve P is obtained. At point o, the voltage is zero and the power is therefore zero. At point a, the current is zero so the power is again zero. In fact the power is zero four times during one cycle. The average power is the difference between the areas of the positive and negative loops, and, since the positive loops are larger than the negative, the average has a definite positive value. The total voltage E is composed of two components, the active component $E_R = RI$, and the reactive component $E_L = 2\pi fLI = XI$. The average power expended in the resistance is $P_R = E_R I$ according to Article 221. The average power expended in the inductance is $P_L = E_L I \cos 90° = 0$ according to Article 225. Therefore the total average power is

$$P = P_R = E_R I. \qquad (62)$$

If θ is the phase angle between the current and the impressed voltage E, then $E_R = E \cos \theta$ and, therefore, the power is

$$P = EI \cos \theta. \qquad (63)$$

With sine waves, $\cos \theta$ is equal numerically to the power factor of the circuit. The same relation holds with a single coil having inductance and resistance. From the foregoing it can be seen that the power is equal to the product of the current and the *active component* ($E \cos \theta$) of the total voltage. For this reason the active component is sometimes called the power component of the impressed voltage.

It should be noted that all the power supplied to the circuit is consumed in the resistance. The inductance does not consume power, since the energy stored in the inductance when the current is increasing, is returned to the supply when the current is decreasing.

The product $E_L I = EI \sin \theta$ (Fig. 205) is called the reactive volt-amperes. Since the apparent power $= EI$, it is evident that apparent power $= \sqrt{(\text{power})^2 + (\text{reactive volt-amperes})^2}$.

Example 1.—Referring to the circuit of Example 1, Article 227,

(*a*) Calculate the power.
(*b*) Calculate the apparent power and the reactive volt-amperes.

(*a*)
$$E_R = 5 \times 50 = 250 \text{ volts.}$$
$$P = E_R I = 5 \times 250 = 1250 \text{ watts.}$$

The power, according to Equation (63) is

$$P = EI \cos \theta = 313 \times 5 \times 0.798 = 1250 \text{ watts.}$$

(*b*) The apparent power is

$$EI = 313 \times 5 = 1565 \text{ volt-amperes.}$$

The reactive volt-amperes is

$$E_L I = 188.5 \times 5 = 942.5 \text{ volt-amperes.}$$

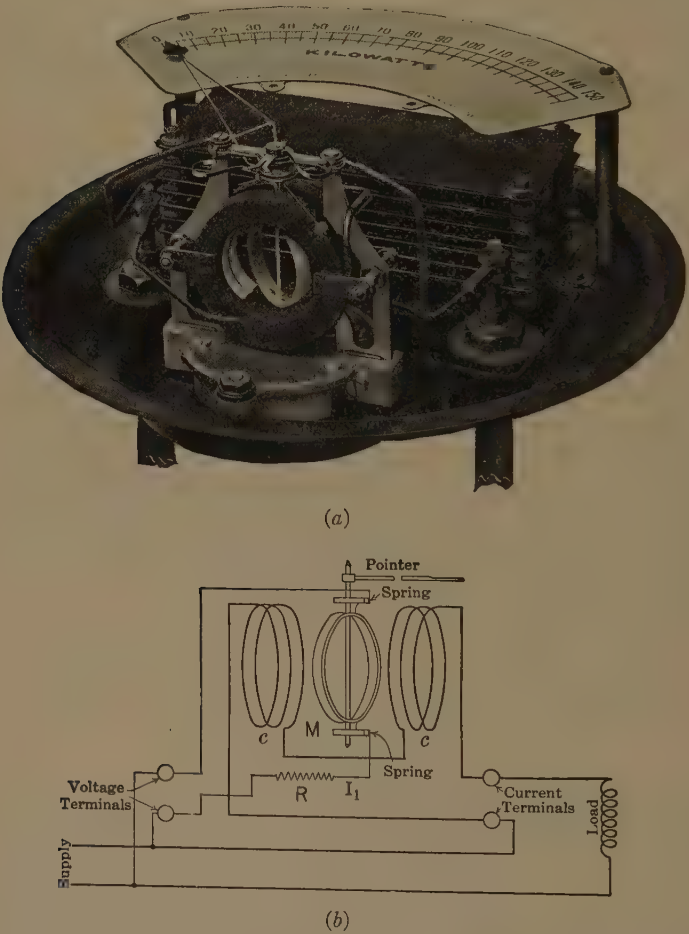
Or it is

$$EI \sin \theta = 313 \times 5 \times 0.602 = 942.5 \text{ volt-amperes.}$$

Also the apparent power is

$$\sqrt{1250^2 + 942.5^2} = 1565 \text{ volt-amperes.}$$

229. The Wattmeter and Power Measurement.—It has been shown in a previous article that the power in an a.c. circuit is $P=EI\cos\theta$. In general, it is not convenient to measure separately the three quantities involved in this equation, therefore a single instrument, called a wattmeter, is used for power measurement. One type, which is in common use, is similar in construction to the electrodynamometer instrument described in Article 210. The stationary coils c, c (Fig. 206 b) are connected in series with the load to be measured and, therefore, carry the load current, I. The moving coil M



(a)

(b)

FIG. 206.—Electrodynamometer Type of Wattmeter. *The Weston Electrical Instrument Co.*

has a non-inductive resistance in series with it, so that the current I_1, which flows through this coil, is limited to a small amount. Since the moving coil circuit is connected to the supply terminals the current I_1 is proportional to the voltage E and is in phase with it because the circuit is non-inductive. The force tending to turn coil M, at any instant, is proportional to the product of the instantaneous current in the fixed and movable coils and, therefore, is proportional to $e \times i$. The instantaneous value of the power supplied to the load is $p = e \times i$ according to Article 228, and, therefore, the force on the moving coil is proportional to the instantaneous value of the power. The force on the coil is balanced by control springs, as in the voltmeter (Article 210), so that the deflection of the pointer is proportional to the average force on the coil. Therefore, the wattmeter reads the average power which, according to Equation (63), is $P = EI \cos \theta$. When a wattmeter is connected into a circuit carrying a large current a transformer is used to reduce the current in the coils c, c. For high voltages a voltage transformer is used to reduce the voltage applied to the instrument.

230. Power Factor.—The power factor of an alternating current circuit is defined as the ratio $\frac{\text{true power}}{\text{apparent power}}$ or power factor $= \frac{P}{EI}$

For sine waves it was shown (Equation 63) that $P = EI \cos \theta$ and, therefore, $\cos \theta$ is numerically equal to the power factor. With a non-inductive load, $\theta = 0$, E and I are in phase and the power factor is one or unity. As the angle θ increases due to a lead or a lag of I with respect to E, the power factor decreases and becomes zero when $\theta = 90°$. Therefore, the power factor may have any value between zero and one. Since the numerical value of the power factor is the same whether the current lags or leads θ degrees, it is always necessary to state if the current is leading or lagging. In general, the term "lagging power factor" means that the *current lags behind the voltage.*

The power factor may be measured by an instrument called a power factor meter but it is frequently calculated from readings of the voltage, current, and power.

Example 1.—The transformer of Example 1, Article 198, carries a current of 1.82 amperes at 2500 volts, with a power input of 1520 watts. The power factor is

$$\text{Cos } \theta = \frac{1520}{2500 \times 1.82} = 0.334.$$

The power factor can also be found from the impedance triangle.

Example 2.—A circuit has a resistance of 10 ohms and a reactance of 23 ohms. What would be the power factor of the circuit? The impedance triangle is shown in Fig. 207.

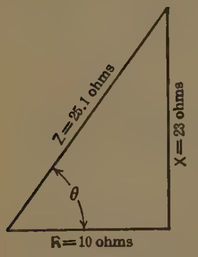


FIG. 207.—Impedance Triangle for a Circuit with Resistance and Reactance.

$$\text{Tan } \theta = \frac{23}{10} = 2.3.$$

$$\theta = 66.5°.$$

$$\text{Power factor} = \cos \theta = 0.399.$$

For a circuit such as that in Fig. 205 it is apparent that the power factor is

$$\text{Cos } \theta = \frac{E_R}{E}.$$

This can be applied to any circuit where the active component (E_R) of the voltage is known. The methods described above apply only to single-phase circuits and require modification for polyphase circuits. (See Chapter XXVIII.)

231. Detrimental Effects of Low Power Factors.—The power factor of an a.c. system is maintained as near unity as is practicable because a low power factor, particularly if it is lagging, has a detrimental effect upon the operation of the system. Since power equals $EI \cos \theta$, it is apparent that for a given amount of power, the current is larger for a low power factor than for unity power factor. Therefore, with low power factors, larger generators and transmission lines are required, and the cost of a system to supply a given amount of load is greater. A load, having a low lagging power factor, causes an excessive voltage drop in transmission lines and is likely to produce poor voltage regulation in the system. With low power factors it is difficult for the alternators to maintain the required voltage.

Incandescent lamps have a unity power factor. An induction motor load gives a station power factor of from 70 to 80 per cent lagging. Single-phase railway systems have a lagging power factor of about 60 to 70 per cent. Long-distance, high-voltage transmission lines, when lightly loaded have a leading power factor at the generating end, due to the condenser effect of the line.[4] Methods of improving the power factor of a system will be discussed later. (Article 322.)

232. Single-phase Transmission Line.—The wires of an overhead line have inductance as well as resistance, due to the field produced by the line current, so that such a line is an example of a series circuit containing resistance and inductance.

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[4] This is discussed more in detail in Article 235.

Example 1.—A single-phase line, 5 miles long is composed of No. 0 stranded wires having a resistance of 0.100 ohm per 1000 feet and a reactance at 25 cycles of 0.0431 ohm per 1000 feet.

(*a*) If the line supplies a lighting load (unity power factor) of 100 kw. at 2200 volts, calculate the voltage required at the power-house end of the line.

(*b*) If the line supplies a load of 100 kv.-a. at 2200 volts, at a power factor of 0.8 *lagging*, calculate the voltage required at the power-house end of the line.

(*c*) If the line supplies a load of 100 kv.-a. at 2200 volts, at a power factor of 0.8 *leading*, calculate the voltage required at the power-house end of the line.

The equivalent circuit is represented by Fig. 208 *a*. Each line wire which is 5 miles long is equivalent to an inductive circuit having the following constants:

$$R_1 = 0.1 \times 5 \times 5.28 = 2.64 \text{ ohms.}$$

$$X_1 = 0.0431 \times 5 \times 5.28 = 1.14 \text{ ohms.}$$

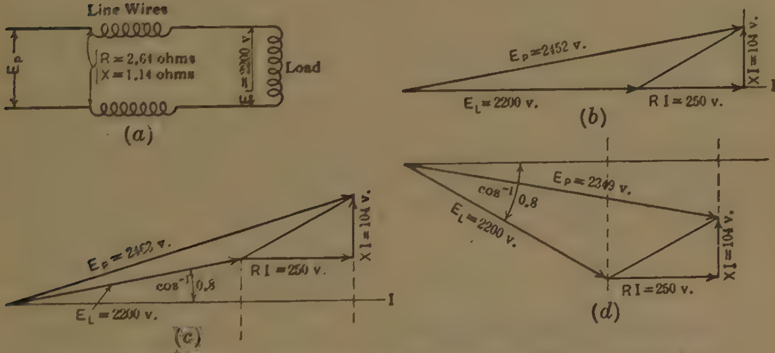

FIG. 208.—Vector Diagrams for a Single-phase Transmission Line.

The total resistance and reactance for the line is

$$R = R_1 + R_2 = 2.64 + 2.64 = 5.28 \text{ ohms,}$$

and

$$X = X_1 + X_2 = 1.14 + 1.14 = 2.28 \text{ ohms.}$$

(*a*) The vector diagram for a unity power factor load is shown in Fig. 208 *b*. The current is

$$\frac{100,000}{2200} = 45.5 \text{ amperes.}$$

Then

$$RI = 5.28 \times 45.5 = 250 \text{ volts}$$

and

$$XI = 2.28 \times 45.5 = 104 \text{ volts.}$$

The power-house voltage is

$$E=\sqrt{(2200+250)^2+104^2}=2452 \text{ volts.}$$

(*b*) For a lagging load of 100 kv.-a., the current is 45.5 amperes as in (*a*). Therefore, the voltage lost in the line is the same as before. The vector diagram is shown in Fig. 208 *c*, since the current lags behind the load voltage by an angle whose cosine is 0.80. The components of the load voltage are

$$H=2200\times0.8=1760 \text{ volts.}$$

$$V=2200\times0.6=1320 \text{ volts.}$$

The total voltage is

$$E=\sqrt{(1760+250)^2+(1320+104)^2}=2463 \text{ volts.}$$

(*c*) For a leading load of 100 kv.-a., the current is 45.5 amperes and the line loss is the same as for (*a*). The vector diagram is shown in Fig. 208 *d*. The components of the load voltage are

$$H=2200\times0.8=1760 \text{ volts.}$$

$$V=-2200\times0.6=-1320 \text{ volts.}$$

The total components are

$$H=1760+250=2010 \text{ volts.}$$

$$V=-1320+104=-1216 \text{ volts.}$$

The V component of the load voltage is negative because it points downward. The V component of the line voltage loss is positive because it points upward. The total V component is, therefore, the difference of these two components and is negative. Therefore

$$E=\sqrt{(2010)^2+(-1226)^2}=2349 \text{ volts.}$$

233. A.C. in a Capacity Circuit.—We have seen in Article 67 that a circuit having appreciable electrostatic capacity has the property of storing electricity, when a voltage is applied to the circuit, and of discharging this electricity, when the voltage is removed. If an alternating potential is applied to such a circuit, the condenser (Fig. 209) will be alternately charged and discharged, and an alternating current of the same frequency will flow in the circuit. Thus in Fig. 209 *a* the two ammeters A_1 and A_2 would indicate a steady value of current. This is called the charging current of the condenser. Since the charge of a condenser depends upon its capacity and the voltage applied, the charging current will be proportional to the voltage of the supply and the capacity of the condenser. When a condenser is connected to an a.c. supply, it becomes fully charged at each half cycle; therefore, if the frequency is increased, the time for flow of the charge must be decreased and, therefore, the charge must flow at a higher rate; that is, the current would be greater with the increased frequency. Hence, for a definite a.c. voltage, which would determine the maximum charge, the charging current is directly proportional to the electrostatic capacity and to the frequency.

234. Phase Position of Current in a Capacity Circuit.—Suppose that a sine wave of voltage, E, is applied to a condenser (Fig. 209). The charge on the plates of the condenser, at any instant, is proportional to the voltage e at that instant since, according to Equation (21), $Q = Ce$. The charge can, therefore, be represented by the curve Q (Fig. 209), and the point of maximum charge will coincide with the point of maximum voltage. Since the voltage is alternating, the condenser is charged alternately in opposite directions, and an alternating charging current will flow. Between the points o and b the charge on the plate a may be called positive and between points b and d, negative. At points a and c the voltage is not changing; therefore, the current flow is zero, although the charge on the condenser is a maximum. During the time from o to a the condenser is being charged in a positive direction and current is flowing towards plate 1 (Fig. 209 *d*) at a gradually decreasing rate until point a is reached, when the current becomes zero. This gives the portion of the current curve between o and a. Between points a and b the voltage is decreasing; therefore, the charge on the condenser must decrease or the condenser must discharge. To do this a current must flow in a reversed direction as in Fig. 209 *e*. Hence, the current flow between a and b must be represented by a curve below the line, as in Fig. 209 *c*. Between points b and d the condenser charges in the negative direction and then discharges so that the current curve is the exact reverse of that

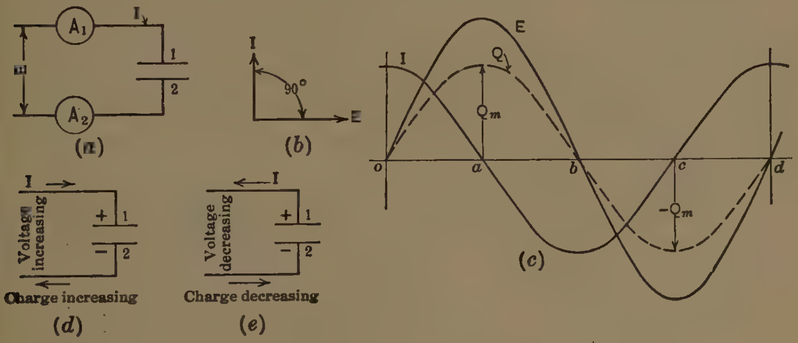

FIG. 209.—Alternating Current in a Capacity Circuit.

between *o* and *b*. An examination of the curves (Fig. 209 *c*) shows that the charging current of a condenser leads the voltage at the condenser terminals by an angle of 90 degrees. Therefore, a condenser is said to have a leading power factor.

235. Examples of Charging Current in Capacity Circuits.—In practice, the charging current caused by the electrostatic capacity of the circuit is frequently of considerable importance, especially in the case of long-distance overhead lines which are operated at high voltages, or in extensive underground cable systems. For example, a certain three-phase, 240-mile line consists of aluminum-steel cable 0.95 inch in diameter spaced 17.5 feet apart. When operated at 150,000 volts, 60 cycles, with no load on the line, the charging current due to the electrostatic capacity of the line wires is 81 amperes; hence, it requires a generator capacity of 21,000 kv.-a. to supply this charging current. For power transmission, multiple conductor cables are very commonly employed, and these have relatively much greater charging current than overhead lines of the same length and voltage because the conductors are nearer each other and also because the dielectric of the cables has a specific inductive capacity [5] several times as great as air.

The capacity current of an a.c. system also has an important bearing upon the danger of shock from touching a conductor. Even if an a.c. system were perfectly insulated from the ground, which, of course, in practice is never actually the case, it would still be extremely dangerous to touch any of the high-voltage conductors when one was in contact with any grounded conductor. This is illustrated in Fig. 210, which shows a single-phase high-voltage line. The condensers *a* and *b* represent the electrostatic capacity between wires 1 and 2 and the ground. Normally the charging current of the line flows through this capacity. If a contact is made between a point *c* on one of the lines and the ground *d*, a current will flow between 2 and 1, by way of *c*, *d*, and condenser *a*. In a system of considerable extent, for example the 2200-volt circuits supplying a city, the amount of current which would flow through a person who completed the circuit from *c* to *d* would be sufficient to give a very severe and probably fatal shock. The voltage between *c* and *d* would depend upon the resistance of the portion of the circuit between *c* and *d*, but, if this were a human body,

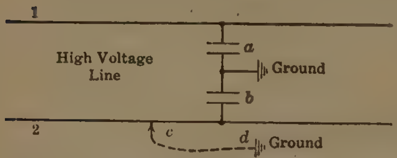

FIG. 210.—Capacity Current of a High-voltage Line.

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[5] See Article 68 for definition and table.

the resistance would be so high that the person would receive approximately one-half the voltage between wires 1 and 2. For a three-phase system, the corresponding voltage would be about six-tenths of the voltage between wires.

236. Voltage Relations in a Capacity Circuit.—If a sine wave of voltage E, having a frequency of f cycles per second, is applied to a condenser, the charge will change from zero to a maximum value in one-quarter of a cycle or $\frac{1}{4f}$ second. See Fig. 209 *c*. During this time, the condenser receives a total charge of $Q_m = CE_m$ coulombs. But since charge = average current × time, the total charge is

$$Q_m = CE_m = I_{\text{av.}} \times \frac{1}{4f}.$$

Therefore the average charging current is

$$I_{\text{av.}} = 4fCE_m \text{ amperes. } \quad \ldots\ldots\ldots \quad (64)$$

Since the quantities are sine waves,

$$I_m = I_{\text{av.}} \times \frac{\pi}{2}.$$

Substituting the value of $I_{\text{av.}}$ from Equation (64) gives

$$I_m = 2\pi fCE_m.$$

Therefore

$$I_{\text{eff.}} = 2\pi fCE_{\text{eff.}},$$

or dropping the subscripts and transposing,

$$E = \frac{1}{2\pi fC} I. \quad \ldots\ldots\ldots\ldots \quad (65)$$

For an inductive circuit,

$$E = 2\pi fLI = X_L I \quad \text{and} \quad X_L = 2\pi fL,$$

called the inductive reactance, is expressed in ohms. For capacity circuits

$$E = X_C I,$$

where $X_C = 1 \div 2\pi fC$ is called the capacity reactance or capacitance and is also expressed in ohms. The capacity C in Equation (65) is expressed in farads. (See Article 68.)

Example 1.—A condenser having a capacity of 50 micro-farads has 500 volts at 60 cycles impressed upon it. Calculate the charging current

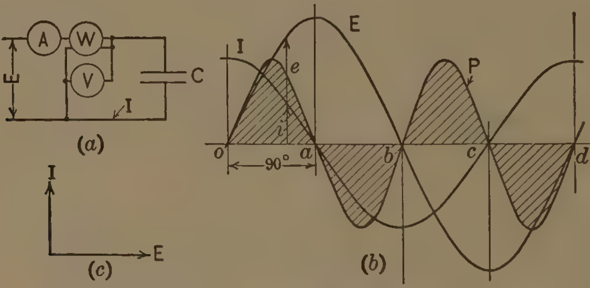
$$I = 2\pi \times 60 \times \frac{50}{10^6} \times 500 = 9.43 \text{ amperes.}$$

Example 2.—If the frequency for the condenser in Example 1 is 25 cycles, calculate the current

$$I = 2\pi \times 25 \times \frac{50}{10^6} \times 500 = 3.92 \text{ amperes.}$$

It will be seen that as the frequency decreases, with constant voltage impressed on the condenser, the charging current decreases.

In the above examples, leakage currents and losses in the condenser have been neglected. In practice, these are generally negligibly small. Both leakage currents through the dielectric and losses due to dielectric hysteresis produce a small component of current which is *in phase* with the impressed voltage, while the true charging current, as calculated by means of Equation (65), is 90 degrees *ahead* of the impressed voltage as shown in Article 234. Where losses occur in a condenser, the result is the same as if a resistance were introduced into the circuit. This is discussed in Article 239. In the problems which follow, the condensers are assumed to have negligible losses so that the current leads the voltage by 90 degrees.



Wattmeter reads zero, Ammeter reads I amperes
Voltmeter reads E volts

FIG. 211.—Power in a Capacity Circuit. Power curve is drawn to a smaller scale.

237. Power in a Capacity Circuit.—If an alternating voltage E is applied to a circuit consisting of a condenser with negligible losses, the entire voltage E will be used to charge the condenser and the current I which will flow will lead the voltage E by 90 degrees. A wattmeter connected in the circuit (Fig. 211 *a*) will read zero, indicating that the *average* power input to the condenser is zero. In Fig. 211 *b* is shown the sine curves representing the current and voltage, the effective values of which are I and E, as determined by the instrument readings. The power in the circuit at any instant is equal to the product $e \times i$ at that instant. At point *o* the voltage is zero and the current is a maximum; similarly, at point *a* the current is zero and the voltage is a maximum. Hence, at each of these points, the power is zero. Similarly, the power is zero at points *b*, *c*, and *d*. Between points *o* and *a* the product $e \times i$ is positive, while, between *a* and *b*, it is negative since *e* is positive and *i* is negative. By similar reasoning, between *b* and *c* the product is positive and between *c* and *d*, negative. Curve *P* (Fig. 211 *b*) is obtained by plotting the values of $e \times i$. The ordinates of this curve represent the value of the power at each instant during a cycle. The *average* value of the power loop between *o* and *a* represents the average power supplied to the condenser during the first quarter of a cycle when energy is being taken from the supply to charge the condenser. The area of this power loop represents the total amount of energy stored in the condenser when it is fully charged. During the next quarter of the cycle (between points *a* and *b* on the curve) the power is negative; that is, the condenser is discharging energy into the supply. The *average* value of the ordinate of this part of the curve is the average power during this part of the cycle, and the area of the power loop between points *a* and *b* represents the total energy delivered to the supply by the condenser. Since the condenser is assumed to have no losses, the area of the positive power loop (*o* to *a*) is equal to the area of the negative power loop (*a* to *b*). That is, the condenser receives a certain amount of energy from the supply during a quarter of a cycle and returns to the supply *exactly the same* amount of energy during the succeeding quarter of a cycle. Between points *b* to *c* and *c* to *d* the same process is repeated so that, during a complete cycle, the condenser is charged and discharged twice. Since the negative power loops are equal to the positive power loops, the *average* power for the entire cycle is zero. Therefore, the wattmeter, which indicates the average power, will read zero. This can also be expressed by the equation

$$P = EI \cos \theta = EI \cos 90° = 0, \quad . \quad . \quad . \quad . \quad . \quad (66)$$

since, in a condenser having no losses, the current leads the voltage by 90 degrees. The power factor is therefore zero.

238. Resistance and Electrostatic Capacity in Series.—Assume that a voltage E is required to force a current I through a circuit consisting of a resistance and a condenser in series (Fig. 212 *a*). The voltage E consists of two components. One, $E_R = IR$, is required to overcome the resistance R, and the other, $E_C = I \div 2\pi f C = X_C I$, is required to charge the condenser. The voltage E_R is in phase with, and the voltage E_C is 90 degrees behind, the current. The relations are shown by the sine curves and vector diagrams of Fig. 212 *b* and *c*. Evidently, $E=\sqrt{E_R^2+E_C^2}$ and the impedance is

$$Z=\frac{E}{I}=\frac{\sqrt{E_R^2+E_C^2}}{I}$$

$$=\sqrt{R^2+X_C^2}. \quad \dots \dots \quad (67)$$

A comparison of Fig. 212 with the relations for resistance and inductance in series, as given in Fig. 202, will show that they are similar, the principal difference being that the current in an inductive circuit *lags* θ degrees behind the impressed voltage E, while in a capacity circuit the current *leads* by θ degrees.

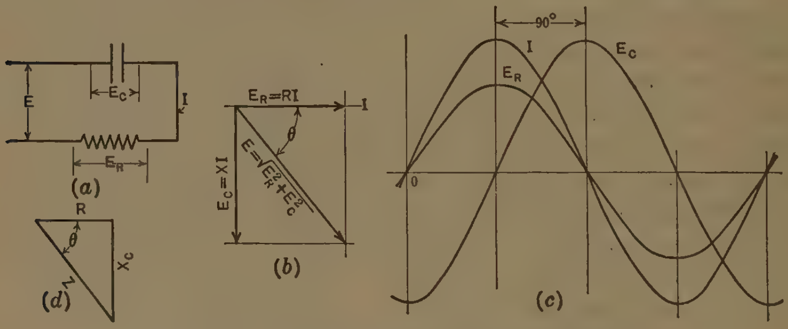


FIG. 212.—Resistance and Capacity in Series.

239. The power in a circuit with resistance and capacity can be found as follows:

For the resistance $P_r=E_RI$, by Article (221).
For the condenser $P_c=E_CI\cos 90°=0$, by Article 237.
The total power is therefore $P=E_RI=I^2R=EI\cos\theta$,

and $\cos\theta$ is the power factor for the entire circuit. Since the current leads the resultant voltage E, the circuit is said to have a *leading* power factor. The condenser does not consume any energy, but the entire power input is lost in the resistance portion of the circuit.

In the discussion of condensers it has been assumed that they have no losses and, therefore, the voltage at the terminals of the condenser will be 90 degrees behind the current in the condenser, according to Article 234. The phase angle for commercial condensers is, however, less than 90 degrees because there is always a loss in the dielectric. In a well-made condenser this loss is small so that the power factor is nearly zero. A condenser with losses can be treated as if it were a perfect condenser in series with a resistance R. The value of this resistance is such that $R = P \div I^2$ where P is the power loss in the condenser and I is the current.

240. Resistance, Inductance, and Electrostatic Capacity in Series.—Consider a series circuit (Fig. 213 *a*) having a non-inductive resistance R, in series with a condenser, and an inductance of negligible resistance, through which a current of I amperes is flowing. The voltages across the parts of the circuit are

For the resistance (1), $E_R = RI$.

For the inductance (2), $E_L = 2\pi f L I = X_L I$.

For the condenser (3), $E_C = \dfrac{I}{2\pi f C} = X_C I$.

The voltage E_R is in phase with the current I, the voltage E_L leads by 90 degrees and E_C lags by 90 degrees as shown in the vector diagram and the sine curves (Fig. 213 *b* and *c*). The voltage between the terminals *m* and *n* (Fig. 213 *a*) is, at any instant, the vector sum of these voltages. Hence

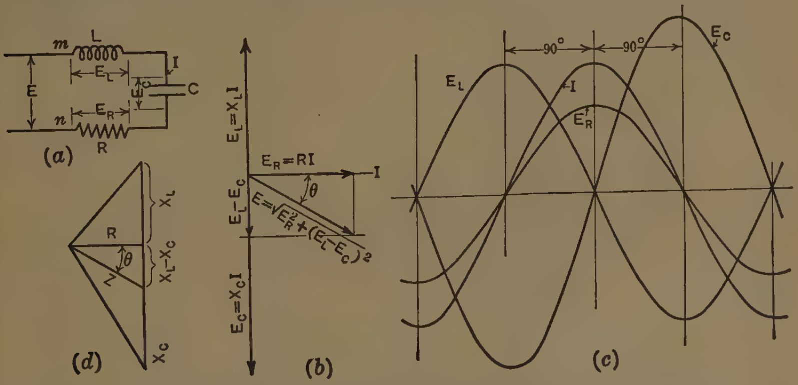


FIG. 213.—Resistance, Inductance and Capacity in Series.

$$E = \sqrt{E_R^2 + (E_L - E_C)^2}. \quad \ldots\ldots \quad (68)$$

By definition the total impedance is

$$Z = \frac{E}{I} \quad \text{or} \quad E = ZI.$$

Substituting the values of E_R, E_L, and E_C in Equation (68), the impedance is found to be

$$Z = \frac{E}{I} = \sqrt{\frac{E_R^2 + (E_L - E_C)^2}{I}} = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

$$= \sqrt{R^2 + (X_L - X_C)^2}. \quad \ldots\ldots \quad (69)$$

The impedance triangle is shown in Fig. 213 *d*. The angle θ is the phase angle, and cos θ is the power factor for the entire circuit.

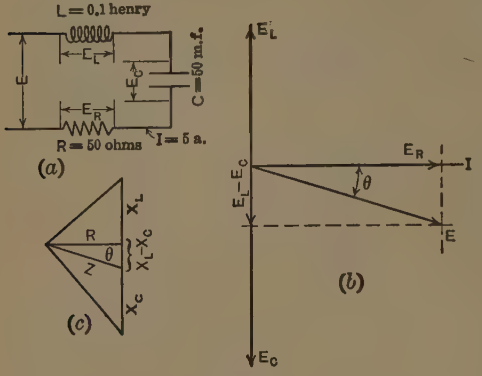


FIG. 214.—Vector Diagram for a Series Circuit with Resistance, Inductance and Capacity.

Example 1.—A current of 5 amperes, at 60 cycles, flows in a series circuit consisting of a resistance of 50 ohms, an inductance of 0.1 henry, and a condenser of 50 microfarads (Fig. 214).

(*a*) Calculate the voltage across each part of the circuit.
(*b*) Calculate the total voltage.
(*c*) Calculate the impedance.
(*d*) Calculate the power and power factor.

(*a*)

$$X_L = 2\pi \times 60 \times 0.1 = 37.7 \text{ ohms}$$

$$X_C = \frac{10^6}{2\pi \times 60 \times 50} = 53 \text{ ohms.}$$

$$E_R = 50 \times 5 = 250 \text{ volts.}$$

$$E_L = 37.7 \times 5 = 188.5 \text{ volts.}$$

$$E_C = 53 \times 5 = 265 \text{ volts.}$$

(*b*)

$$E = \sqrt{250^2 + (188.5 - 265)^2}$$

$$= \sqrt{250^2 + (-76.5)^2} = 261.5 \text{ volts.}$$

The total voltage is only slightly greater than E_R because the condenser and inductance voltages nearly neutralize each other.

(*c*) The impedance is

$$Z = \sqrt{50^2 + (37.7 - 53)^2} = 52.3 \text{ ohms.}$$

(*d*) The power in the condenser and the inductance is zero, according to Articles 225 and 237.

The total power is therefore:

$$P = E_R I = 250 \times 5 = 1250 \text{ watts.}$$

The power factor is

$$\text{Cos } \theta = \frac{250}{261.5} = 0.957.$$

The total power factor is leading (i.e., the current leads the impressed voltage), since the V component is negative. The power factor could also be calculated from the impedance triangle, Fig. 214 *c*, thus:

$$\text{Cos } \theta = \frac{R}{Z} = \frac{50}{52.3} = 0.957.$$

The power can also be calculated as follows:

$$P = EI \cos \theta = 261.5 \times 5 \times 0.957 = 1250 \text{ watts},$$

which checks with the value found previously.

If a certain device has both resistance and inductance, the same principles can be applied. In this case the voltage across this part of the circuit can be resolved into two components, one due to the resistance of the device and the other due to the self-induction.

Example 2.—A series circuit consists of a coil of wire having an effective resistance of 15 ohms and an inductance of 0.1 henry, which is in series with a condenser of 50 microfarads capacity. Calculate the total voltage which must be impressed on this circuit to cause 5 amperes to flow. The frequency is 60 cycles (Fig. 215). According to Article 227, the voltage at the terminals of the coil would be made up of two components:

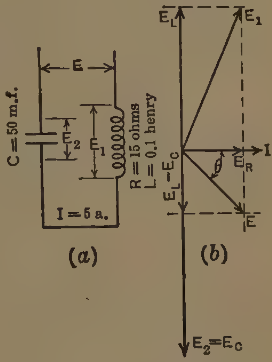


FIG. 215.—Vector Diagram for a Coil in Series with a Condenser.

The resistance component $E_R = IR = 5 \times 15 = 75$ volts.

The reactance component $E_L = 2\pi f L I = 37.7 \times 5 = 188.5$ volts.

The voltage across the condenser is

$$E_C = \frac{I}{2\pi f C} = \frac{5 \times 10^6}{2\pi \times 60 \times 50} = 265 \text{ volts.}$$

This voltage is 90° behind the current, since the condenser is assumed to have no losses. The total voltage is

$$E = \sqrt{E_R^2 + (E_L - E_C)^2} = \sqrt{75^2 + (188.5 - 265)^2} = 107.1 \text{ volts.}$$

If the power factor of each part of the circuit is known the terminal voltage of each part can be resolved into components in phase with and at right angles to the current. The total voltage impressed on the circuit can then be found by combining the components.

Example 3.—A series circuit consisting of four parts has the following characteristics: Part A has a voltage at its terminals of 110 volts with a power factor of unity; Part B, a voltage of 150 volts with a power factor of 0.8 lagging; Part C, a voltage of 200 volts with a power factor of 0.1 leading; and Part D, a voltage of 50 volts with a power factor of 0.65 lagging. Calculate the total voltage impressed on the circuit (Fig. 216).

The voltage E_A is in phase with I. Since the power factor of Part B is 0.8 lagging, $\cos \theta = 0.8$, and $\sin \theta = 0.6$. For Part C, $\cos \phi = 0.1$ and $\sin \phi = 0.995$; for Part D, $\cos \beta = 0.65$ and $\sin \beta = 0.76$.

The components are

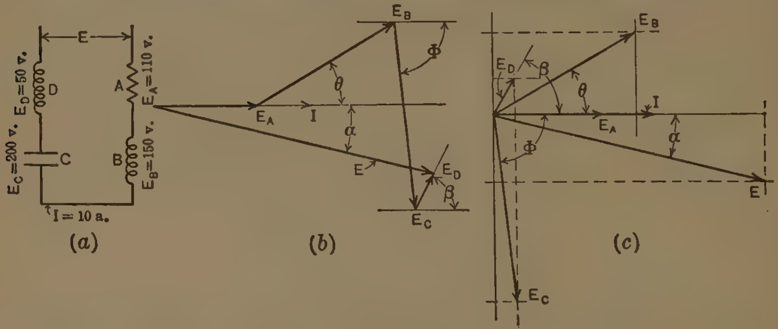
H	V
For A, $E_A \cos 0° = 110 \times 1 = 110$	$E_A \sin 0° = 110 \times 0 = 0$
For B, $E_B \cos \theta = 150 \times 0.8 = 120$	$E_B \sin \theta = 150 \times 0.6 = 90$
For C, $E_C \cos \phi = 200 \times 0.1 = 20$	$E_C \sin \phi = -200 \times 0.995 = -199$
For D, $E_D \cos \beta = 50 \times 0.65 = 33$	$E_D \sin \beta = 50 \times 0.76 = 38$
$H = 283$	$V = -71$

$$\tan \alpha = \frac{71}{283} = 0.251. \quad \alpha = 14.10°.$$

$$\cos \alpha = 0.97.$$

$$E = \frac{283}{0.97} = 292 \text{ volts.}$$

Since the V component is negative, the resultant voltage is 14.1° behind the current; that is, the entire circuit has a leading power factor of 0.97.



(a) (b) (c)

FIG. 216.—Vector Diagram for a Series Circuit.

The total power in any circuit is equal to the sum of the power for the different parts. The total apparent power is equal to the product of the current and the total voltage. It is not equal to the sum of the apparent power in the different parts.

Example 4.—Referring to the circuit of Example 3 (Fig. 216):

(a) Calculate the power for each part and the total power with a current of 10 amperes.

(b) Calculate the apparent power, for each part, and the total apparent power.

(a) The power is

For A, $P_A = 110 \times 10 = 1100$ watts.
For B, $P_B = 150 \times 10 \times 0.8 = 1200$ watts.
For C, $P_C = 200 \times 10 \times 0.1 = 200$ watts.
For D, $P_D = 50 \times 10 \times 0.65 = 325$ watts.

The total power is $1100+1200+200+325=2825$ watts.
Also, the total power is $EI\cos\alpha=292\times10\times0.97=2825$ watts.

(*b*) The apparent power is

For A, $110\times10=1100$ volt-amperes.
For B, $150\times10=1500$ volt-amperes.
For C, $200\times10=2000$ volt-amperes.
For D, $50\times10=500$ volt-amperes.

The total apparent power is

$$292\times10=2920 \text{ volt-amperes.}$$

It is apparent that the sum of the apparent power of the four parts, which is 5100, is considerably greater than the total apparent power as calculated.

241. Resistance, Inductance, and Electrostatic Capacity in Parallel.—When a voltage E is impressed upon a circuit having resistance, inductance, and electrostatic capacity in parallel (Fig. 217 *a*) the currents in the three branches are

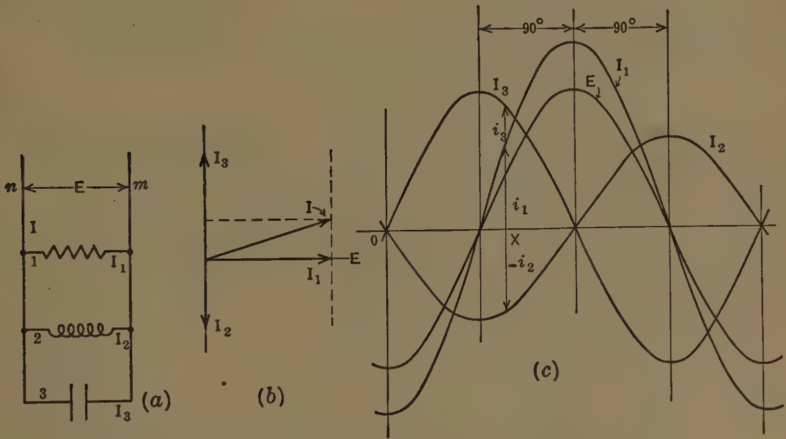


FIG. 217.—Resistance, Inductance, and Capacity in Parallel.

For the resistance (1), $I_1=\frac{E}{R}$.

For the inductance (2), $I_2=\frac{E}{2\pi fL}=\frac{E}{X_L}$.

For the condenser (3), $I_3=2\pi fCE=\frac{E}{X_C}$.

The current I_1 is in phase with E, I_2 lags 90 degrees, and I_3 leads 90 degrees, as shown by the vector diagram, and sine curves (Fig.

217 *b* and *c*). The total current taken by the circuit is, at any instant, the algebraic sum of the currents in the three branches. Thus, at point *x* in the cycle, the currents in the branches are $i_1, -i_2$, and i_3, respectively. The total current is i. It will be seen that i_2 and i_3 are opposite in sign so that $i = i_1 - i_2 + i_3$. A similar relation exists at other points in the cycle. The effective value of the current I which flows between m and n is the vector sum of I_1, I_2, and I_3, the effective values of the currents in the three branches. (Article 217.) This is shown in the vector diagram. Since I_2 and I_3 are in opposition, they tend to neutralize each other. The resultant vertical component will be above or below the horizontal, according as I_C or I_L is the greater. The total current is

$$I = \sqrt{I_1^2 + (I_3 - I_2)^2}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (70)$$

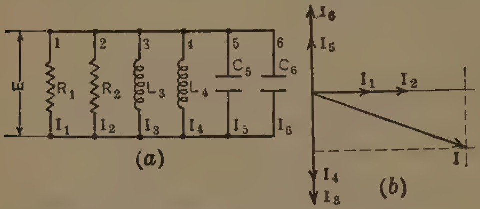


FIG. 218.—Diagram for a Parallel Circuit.

For a circuit having a larger number of branches, such as that in Fig. 218, the total current is

$$I = \sqrt{(I_1 + I_2)^2 + \left((I_5 + I_6) - (I_3 + I_4) \right)^2}. \quad . \; . \; . \; . \quad (71)$$

Example 1.—Calculate the total current for the circuit shown in Fig. 219. The frequency is 60 cycles. The branch currents are

$$I_1 = \frac{250}{50} = 5 \text{ amperes.}$$

$$I_2 = \frac{250}{2\pi \times 60 \times 0.1} = 6.63 \text{ amperes.}$$

$$I_3 = \frac{250}{2\pi \times 60 \times 0.2} = 3.32 \text{ amperes.}$$

$$I_4 = \frac{2\pi \times 60 \times 50 \times 250}{10^6} = 4.71 \text{ amperes.}$$

$$I_5 = \frac{2\pi \times 60 \times 25 \times 250}{10^6} = 2.36 \text{ amperes.}$$

$$I = \sqrt{5^2 + \left((4.71 + 2.36) - (6.63 + 3.32) \right)^2}$$

$$= \sqrt{5^2 + (7.07 - 9.95)^2} = \sqrt{5^2 + (-2.88)^2} = 5.77 \text{ amperes.}$$

The reactive component is greater than the condenser component. Therefore the total vertical component V is negative and the total or resultant current I, lags behind the impressed voltage E. Equation (71) applies to circuits where the branch currents are either in phase with or 90 degrees from the impressed voltage. When the phase angle is less than 90 degrees, the branch currents must be resolved into H and V components and combined by the principles given in Article 217.

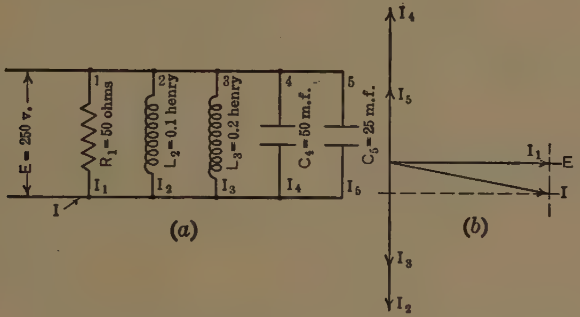


FIG. 219.—Vector Diagram for a Simple Parallel Circuit.

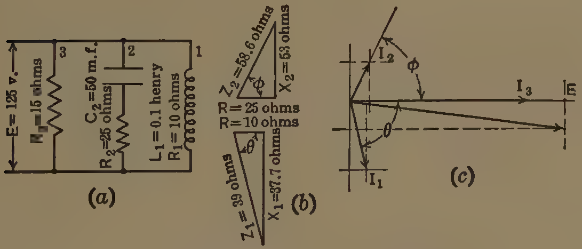


FIG. 220.—Vector Diagram for a Parallel Circuit.

Example 2.—Calculate the total current for the circuit shown in Fig. 220, assuming a frequency of 60 cycles.

The impedances for the branch circuits are

$$Z_1=\sqrt{10^2+(2\pi\times60\times0.1)^2}=\sqrt{10^2+37.7^2}=39 \text{ ohms.}$$

$$Z_2=\sqrt{25^2+\left(\frac{10^6}{2\pi\times60\times50}\right)^2}=\sqrt{25^2+53^2}=58.6 \text{ ohms.}$$

$$Z_3=15 \text{ ohms.}$$

The branch currents are

$$I_1=\frac{E}{Z_1}=\frac{125}{39}=3.21 \text{ amperes.}$$

$$I_2=\frac{E}{Z_2}=\frac{125}{58.6}=2.13 \text{ amperes.}$$

$$I_3=\frac{E}{Z_3}=\frac{125}{15}=8.33 \text{ amperes.}$$

Since branch 1 is inductive, the current I_1 lags behind E by the angle θ.

$$\cos\theta=\frac{10}{39}=0.257,\qquad \sin\theta=\frac{37.7}{39}=0.967.$$

Branch 2 has condenser reactance and therefore I_2 leads E by the angle ϕ.

$$\cos\phi=\frac{25}{58.6}=0.426,\quad \sin\phi=\frac{53}{58.6}=0.905.$$

The current in branch 3 is in phase with E.

The components are

	H	V
For I_1,	$3.21\times0.257=0.825$	$-3.21\times0.967=-3.10$
For I_2,	$2.13\times0.426=0.908$	$2.13\times0.905=1.93$
For I_3,	$=8.330$	0
	$H=10.063$	$V=-1.17$

The total current is

$$I=\sqrt{10.063^2+1.17^2}=10.13\text{ amperes.}$$

The method illustrated in Example 2 could be used if there were capacity, inductance, and resistance in the same branch. It is, in fact, a general method which can be applied to any parallel circuit. Each branch is treated like a series circuit and the branch current determined. The total current is then found by combining the horizontal and vertical components as was illustrated in Example 2.

By definition the total impedance of a parallel circuit is $Z=\frac{E}{I}$. Substituting the value of I given in Equation (71), the impedance is

$$Z=\frac{E}{\sqrt{(I_1+I_2)^2+(\,(I_5+I_6)-(I_3+I_4)\,)^2}}. \quad \text{. . . . . (72)}$$

But

$$I_1=\frac{E}{R},\quad I_3=\frac{E}{X_3},\ \text{etc.}$$

Substituting in Equation (72), the impedance becomes

$$Z=\frac{1}{\sqrt{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2+\left(\left(\frac{1}{X_5}+\frac{1}{X_6}\right)-\left(\frac{1}{X_3}+\frac{1}{X_4}\right)\right)^2}}. \quad \text{. . . (73)}$$

For parallel circuits, the reciprocal of the impedance, called the admittance, is more generally used. The admittance is

$$Y=\frac{1}{Z}=\sqrt{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2+\left(\left(\frac{1}{X_5}+\frac{1}{X_6}\right)-\left(\frac{1}{X_3}+\frac{1}{X_4}\right)\right)^2}. \quad \text{. . (74)}$$

Example 3.—Calculate the total impedance for the circuit shown in Fig. 219 (Example 1). The frequency is 60 cycles.

$$R_1=50 \text{ ohms.}$$

$$X_2=2\pi\times60\times0.1=37.7 \text{ ohms.}$$

$$X_3=2\pi\times60\times0.2=75.4 \text{ ohms.}$$

$$X_4=\frac{10^6}{2\pi\times60\times50}=53 \text{ ohms.}$$

$$X_5=\frac{10^6}{2\pi\times60\times25}=106 \text{ ohms.}$$

$$Z=\frac{1}{\sqrt{\left(\frac{1}{50}\right)^2+\left(\left(\frac{1}{53}+\frac{1}{106}\right)-\left(\frac{1}{37.7}+\frac{1}{75.4}\right)\right)^2}}=43.4 \text{ ohms.}$$

This can be checked by using the results from Example 1.

The total impedance is

$$Z=\frac{250}{5.77}=43.4 \text{ ohms.}$$

Equations (73) and (74) are correct only for circuits like that shown in Fig. 218, where the currents are in phase or in quadrature with the voltage. These equations cannot be used for circuits like the one shown in Fig. 220, where the phase angles are less than 90 degrees. The impedance of a circuit like this can be found by dividing the voltage by the total current.

Example 3.—Calculate the total impedance of the circuit shown in Fig. 220, Example 2.

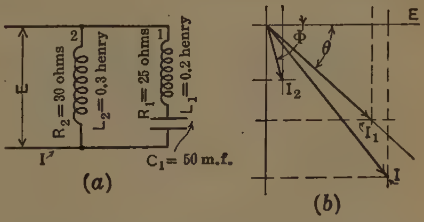


FIG. 221.—Vector Diagram for a Parallel Circuit with Inductive and Capacity Reactance in the Same Branch.

$$Z=\frac{125}{10.13}=12.33 \text{ ohms.}$$

Even if no voltage is given, a voltage can be assumed, and the impedance calculated.

Example 4.—Calculate the impedance and admittance at 60 cycles for the circuit shown in Fig. 221.

For branch 1, the impedance is

$$Z_1=\sqrt{25^2+\left((2\pi\times60\times0.2)-\left(\frac{10^6}{2\pi\times60\times50}\right)\right)^2}=\sqrt{25^2+(75.4-53)^2}$$

$$=\sqrt{25^2+22.4^2}=33.6 \text{ ohms.}$$

For branch 2, the impedance is

$$Z_2=\sqrt{30^2+(2\pi\times60\times0.3)^2}=\sqrt{30^2+113.1^2}=117 \text{ ohms.}$$

Assuming 100 volts applied, the branch currents are

$$I_1=\frac{100}{33.6}=2.98 \text{ amperes.}$$

$$I_2=\frac{100}{117}=0.855 \text{ ampere.}$$

In branch 1, the inductive reactance (75.4 ohms) is greater than the capacity reactance (53 ohms); therefore, I_1 lags behind E by θ degrees. Branch 2 is inductive and I_2 lags ϕ degrees behind E.

$$\cos\theta=\frac{25}{33.6}=0.744,\quad \sin\theta=\frac{22.4}{33.6}=0.667.$$

$$\cos\phi=\frac{30}{117}=0.256,\quad \sin\phi=\frac{113.1}{117}=0.968.$$

The components for the branch currents are

H	V
For branch 1, $2.98\times0.744=2.22$	$2.98\times0.667=1.98$
For branch 2, $0.855\times0.256=0.22$	$0.855\times0.968=0.83$
$H=2.44$	$V=2.81$

The total current is

$$I=\sqrt{2.44^2+2.81^2}=3.72 \text{ amperes.}$$

The total impedance is

$$Z=\frac{E}{I}=\frac{100}{3.72}=26.9 \text{ ohms.}$$

The admittance is

$$Y=\frac{1}{Z}=\frac{1}{26.9}=0.0372 \text{ mho.}$$

242. Power in a Parallel Circuit.—The total power for a parallel circuit is found by adding the power for the branch circuits. The power in a branch circuit can be found by treating each branch as a series circuit. The total power factor is cos θ where θ is the phase angle between the total current and the impressed voltage.

Example 1.—Calculate the total power in the circuit of Fig. 219 (Example 1, Article 241). The currents in branches 2 and 3 lag the voltage by 90°; therefore the power in these branches is zero. Similarly, I_4 and I_5 lead E by 90° and the power is therefore zero.

The power in branch 1 is

$$P=P_1=EI_1=250\times5=1250 \text{ watts;}$$

This is also the total power, since the power in the other branches is zero.

$$\cos\theta=\frac{5}{5.77}=0.867.$$

The total power is also

$$P=EI\cos\theta=250\times5.77\times0.867=1250 \text{ watts}$$

as before.

Example 2.—Calculate the power and power factor for the circuit shown in Fig. 220 (Example 2, Article 241).

The power in branch 1 is

$$P_1=EI_1\cos\theta=125\times3.21\times0.257=103 \text{ watts.}$$

For branch 2

$$P_2=EI_2\cos\phi=125\times2.13\times0.426=113.5 \text{ watts.}$$

For branch 3

$$P_3=EI_3\cos 0^\circ=125\times8.33=1040 \text{ watts.}$$

The total power is

$$P=P_1+P_2+P_3=103+113.5+1040=1256.5 \text{ watts.}$$

Check: The power factor of the entire circuit is

$$\cos\beta=\frac{10.063}{10.13}=0.992.$$

The total power is

$$P=EI\cos\beta=125\times10.13\times0.992=1256.5 \text{ watts,}$$

which checks with the above result.

The apparent power for a parallel circuit is EI volt-amperes, where I is the total current and E the voltage. The apparent power for each branch is EI_1, EI_2, etc. The sum EI_1+EI_2, etc., is not, however, equal to the total apparent power EI.

Example 3.—Referring to Fig. 220 and the calculations from Example 2, Article 241, the apparent power for the entire circuit is

$$\text{Apparent power}=EI=125\times10.13=1266 \text{ volt-amperes.}$$

The apparent power for each branch is

For branch 1, $EI_1=125\times3.21=$ 401 volt-amperes.

For branch 2, $EI_2=125\times2.13=$ 266 volt-amperes.

For branch 3, $EI_3=125\times8.33=$ 1040 volt-amperes.

It will be seen that the numerical sum of the apparent power for the three branches is considerably greater than the total apparent power which is 1266 volt-amperes.

Example 4.—A certain load, totalling 1000 kv-a., has a power factor of 0.7 lagging. Assuming that another load having a power factor of 0.6 leading is connected in parallel with this lagging load, calculate the required kv-a. of leading load to give a power factor of unity for the entire load (Fig. 222). If the total load is to have unity power factor, the reactive kv-a. for the lagging and leading loads must be equal. For the lagging load, this is $1000 \times \sin \theta = 1000 \times 0.715 = 715$ kv-a. Then the reactive component of the leading load is 715 kv-a., and since $\sin \phi = 0.8$ the total kv-a. is

$$\frac{715}{\sin \phi} = \frac{715}{0.8} = 895 \text{ kv-a.}$$

Therefore a load of 895 kv-a., having a leading power factor of 0.6, will entirely neutralize the lagging load and produce unity power factor for the entire load.

243. The Kv-a. Method of Solving Circuit Problems.—In practice, the characteristics of the circuit may be given in terms of kv-a. and power factor. When this is the case, the values can be combined by adding their *H* and *V* components as if they were voltages or currents.

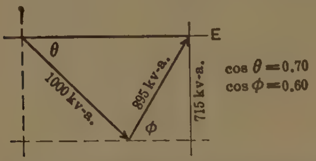


FIG. 222.—Neutralizing Inductive Reactance by Capacity Reactance.

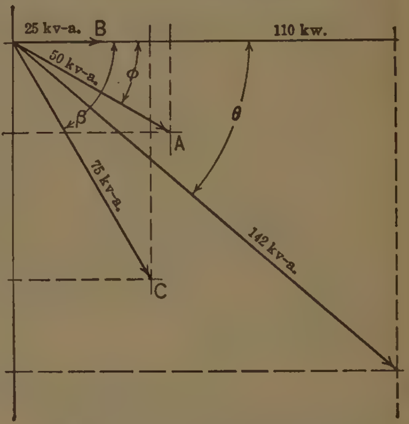


FIG. 223.—Solving Problems by the Kv-a. Method.

Example 1.—A single-phase system supplies three loads as follows:

$A = 50$ kv-a. at 0.8 power factor lagging;

$B = 25$ kv-a. at unity power factor;

$C = 75$ kv-a. at 0.6 power factor lagging.

(*a*) Calculate the total kv-a. load.
(*b*) Calculate the power factor of the entire load.
(*c*) Calculate the total kw. load.

The components are (see Fig. 223):

H	*V*
A, $50 \times 0.8 =$ 40 kw.	$50 \times 0.6 = 30$ kv-a.
B, $25 \times 1 =$ 25 kw.	$25 \times 0 = 0$ kv-a.
C, $75 \times 0.6 =$ 45 kw.	$75 \times 0.8 = 60$ kv-a.
$H = 110$ kw.	$V = 90$ kv-a.

The total kv-a. is

$$\sqrt{110^2 + 90^2} = 142 \text{ kv-a.}$$

The power factor is

$$\cos\theta=\frac{110}{142}=0.775.$$

The total power is

$$\text{kv-a.}\times\cos\theta=142\times0.775=110\text{ kw.},$$

or it is

$$(50\times0.8)+(25+1)+(75\times0.6)=110\text{ kw.}$$

244. Resonance in Series Circuits.—In Article 240 it was shown that for a series circuit containing inductance and electrostatic capacity, the condenser reactance X_c is opposite in its effect to the inductive reactance X_L, so that one tends to neutralize the other and the total reactance is X_L-X_C. (See Equation (69), Article 240.) Since

$$X_L=2\pi fL,\quad\text{and}\quad X_C=\frac{1}{2\pi fC},$$

the inductive reactance X_L *increases*, and the capacity reactance X_C *decreases*, as the frequency f is increased. By variation of the frequency of such a circuit, it is possible, therefore, to obtain a condition such that $X_L=X_C$. When this occurs, the capacity reactance X_C is exactly neutralized by the inductive reactance X_L so that the relation for current and voltage becomes

$$E=I\sqrt{R^2+(X_L-X_C)^2}=IR\quad.\;.\;.\;.\;.\quad(75)$$

and the current which flows is determined entirely by the resistance of the circuit. Whenever, in a series circuit, $X_L=X_C$ the circuit is said to be in resonance and the frequency at which this occurs is called the resonant frequency. This frequency can easily be determined. Since

$$X_L=2\pi fL\quad\text{and}\quad X_C=\frac{1}{2\pi fC}.$$

Therefore for resonance,

$$2\pi fL=\frac{1}{2\pi fC},$$

and

$$f=\frac{1}{2\pi\sqrt{LC}}.\quad.\;.\;.\;.\;.\;.\;.\;.\quad(76)$$

The phenomenon of resonance may occur when a pendulum is acted upon periodically by a small force. If the force is applied at regular intervals and with a frequency corresponding to the natural period of vibration of the pendulum, even a very small force will produce a large amplitude of vibration of the pendulum. At any other frequency, the amplitude is very much less.

Example 1.—The effect of the variation of frequency in the series circuit shown in Fig. 224 *a* is indicated by the curves Fig. 224 *b* and the tabulation below. The resistance $R=6$ ohms, $L=0.15$ henry, and $C=50$ microfarads. The impressed voltage is 125 volts.

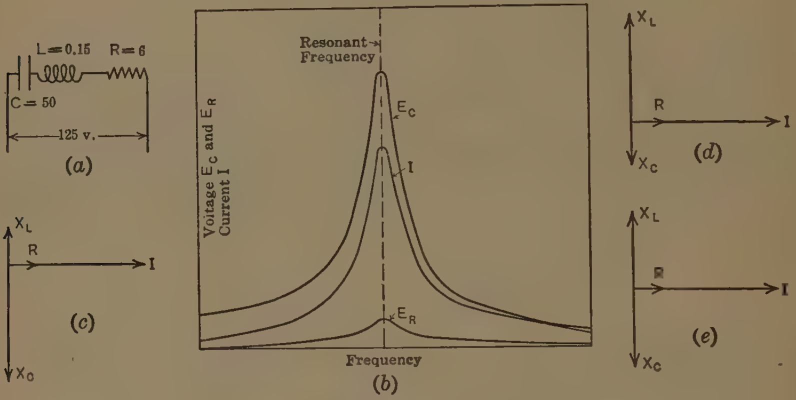


FIG. 224.—Resonance in a Series Circuit.

RESONANCE IN A SERIES CIRCUIT

Frequency f	R	$X_C=\frac{1}{2\pi fC}$	$X_L=2\pi fL$	X_L-X_C	$Z=\sqrt{R^2+(X_L-X_C)^2}$	$I=\frac{E}{Z}$	$E_R=IR$	$E_C=IX_C$	$E_L=IX_L$
20	6	159	18.8	−140.2	140.3	0.89	5.3	142	16.8
30	6	106	28.2	− 77.8	78	1.60	9.6	170	45
40	6	79.5	37.7	− 41.8	42.2	2.96	17.8	235	115
50	6	63.6	47.1	− 16.5	17.6	7.1	42.6	452	335
58.1	6	54.7	54.7	0	6	20.83	125	1140	1140
70	6	45.5	65.9	20.4	21.3	5.9	35.4	268	388
80	6	39.8	75.3	35.5	36	3.5	21	139	264
90	6	35.4	84.7	49.3	49.7	2.5	15	89	212
100	6	31.8	94.2	62.4	62.7	2.0	12	64	188

At low frequencies the current is limited principally by the capacity reactance X_C, and the current leads the impressed voltage E as shown in Fig. 224 *c*. At high frequencies the current is limited principally by the inductive reactance X_L, and the current lags behind the impressed voltage, as shown in Fig. 224 *d*. At the resonant frequency of 58.1 cycles $X_L = X_C$, the impedance is a minimum, and the current reaches a maximum of 20.83 amperes. The voltages across the condenser and inductance are then 1140 volts each, and the impressed voltage equals the drop across the resistance. At resonance the current and impressed voltage E are in phase, as is shown by the vector diagram, Fig. 224 *e*.

Circuits containing inductance and electrostatic capacity in series are very likely to have excessive voltages across some part of the circuit even if they are not operated at the resonant frequency. This can be seen by examination of the tabulation in Example 1. Thus, at 40 cycles, the voltage across the condenser is 235 volts, nearly double the impressed voltage E. At 80 cycles the voltage across the condenser is low, but the voltage across the inductance is 264 volts. The amount of resistance in the circuit influences the voltage across the different parts. At resonance, the current is limited only by this resistance so that, if this were halved, the current would be doubled and the voltage across the inductance and condenser would be doubled. Unsafe values of voltage are, therefore, likely to exist where the resistance of the circuit is low. In Example 1, resonance was obtained by varying the frequency, but resonance can also be produced at any fixed value of frequency by varying either L or C or both in such a manner as to make $X_L = X_C$. This method is frequently used in practice, particularly in the adjustment of apparatus for radio communication where the process is called "tuning the circuit."

245. Resonance in Parallel Circuits.—In a circuit consisting of resistance, inductance, and electrostatic capacity in parallel (Fig. 225) the current through the inductance (I_L) is 180 degrees with respect to the current through the condenser (I_C); hence, these currents tend to neutralize each other and the total current is, according to Article 241:

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}. \quad \dots \dots \dots \quad (77)$$

When the frequency is of such a value that $X_L = X_C$ the circuit is in resonance and the total current taken from the line is $I = I_R$.

Example 1.—The effect of variation of frequency in a parallel circuit, such as is shown in Fig. 225 *a*, is indicated by the curves (Fig. 225 *c*). The resistance is 800 ohms, $L = 0.15$ henry and $C = 50$ microfarads. The impressed voltage $E = 550$ volts.

At low frequencies the capacity reactance X_C is high and the inductive reactance X_L low, so that the total current is determined principally by the inductive branch, as is shown by the table.

RESONANCE IN A PARALLEL CIRCUIT

f	R	X_C	X_L	I_R	I_C	I_L	$I_C - I_L$	I
20	800	159	18.8	0.69	3.46	29.3	−25.8	25.8+
30	800	106	28.2	0.69	5.19	19.5	−14.3	14.3+
40	800	79.5	37.7	0.69	6.91	14.6	− 7.7	7.7+
50	800	63.6	47.1	0.69	8.65	11.7	− 3.0	3.08
58.1	800	54.7	54.7	0.69	10.05	10.05	0	0.69
60	800	53.0	56.5	0.69	10.40	9.8	0.6	0.92
70	800	45.5	65.9	0.69	12.10	8.4	3.7	0.376
80	800	39.8	75.3	0.69	13.80	7.3	6.5	6.54
90	800	35.4	84.7	0.69	15.50	6.5	9.0	9.03
100	800	31.8	94.2	0.69	17.30	5.8	11.5	11.52

At high frequencies X_C is low and X_L high, and the total current depends principally upon the amount of current in the capacity branch. At the resonant frequency of 58.1 cycles, the total current taken from the supply is only 0.69 ampere, although the capacity and inductance are each carrying 10.05 amperes.

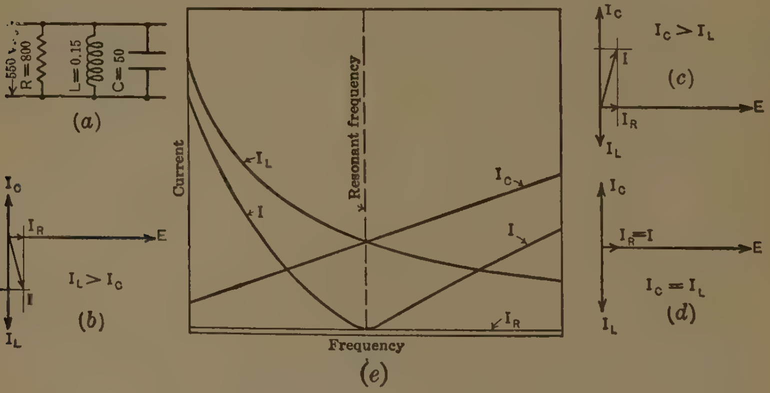


FIG. 225.—Resonance in a Parallel Circuit.

In parallel circuits there is no danger of an excessive voltage, since all branches are subjected to the line potential. The amount of current taken from the supply may be very small, however, even when very large currents are flowing in some of the branch circuits.

246. Applications of the principle of resonance as described in the two articles preceding are of importance. In radio communication, the energy sent out by a transmitting station has a definite frequency which may be anywhere from 12,000 to 3,000,000 cycles, depending upon the requirements of the service. A very small fraction of the energy sent out by the transmitting station reaches the antenna (Fig. 226) located at the receiving station and sets up an alternating e.m.f. between the wires of the antenna and the ground. As a result of this e.m.f., a current flows in the antenna wires which form one terminal of a condenser, the other terminal being the earth. In series with the antenna is an inductance L which can be varied. The arrangement is the equivalent of a series circuit with an inductance L, a capacity C, which is equal to the capacity of the antenna circuit measured to ground, and to a resistance R equal to the resistance of the antenna circuit. The inductance L is adjusted until the antenna circuit is resonant at the frequency of the incoming signal, when the current flowing in the inductance L will be a maximum and the voltage across the inductance will also have its maximum value. This voltage is, however, too small to be detected by an ordinary voltmeter, but, by using a telephone receiver in series with a sensitive detecting device (Fig. 226), the signals may be heard. When the inductance has been adjusted to give a resonant condition the circuit is said to be tuned, and the loudness of the signals heard in the telephone receiver will be a maximum. Increasing or decreasing the value of the inductance L will throw the circuit "out of tune," and the signals will become fainter. There are many modifications of the simple receiving circuit shown, but all of these are based upon the principle of resonance.

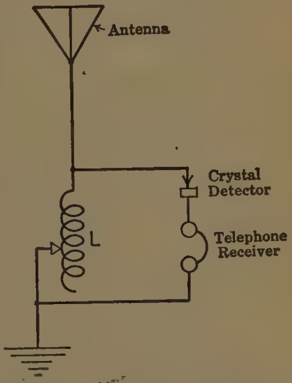


FIG. 226.—Application of the Principle of Resonance. Showing elementary receiving circuit for radio communication.

PROBLEMS ON CHAPTER XXVI

A vector diagram should be drawn for each problem

1-26. A 20-kv-a., 2200-volt transformer takes 0.432 ampere, in the primary or 2200-volt winding at normal voltage, with no load on the secondary. A wattmeter placed in the primary circuit reads 164 watts. The frequency is 60 cycles.

(*a*) What is the effective resistance of this primary coil?

(*b*) When the primary winding is measured with direct current, the resistance is found to be 1.43 ohms. Account for the difference when alternating current is used. What is the conductor resistance of this coil?

2-26. A coil which is wound non-inductively has a resistance of 5 ohms, and is connected to a 125-volt, a.c. supply.

(*a*) Calculate the current for frequencies of 25 and 50 cycles.

(*b*) What is the phase angle between the current and impressed voltage in each case?

3–26. Referring to the coil of Prob. 2–26:

(*a*) Calculate the average power for the two frequencies.

(*b*) Calculate the maximum value of the power in the coil, for the two frequencies.

4–26. Suppose that the 2200-volt coil of the transformer of Prob. 1–26 was unwound from the core and the wire formed into a loop which is practically non-inductive.

(*a*) Calculate the a.c. voltage required to force 0.432 ampere through this wire.

(*b*) If a wattmeter were connected to the wire when arranged in a loop, as above described, what would the reading be when the current is 0.432 ampere?

(*c*) What would the power factor be under these conditions?

5–26. A coil with a negligible resistance has a coefficient of self-induction of 0.25 henry. The coil is connected to a 250-volt, 60-cycle supply.

(*a*) Calculate the current which would flow.

(*b*) What would the current be, if the coil were connected to a 250-volt, 30-cycle supply?

(*c*) Explain why the current is inversely proportional to the frequency, when the impressed voltage is constant.

6–26. Referring to Prob. 5–26, what would the current become if the coil were connected to a 500-volt, 60-cycle supply?

7–26. The coil specified in Prob. 5–26 is to be connected to a 30-cycle supply. What should the voltage be, if it is to take the same current as it does when connected to 250 volts at 60 cycles?

8–26. A 25-cycle, 220-volt circuit is applied to a coil without iron having a negligible resistance. The current which flows is 8 amperes.

(*a*) What is the reactance of this coil? What is the impedance?

(*b*) What is the coefficient of self-induction?

(*c*) If the coil were connected to a 220-volt, 50-cycle circuit, what current would flow?

(*d*) Calculate the reactance and impedance when the coil is connected to a 220-volt, 50-cycle circuit.

9–26. Referring to the coil of Prob. 8–26:

(*a*) What is the power and power factor?

(*b*) What is the phase angle between the voltage and current?

(*c*) What is the maximum value of the power in the coil, during each half cycle?

10–26. A series-circuit is made up of a non-inductive resistance of 15 ohms and an inductance of 0.3 henry the resistance of which is negligible. A current of 5 amperes flows in the circuit when connected to a 60-cycle supply.

(*a*) What would be the voltage drop across each part of the circuit?

(*b*) What would be the total voltage required to force the current through the entire circuit?

(*c*) Calculate the phase angle between the current and the total voltage.
(*d*) Calculate the impedance of the entire circuit.

11–26. Referring to the series circuit of Prob. 10–26:

(*a*) Calculate the reactance and impedance at 30 cycles.

(*b*) What current would flow if the circuit were connected to a 250-volt, 30-cycle supply?

12–26. Calculate the coefficient of self-induction for the 2200-volt winding of the transformer of Prob. 1–26.

13–26. The primary circuit of a 1000-volt, 2-kv-a. transformer has an effective resistance of 9 ohms and an inductance of 19 henries.

(*a*) What is the impedance when operated on 60 cycles?

(*b*) Calculate the current which would flow if the secondary is without load and 1000 volts is applied to the primary.

14–26. A coil without iron, having a resistance of 3 ohms, is connected to a 220-volt, 60-cycle circuit. A current of 8 amperes flows.

(*a*) Calculate the impedance.

(*b*) What is the reactance?

(*c*) What effect would an increase in voltage have upon the impedance, and the reactance?

15–26. A coil takes 15 amperes when connected to a 125-volt d.c. circuit, and 10.7 amperes when connected to a 125-volt, 60-cycle circuit.

Calculate the resistance, inductive reactance and the coefficient of self-induction.

16–26. A series circuit consists of four parts, having the following constants:

Part A, resistance = 5 ohms, inductance = 0 henry.
Part B, resistance = 0 ohm, inductance = 0.15 henry.
Part C, resistance = 23 ohms, inductance = 0.05 henry.
Part D, resistance = 100 ohms, inductance = 0.25 henry.

(*a*) Calculate the impedance of the entire circuit at 25 cycles.

(*b*) What voltage is required to force 3.5 amperes through this circuit?

17–26. Referring to the circuit of Prob. 16–26:

(*a*) Calculate the impedance of each part of the circuit.

(*b*) Calculate the voltage across the terminals of each part.

18–26. Referring to the transformer of Prob. 1–26 (see also Prob. 12–26):

(*a*) What is the active component of the impressed voltage?

(*b*) What is the reactive component of the impressed voltage?

(*c*) What portion of the active component, found in (*a*), is used to overcome the conductor resistance of the winding?

(*d*) What is the power factor of this transformer when it is operating at no load?

19–26. Referring to Prob. 10–26:

(*a*) What is the total power and the power for each part of the circuit?

(*b*) What is the power factor for the entire circuit and for each part?

20–26. Calculate the power and power factor for the transformer of Prob. 1–26.

21–26. Referring to Prob. 14–26:

(*a*) What is the power factor of the circuit?

(*b*) What is the power in the circuit?

(*c*) What is the phase position of the current?

22–26. Referring to Prob. 16–26, with 3.5 amperes flowing:

(*a*) What is the total power and the power in each part?

(*b*) What is the total power factor and the power factor for each part?

(*c*) What is the phase position of current and voltage for each part?

(*d*) What is the apparent power for the entire circuit?

(*e*) What is the reactive volt-amperes for the entire circuit and each part?

23–26. A coil which is connected to a 250-volt supply takes 6.5 amperes at 25 cycles. The inductance is 0.15 henry. What is the power factor?

24–26. A coil which is connected to a 60-cycle, 125-volt supply takes 15 amperes. The resistance is 5 ohms. What is the power factor?

25–26. A reactance coil is to be constructed to produce a power factor of 0.75 when connected to a 25-cycle circuit. The reactance is to be 25 ohms. What is the required resistance of the coil?

26–26. A reactance coil having 10 ohms resistance is to be designed to produce a power factor of 0.85.

(*a*) Calculate the reactance required.

(*b*) What would be the required inductance for 25 and for 50 cycles?

27–26. An a.c. arc lamp takes 6 amperes when operating on a 110-volt circuit. The voltage measured across the various parts is as follows: Arc 72 volts, reactance coil 50 volts. The power for each part as measured by a wattmeter is: Arc 380 watts, reactance coil 50 watts, entire lamp 430 watts. The reactance coil is in series with the arc.

Calculate the phase angle between the current and the voltage for each part and for the entire lamp.

28–26. An a.c. motor is rated at 5 hp., 220 volts. The efficiency is 80 per cent and the power factor 0.78 lagging.

(*a*) Calculate the full-load current.

(*b*) What would the current be for a 5-hp., 220-volt, d.c. motor with an efficiency of 80 per cent?

(*c*) What would the power input be in each case?

29–26. Referring to the a.c. motor in Prob. 28–26:

(*a*) Calculate the equivalent impedance, resistance, and reactance at full load.

(*b*) Assume that at half load the motor takes 16.5 amperes at a power factor of 0.75. Calculate the equivalent impedance, resistance, and reactance at half load.

30–26. A single-phase transmission line, 10 miles long, has a conductor with a resistance of 0.277 ohm per mile and a reactance of 0.595 ohm per mile. The voltage at the load is 20,000. Calculate the voltage required at the power-house end of the line, when it is carrying a load of 2000 kw. at unity power factor.

31–26. Calculate the voltage required at the power-house end of the line of Prob. 30–26, when the line is carrying a load of 2000 kv-a. at a power factor of 0.75 lagging.

32–26. Calculate the voltage required at the power-house end of the line of Prob. 30–26, when the line is carrying a load of 2000 kv-a. at a power factor of 0.75 leading.

33–26. A condenser having a capacity of 50 microfarads is connected to a 125-volt circuit.

(*a*) What would be the current if the frequency is 60 cycles?
(*b*) What would be the current for a frequency of 30 cycles?

34–26. Referring to the condenser of Prob. 33–26, calculate the current for 500 volts, at 30 and at 60 cycles.

35–26. Referring to Probs. 33–26, and 34–26:

What is the capacity reactance and the impedance at 30 and at 60 cycles?

36–26. What electrostatic capacity in microfarads is required to pass 1 ampere at 100 volts:

(*a*) For a frequency of 25 cycles?
(*b*) For a frequency of 50 cycles?
(*c*) For a frequency of 12,000 cycles?

37–26. Referring to Prob. 33–26 (*a*):

(*a*) Calculate the average power supplied to the condenser.
(*b*) Calculate the maximum power supplied to the condenser.
(*c*) What is the apparent power?

38–26. A non-inductive resistance of 50 ohms is in series with a condenser of 45 microfarads, across a 125-volt, 60-cycle supply.

(*a*) What current will flow?
(*b*) What is the voltage across the resistance and the condenser?
(*c*) What is the total impedance of the circuit?
(*d*) What is the capacity reactance and the resistance for the entire circuit?

39–26. Referring to the circuit of Prob. 38–26:

(*a*) What is the total power, and the power for each part of the circuit?
(*b*) What is the apparent power for the entire circuit?
(*c*) What is the power factor for the entire circuit and for each part?
(*d*) What is the phase position of current and voltage for the entire circuit and for each part?

40–26. A condenser of 100 microfarads capacity is in series with an inductance of 0.1 henry, across a 125-volt, 60-cycle supply.

(*a*) Calculate the current.
(*b*) Calculate the voltage across the condenser and the inductance.

41–26. Referring to the circuit of Prob. 40–26:

(*a*) What is the total power and the power in each part?

(*b*) What is the phase angle between current and voltage of each part and of the entire circuit? Does the current lag or lead the supply voltage?

(*c*) What is the power factor for the entire circuit?

42–26. A current of 2 amperes at 25 cycles flows in a series circuit consisting of a non-inductive resistance of 30 ohms, an inductance of 0.25 henry and a condenser of 100 microfarads.

(*a*) Calculate the voltage across each part of the circuit.

(*b*) Calculate the total voltage.

(*c*) Calculate the impedance by two methods.

(*d*) What is the total power and power factor?

(*e*) Does the current lead or lag with respect to the total voltage?

43–26. A non-inductive resistance of 50 ohms is in series with an inductance of 0.1 henry (resistance assumed zero) and a condenser of 45 microfarads across a 125-volt, 60-cycle circuit.

(*a*) What is the total impedance of the circuit?

(*b*) What is the current in the circuit?

(*c*) What is the power factor of each part of the circuit?

(*d*) What is the total power factor?

(*e*) What is the voltage drop across each part of the circuit?

(*f*) Does the current lead or lag with respect to the total voltage?

44–26. (*a*) What value of electrostatic capacity must be connected in series with the coil of Prob. 14–26 in order that the power factor of the combination shall be unity on a 60-cycle circuit?

(*b*) What would be the total voltage required to force 8 amperes through this circuit?

(*c*) What would be the voltage across the coil and the condenser?

(*d*) What is the total power?

45–26. A series circuit consists of four parts having the following characteristics: Part A, has a terminal voltage of 95 volts at a power factor of 0.95 lagging; Part B, a voltage of 200 volts with a power factor of 0.10 leading; Part C, a voltage of 125 volts with a power factor of 0.7 lagging; and Part D, 250 volts with unity power factor. Calculate the total voltage impressed on the circuit.

46–26. Assume that a current of 2 amperes flows in the circuit of Prob. 45–26:

(*a*) Calculate the total power and power for each part.

(*b*) Calculate the apparent power for the entire circuit and for each part.

(*c*) What is the total power factor?

47–26. A non-inductive resistance of 15 ohms and a coil having an inductance of 0.3 henry are connected in parallel across a 125-volt, 60-cycle supply. (Compare with Prob. 10–26.)

(*a*) What is the current in each branch?

(*b*) What is the total current taken from the supply?

(*c*) Calculate the phase angle between the total current and the supply voltage.

(*d*) What is the impedance of the entire circuit?

48–26. Calculate values as required in (*a*), (*b*), (*c*), and (*d*), Prob. 47–26, assuming a supply of 125 volts at 30 cycles.

49–26. What is the total power and power factor for the circuit of Prob. 47–26?

50–26. A non-inductive resistance of 50 ohms and a condenser of 45 microfarads are connected in parallel to a 125-volt, 60-cycle supply. (Compare with Prob. 38–26.)

(*a*) Calculate the total current and the current in each branch.

(*b*) What is the phase angle between the total current and the voltage?

(*c*) What is the total impedance?

51–26. What is the total power and power factor for the circuit of Prob. 50–26?

52–26. Calculate the total current for the circuit shown in Fig. 219 *a*, when the frequency is 30 cycles.

53–26. Calculate the total power and power factor for the circuit of Prob. 52–26.

54–26. A coil which has a resistance of 8 ohms and an inductance of 0.1 henry is connected in parallel with a bank of incandescent lamps taking 500 watts. The voltage applied to the combination is 125 volts at 25 cycles.

(*a*) What is the current in each part of the circuit?
(*b*) What is the total current taken from the supply circuit?
(*c*) What is the total power taken from the supply circuit?
(*d*) What is the power factor of the entire combination?
(*e*) What is the total apparent power?

55–26. A condenser of 25 microfarads and a coil having a resistance of 100 ohms and a self-induction of 2 henries are connected in parallel to a 60-cycle, 250-volt supply.

(*a*) Calculate the current taken by each part of the circuit and the total current taken from the supply.

(*b*) What is the power in each part of the circuit and the total power taken from the supply?

56–26. A circuit is composed of two branches: *A* has a resistance of 4 ohms and an inductive reactance of 3 ohms, *B* has a resistance of 6 ohms and an inductive reactance of 8 ohms.

(*a*) Find the total impedance of the circuit.
(*b*) What total current would flow if 110 volts were impressed on the circuit?
(*c*) What is the current in each of the two branches?

57–26. A circuit is composed of two branches: *A* has a resistance of 2 ohms and a self-induction of 0.015 henry; *B* has a capacity of 80 microfarads, in series with an inductance of 0.005 henry. The frequency is 60 cycles.

(*a*) Find the total impedance of the circuit.
(*b*) What total current would flow if 200 volts were applied to the circuit?
(*c*) What is the current in each branch?

58–26. A non-inductive resistance and a coil having negligible resistance are connected in parallel across a 250-volt supply. The total current is 25 amperes and the current through the resistance is 20 amperes.

(*a*) What is the current in the coil?
(*b*) Calculate the total impedance by two methods.

59–26. Two reactance coils are connected in parallel across a 125-volt supply. The total current is 15 amperes and the power factor 0.8. One coil takes 8 amperes at 0.85 power factor.

(*a*) What is the current in the second coil?
(*b*) What is the reactance and resistance for each coil?

60–26. Referring to Prob. 42–26:

(*a*) At what frequency would this circuit be resonant?
(*b*) What would the current be at resonance?
(*c*) Calculate the total voltage and the voltage across each part when the circuit is resonant.

61–26. Referring to Prob. 55–26:

(*a*) At what frequency would this circuit be resonant?

(*b*) Calculate the current taken by each branch and the total current, when the circuit is resonant.

62–26. Three reactance coils, connected in series, carry a current of 15 amperes. The power and voltage for each coil are as follows:

Coil 1, $P_1 = 850$ watts; $E_1 = 120$ volts.
Coil 2, $P_2 = 2100$ watts; $E_2 = 156$ volts.
Coil 3, $P_3 = 1250$ watts; $E_3 = 95$ volts.

(*a*) Calculate the resistance, reactance and impedance for each coil.
(*b*) Calculate the total voltage for the circuit.
(*c*) What is the total impedance?
(*d*) What is the total power and power factor?
(*e*) What is the total apparent power?

63–26. A reactance coil having a resistance of 5 ohms and reactance of 10 ohms is to be connected in series with a second coil, to a 250-volt circuit. The second coil is to be designed so that the current will be 8 amperes and the power factor of the entire combination, 0.70.

(*a*) What is the required voltage for the second coil?
(*b*) Determine the resistance, reactance and impedance for each coil.
(*c*) Calculate the total power by two methods.

64–26. A certain a.c. motor has an efficiency of 0.75 and a power factor of 0.80 when delivering a load of 3 hp., the voltage at the motor terminals is 200 volts. The motor is to be operated on a 250-volt circuit.

(*a*) If a coil of negligible resistance is placed in series with the motor, calculate the required reactance of the coil.

(*b*) If a non-inductive resistance is used instead of the coil, calculate the value of this resistance.

65–26. A 5-hp., 220-volt a.c. motor takes a current of 110 amperes when thrown directly on the line at full voltage. The full load running current is 23 amperes and it is desired to limit the current, when starting, to 2.5 times the full-load running current. The power factor of the motor when starting is 0.50. Assume that the impedance of the motor is the same for 2.5 times full-load current, that it is for 110 amperes.

(*a*) If a non-inductive resistance is used to reduce the voltage, calculate the value of this resistance and the power lost.

(*b*) If a reactance coil having a power factor of 0.30 is used instead of the resistance of (*a*), calculate the voltage across the reactance and its resistance and reactance.

(*c*) Compare the power lost by method (*b*) with that of (*a*).

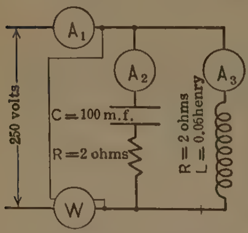


FIG. 66–26.—Problem of a Parallel Circuit.

66–26. Referring to Fig. 66–26, and assuming a frequency of 60 cycles:

(*a*) What would be the reading of each ammeter?
(*b*) What would be the reading of the wattmeter?

67–26. Given four condensers having capacities as follows:

$C_1 = 50$ microfarads.
$C_2 = 100$ microfarads.
$C_3 = 75$ microfarads.
$C_4 = 25$ microfarads.

The frequency is 60 cycles and the supply voltage 250 volts.

(*a*) Calculate the total current when all four condensers are connected in parallel to the supply.

(*b*) Calculate the current when all four condensers are connected in series to the supply.

68–26. A certain coil without iron has a resistance of 30 ohms and an inductance of 0.8 henry.

(*a*) If the coil were placed across a 60-cycle, 110-volt circuit, what current would flow in the coil?
(*b*) What would be the current in the coil with 30 cycles and 110 volts?
(*c*) What would be the current with 110 volts direct current?
(*d*) What is the reactance of this coil with 60 cycles?
(*e*) What is the impedance of the coil with 60 cycles?
(*f*) What effect would a change of voltage have upon the impedance?

69–26. Referring to Prob. 68–26 (*a*):

(*a*) What would be the power factor of the coil?
(*b*) What would be the phase position of the current with respect to the impressed voltage?

70–26. The secondary side of a transformer is connected in series with a non-inductive resistance and sufficient voltage is impressed on the entire circuit to give a normal voltage of 110 volts on the transformer, which is without load. The voltage across the non-inductive resistance is 90 volts and the total voltage is 175 volts. The current is 8 amperes.

(*a*) What is the impedance of the transformer?
(*b*) What is the power in the transformer?

71–26. A bank of lamps is to be provided with a dimmer which will reduce the voltage of the lamps from the normal voltage of 110 volts to 70 volts. The current taken by the lamps at 70 volts is 10 amperes and the frequency 60 cycles. The dimmer is to consist of a reactance which will have a power factor 0.3 with 10 amperes flowing.

(*a*) What is the required voltage across the dimmer?
(*b*) What is the power lost in this dimmer?
(*c*) If a resistance were used instead of a reactance, what would be the power lost in the resistance?

72–26. A non-inductive resistance of 50 ohms and an inductance (resistance negligible) of 0.1 henry are connected in parallel to a 125-volt, 60-cycle circuit.

(*a*) Calculate the current which would flow in each part of the circuit.
(*b*) Calculate the total current.
(*c*) Calculate the total impedance.

73–26. A series circuit of two parts is connected to a 110-volt supply. The voltage across one part is $E_2 = 50$ volts which leads the current by 30°. The supply voltage is 45° ahead of the current.

(*a*) Find the value of the voltage E_1 for the other part.
(*b*) What is the phase position of E_1 with respect to the current?

CHAPTER XXVII

PRODUCTION OF POLYPHASE VOLTAGES

247. Definitions.—A single-phase system is one energized from a single alternating e.m.f.; whereas a polyphase system is energized from two, three, or more e.m.f.'s. The ordinary two-phase or quarter-phase system is energized from two substantially equal voltages, which differ in phase by a quarter of a cycle or 90 degrees. A three-phase system is energized from three substantially equal voltages, which differ in phase by one-third of a cycle or 120 degrees. The usual source of supply for a polyphase power system is a polyphase alternator, either two- or three-phase.

The voltages between terminals of a generator or motor or between line wires of a feeder are called the "line voltages" (Fig. 227). The current in a line wire or the current entering or leaving a terminal of a generator or motor is called the "line current." Each winding of a generator, or motor, or a branch circuit forming part of a polyphase load is known as a "phase." The voltage across a phase of a machine or a load is called the "phase voltage." If a polyphase system has equal voltages differing successively in phase by the same angle, the voltages are said to be balanced. Thus, in the three-phase system

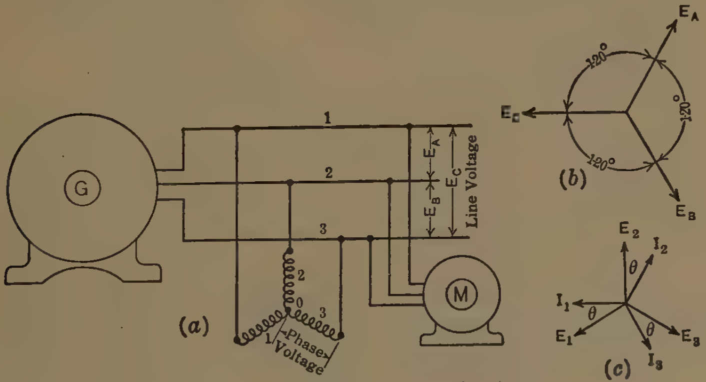


FIG. 227.—Phase, and Line Voltages.

of Fig. 227 *a* the line voltages are E_A, E_B, E_C, and, if balanced, they would be represented by the three vectors of Fig. 227 *b*, which are equal and 120 degrees apart. In practice, polyphase systems have approximately balanced voltages, except where the loads on the different phases are unequal, when there may be considerable unbalance of voltages. If a polyphase system has equal currents in all phases, with equal phase displacements the currents are said to be balanced. Thus, in Fig. 227 *a*, if the currents in the three branches of the load 1, 2, 3 are equal and each lags θ degrees behind the phase voltages E_1, E_2, and E_3 (Fig. 227 *c*), the currents are said to be balanced. When both voltages and currents are balanced, the load due to these currents is balanced and the power input to or output from each of the phases is the same.

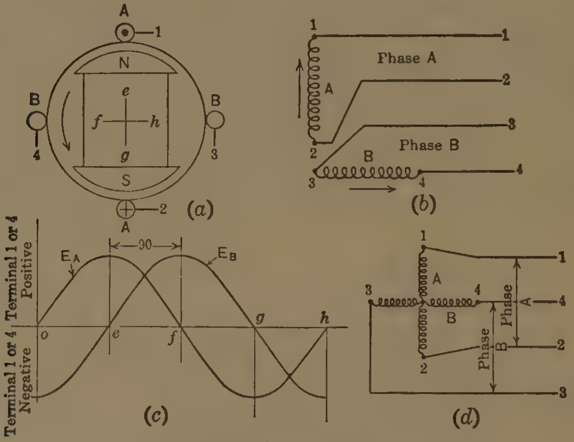
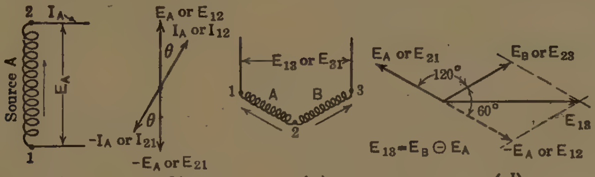


FIG. 228.—Generation of Two-phase Voltages.

248. Generation of Two-phase Voltages.—In a single-phase alternator, all the coils constituting the armature winding are connected together in such a manner as to produce a single source of e.m.f. In a two-phase alternator, the armature coils are divided into two separate groups, with the same number of conductors in each group. The coils for the two groups are so chosen and connected that the machine produces equal voltages at the terminals of each group of coils, and these voltages differ by 90 degrees. This, by definition, constitutes a two-phase alternator. A simple form of two-phase alternator is shown in Fig. 228 *a*, where there is a single coil for each phase, with a two-pole field. Let the curve E_A represent the e.m.f. generated in coil *A* by rotation of the field. It is apparent that, when the voltage of coil A (E_A) is a maximum in a positive direction (terminal 1 positive), the voltage of coil B is zero. Coil B reaches its maximum positive value when the field has turned 90 electrical degrees. Hence, the e.m.f. of B (E_B) is displaced from E_A by 90 degrees as shown in Fig. 228 *c*. These curves indicate that terminal 4 of coil B reaches a positive maximum with respect to terminal 3, 90 degrees or one-quarter of a cycle after terminal 1 reaches a positive maximum. It will be seen that the two coils are located 90 electrical degrees apart on the armature. For a multipolar machine the same holds true. Commercial alternators usually have more than one coil for each phase, in order to use effectively the entire surface of the armature. The two groups of coils are so arranged, however, that the center line of the coils of one phase is 90 electrical degrees from the center line of the coils of the other phase. This construction is explained in Chapter XXXI. The two groups of coils may be kept electrically independent as shown as Fig. 228 *b* or may be interconnected as described in Article 251.



(*a*) (*b*) (*c*) (*d*)

FIG. 229.—Notation for Vectors in Polyphase Circuits.

249. Voltage and Current Relations in a Two-phase, Four-wire Alternator.—Where the terminals of the two groups of coils are kept separate as in Fig. 228 *a* or *b*, four line wires are required to transmit the load supplied by the alternator. Such an arrangement constitutes a two-phase, four-wire system. The two windings may be electrically independent as in Fig. 228 *b*, or the middle points may be connected as in Fig. 228 *d*. In either case, the loads are connected across the phases, as shown. Each phase may be treated like a single-phase circuit and the currents and loads determined by the methods already described.

250. Notation for Vectors in Polyphase Circuits.—In polyphase systems, with several sources of e.m.f., it is essential to employ a systematic method of notation for voltages and currents to avoid errors and indicate plainly the relations between the different quantities. Thus, when the voltage E_A is acting in the direction of the arrow (Fig. 229), we can call this a positive direction and say that the voltage is acting from 1 to 2. This voltage can then be called E_A or E_{12} (read E, one two). If a current I_A flows in the circuit due to the voltage E_{12} acting on the circuit, this current could also be designated as the current I_{12}. If a voltage acts in the direction opposite to the arrow, the voltage can be designated as $-E_A$ or E_{21}. A current flowing opposite to the assumed positive direction can be called $-I_A$ or I_{21}. Actually, the voltage reverses in direction every cycle, but the above notation is convenient to indicate the relative polarity of several voltages or currents in the same system. Thus suppose there are two sources of e.m.f., E_A and E_B in a circuit, and that the positive direction for each of them is as indicated by the arrows

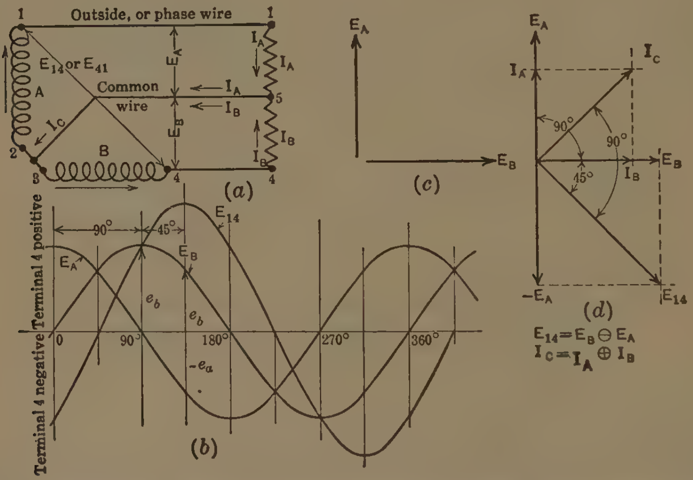


FIG. 230.—Voltage and Current Relations in a Two-phase, Three-wire Alternator.

(Fig. 229 *c*). The phase angle between these voltages is 120 degrees. The voltage between 1 and 3 is the resultant of E_A and E_B, but it is important to determine if E_B is to be subtracted or added vectorially to E_A. According to the notation just given, the voltage E_{13} is the resultant of $-E_A$ and $+E_B$ or $E_{13}=E_B \ominus E_A$. This is shown in the vector diagram (Fig. 229 *d*). Similarly, the voltage acting from 3 to 1 is $E_{31}=E_A \ominus E_B$.

251. Voltage and Current Relations in a Two-phase, Three-wire Alternator.—When the two windings of a two-phase alternator are joined together as in Fig. 230 *a*, only three wires are required to transmit the power. This arrangement is called a two-phase, three-wire system. The wire connected to the junction of the two phase-windings is called the common wire. The other two wires are called the phase or "outside" wires. It is evident that the voltage between the outside wires and the common wire is the phase-voltage E_A or E_B. The voltage between the outside wires is the *resultant* of the two phase-voltages. The value of this resultant voltage can be found by combining algebraically the instantaneous values of the phase-voltages, which are plotted in Fig. 230 *b*, where E_B lags 90 degrees behind E_A. Let the positive directions of these voltages be as indicated by the arrows (Fig. 230 *a*); that is, when terminal 1 is positive with respect to terminal 2 of phase *A*, then the instantaneous values of E_A will be plotted *above* the horizontal axis. Similarly, when terminal 4 of phase *B* is positive with respect to terminal 3, the curve for E_B is plotted above the horizontal axis. It should be noted that these arrows do *not* represent the polarities of the two windings at the *same instant.* Assume that it is required to find the voltage E_{14}, that is the voltage between terminals 1 and 4. In this case, when 4 is positive the voltage would be plotted *above* the horizontal axis or would be *positive.* At 0 degrees, E_A is positive; i.e., lead 1 is positive and E_B is zero. Hence, the total voltage between 1 and 4 is equal numerically to e_a and terminal 4 is *negative* with respect to 1. Therefore, this point should be plotted *below* the horizontal axis because it was agreed to plot points above when 4 was positive. At 45 degrees the voltages are $+0.707e_a$ and $+0.707e_b$, and terminals 1 and 4 are both *positive.* Hence, the voltage between 1 and 4 is zero. At 90 degrees E_A is zero, the total voltage between 1 and 4 is e_b, and terminal 4 is positive with respect to 1. Therefore the point is above the horizontal axis. At 135 degrees, the voltages are $+e_b$ and $-e_a$, that is, terminal 4 is positive with respect to 3, and 2 is positive with respect to 1. Hence, the total instantaneous voltage between 1 and 4 is $e_{14}=e_a+e_b$. This gives point p on the curve for voltage E_{14}. Other points are determined similarly, giving the curve E_{14} lagging 45 degrees behind E_B and 135 degrees behind E_A as shown in the vector diagram (Fig. 230 *d*).

It is not necessary to plot the sine curves as in Fig. 230 *b* in order to determine the value and phase position of E_{14}; instead, it can be found by taking the resultant of the vectors E_a and E_b which are drawn 90 degrees apart as in Fig. 230 *c*. It is first necessary to write an equation to indicate the way in which these two vectors are to be combined. This must be determined from the diagram of connections (Fig. 230 *a*). In this diagram, the positive direction of voltage is taken in the direction of the arrows; that is, away from the common junction. Any other direction might be assumed at the start of a problem, without affecting the result, provided the assumed positive direction is kept the same throughout. This assumed direction indicates the direction of the e.m.f. in the two windings at an interval of one-quarter of a cycle. Thus, referring to the two-pole alternator of Fig. 228, suppose terminal 1 of coil A were positive with respect to terminal 2, when the N pole of the field was in the position shown. When the field has revolved 90 degrees or one-quarter of a cycle, the N pole would be in line with terminal 4 of coil B and this terminal would be positive with respect to terminal 3. These conditions are represented in the diagram of connections (Fig. 230 *a*) by the arrows. It is necessary to identify the polarities of the phases of a polyphase machine in this manner before attempting to get the resultant. If it is desired to obtain the voltage E_{14}, the positive direction would be from 1 to 4 or in the direction of the arrow in Fig. 230 *a*, according to the rules stated in Article 250. Hence, the equation to represent the vectors is $E_{14} = \ominus E_A \oplus E_B$. E_A is taken negative because we are proceeding from 1 to 4 in a *negative* direction through phase A and a *positive* direction through B. The vector equation indicates that E_{14} is the resultant of $+E_B$ and $-E_A$. Hence, the vector E_A is reversed as shown in Fig. 230 *d* and the resultant E_{14} is found in the regular manner. If the voltage E_{41} is to be found, the equation is $E_{41} = \ominus E_B \oplus E_A$. Since $E_A = E_B$ the numerical value of E_{14} or E_{41} is $\sqrt{2}E_A$ or $1.41E$ where E is the phase-voltage of the machine. Suppose that two equal non-inductive loads are connected across the phases as in Fig. 230 *a*. The currents in these loads are I_1 and I_4 and are equal. The current in an outside wire is the same as the load current for that phase, while the current in the common wire is the *resultant* of these phase-currents. Referring to Fig. 230 *a*, the current I_1 is forced through the circuit by the voltage E_A in the direction of the arrows. That is, when terminal 1 is positive, current I_1 flows in the direction shown. These arrows do not show the instantaneous directions of currents, but only the positive direction corresponding to the positive direction of the voltage. The current in the common wire could be determined by plotting the sine curves representing these two currents and combining their instantaneous values as was done for voltages E_A and E_B, in Fig. 230 *b*. It is not necessary to do this, however, as the current can be found by combining the current vectors, using the same method as for the voltages. Since the positive directions of two currents in the common wire are the same, the vector equation can be written $I_C = I_1 \oplus I_4$ and the vectors take the position shown in Fig. 230 *d*. The resultant current I_c is evidently $\sqrt{2}I_1$, or $1.41I$, where I is the current in each phase wire. It will be seen by inspection of the vector diagram that I_c, the current in the common wire, is 90 degrees out of phase with the resultant voltage E_{14} across the two outside wires. This holds true only for the case of two equal phase-currents at unity power factor. The method of dealing with unbalanced loads will be given in the next chapter.

252. Generation of Three-phase Voltages.—A three-phase alternator has three separate groups of coils, each with one-third the total number of conductors. The three groups of coils are so arranged that the machine produces three equal voltages having a phase displacement of 120 degrees. This, by definition, gives a three-phase source. Referring to Fig. 231 *c*, curves E_A, E_B, and E_C represent the e.m.f.'s generated in coils A, B, and C, respectively. In the position shown, coil A is producing its maximum voltage in a positive direction, as indicated by the arrow in Fig. 231 *b*. Coil B does not reach its positive maximum until one-third of a cycle later or until the field has revolved 120 degrees. Coil C reaches its positive maximum after a further rotation of 120 degrees. As in the case of two-phase machines, the commercial alternators have more than one coil per phase, but the same angular relation holds true so that Fig. 231 *b* and *c* can be used to illustrate the voltage relations for a machine of any number of poles, since the angular distance is measured in electrical degrees. If the three windings were kept independent, six wires would be required to distribute the energy. Hence, in practice, the coils constituting the three phases are connected together in such a way that the machine requires only three wires. This is discussed in the next two articles.

253. Voltage and Current Relations in a Three-phase, Y-connected Alternator.—If the three-phase windings have an end of each connected together, leaving only three free terminals, the machine is said to be star- or Y-connected. To produce a true three-phase system, however, the voltages between terminals of the alternator must all be alike and these voltages must have a phase difference of 120 degrees. This result will not be accomplished unless the windings are interconnected in a certain manner. Suppose that the terminals 2, 4, and 6 are connected together as in Fig. 231 *b*. To find the voltages between terminals E_{13}, E_{35}, and E_{51}, it is necessary to obtain the resultant of the phase-voltages, E_A, E_B, and E_C. This can be done by combining algebraically the instantaneous values of the phase voltages in the same manner as for a two-phase machine. (See Article 251.)

In Fig. 231 *d* the sine curves for E_A and E_B are plotted, with positive values, when leads 1 or 3 are positive. The instantaneous voltage between terminals 1 and 3 will be the algebraic sum of the instantaneous voltages of E_A and E_B. At

0° these are $e_a = E_m$ and $e_b = -0.5\ E_m$; that is, at this instant terminal 1 is *positive* with respect to 2 and terminal 3 is *negative* with respect to 4. The actual polarities at this instant are shown in Fig. 231 *e*. It will be seen that terminal 3 is *negative* with respect to 1 and the voltage E_{13} is equal to $-1.5\ E_m$. This point on the curve for E_{13} is therefore plotted *below* the horizontal axis. At 30° (point *n*) the voltage of phase *A* is $+0.866E_m$ and of phase *B* is zero. Hence, terminal 3 is *negative* with respect to 1, and the voltage between 1 and 3 is $-0.866E_m$. At 60° the phase-voltages are $+0.5E_m$ and $+0.5E_m$; hence, the voltage between 1 and 3 is zero. At 90° the voltage of *A* is zero and of *B* is $+0.866E_m$, and the voltage between 1 and 3 is $+0.866E_m$. Values for other points on the curve E_{13} are given in the tabulation of Fig. 231, and the entire curve is plotted in Fig. 231 *d*. It will be seen that the maximum value of this curve is $\sqrt{3}$ or 1.73 times the maximum value of the phase voltages. Therefore if the effective value of the phase voltages of windings *A* and *B* are E_A and E_B respectively, the effective value of the terminal voltage E_{13} is $\sqrt{3}E_A$, since $E_A = E_B$. Similar curves could be plotted to obtain the voltage E_{35}

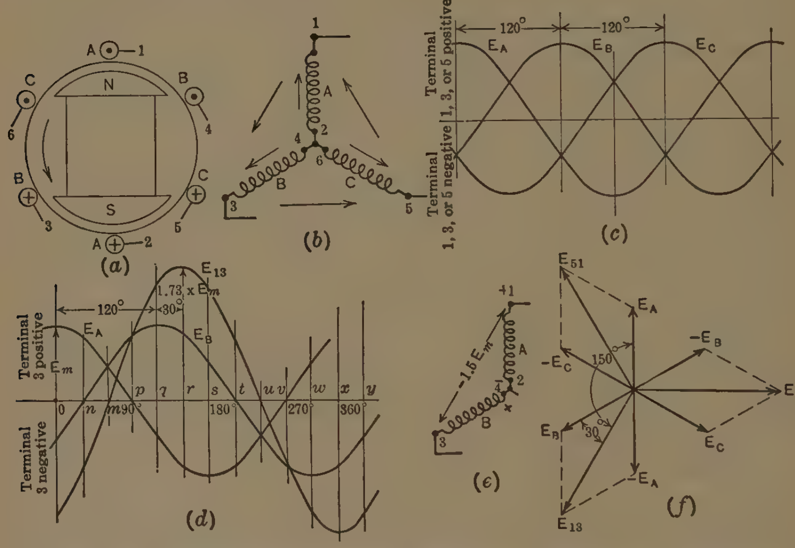


FIG. 231.—Voltage Relations in a Three-phase, Y-connected Alternator.

INSTANTANEOUS VALUES OF E_{13}

Point	Deg.	E_A	E_B	E_{13}	Point	Deg.	E_A	E_B	E_{13}
o	0	E_m	$-0.5E_m$	$-1.5E_m$	*s*	180	$-E_m$	$0.5E_m$	$1.5E_m$
n	30	$0.866E_n$	0	$-0.866\ E_m$	*t*	210	$-0.866\ E_m$	0	$0.866\ E_m$
m	60	$0.5E_m$	$0.5E_m$	0	*u*	240	$-0.5E_m$	$-0.5E_m$	0
p	90	0	$0.866E_m$	$0.866\ E_m$	*v*	270	0	$-0.866\ E_m$	$-0.866\ E_m$
q	120	$-0.5E_m$	E_m	$1.5E_m$	*w*	300	$0.05\ E_m$	$-E_m$	$-1.5E_m$
r	150	$-0.866\ E_m$	$0.866\ E_m$	$1.732\ E_m$	*x*	330	$0.866\ E_m$	$-0.866\ E_m$	$-1.732\ E_m$
					y	360	E_m	$-0.5E_m$	$-1.5E_m$

and E_{51} and it would be found that these voltages would be 120° from E_{13}. Therefore, by connecting the three phases in the manner shown in the diagram (Fig. 231 *b*), a true three-phase system is produced. It should be noted that the voltage E_{13} is 150° from E_A and 30° from E_B (Fig. 231 *f*).

The resultant voltage between terminals of a three-phase, Y-connected machine can be determined by combining the vectors representing the phase-voltages in a manner similar to that employed for a two-phase machine (Article 251). In doing this, the positive direction of the voltages should first be indicated on the diagram of connections (Fig. 231 *b*). This positive direction is usually taken away from the common junction point *O* and is determined in the manner already described in the previous paragraph; that is, terminals 1, 3, and 5 would become positive with respect to *O*, at successive intervals corresponding to one-third of a cycle or 120 degrees. In considering the voltages between terminals, the order can be taken as E_{13}, E_{35}, E_{51} or E_{15}, E_{53}, and E_{31}. It is generally customary to consider the voltages in a counter-clockwise direction as positive, and this will be done in the following illustrations. This direction is indicated by the arrows in the diagram (Fig. 231 *b*). To find the terminal voltages it is first necessary to write the three vector equations, which are:

$$E_{13} = \ominus E_A \oplus E_B, \quad E_{35} = \ominus E_B \oplus E_C, \quad E_{51} = \ominus E_C \oplus E_A.$$

The positive direction of the three phase-voltages is indicated in Fig. 231 *b*. The vector diagram is then drawn as in Fig. 231 *f*. For E_{13} the equation indicates that the resultant is found by adding $-E_A$ and $+E_B$. These vectors are, therefore, drawn in the direction indicated and the resultant determined in the usual manner. It will be seen that the angle between $-E_A$ and $+E_B$ is 60 degrees and since $E_A = E_B$ the value of E_{13} is $\sqrt{3}E_A$. The voltages E_{35} and E_{51} are found similarly. It will be seen that $E_{13} = E_{35} = E_{51}$ and they are displaced 120 degrees; hence they give a balanced three-phase system. It should be noted that the terminal voltages are not in phase with the voltage of the three windings but are displaced 30 degrees. If the connections of one phase of a three-phase alternator are reversed, the three terminal voltages will no longer be equal, and the phase angles will not be 120 degrees.

The currents in the line wires of a three-phase, Y-connected system are evidently the same as the currents in the corresponding phase of the windings. The current and voltage relations for a Y-connected alternator may be summarized as follows:

$$E_l = \sqrt{3}E_p, \quad I_l = I_p,$$

where E_l = voltage between any two line wires or terminals;
E_p = phase voltage of one group of coils, or the voltage between the terminal and the common junction or neutral point;
I_l = current in a line wire;
I_p = phase current.

254. Voltage and Current Relations in a Three-phase, Delta-connected Alternator.—Instead of connecting the three windings in Y, as described in the preceding article, they may all be combined into a single circuit as shown in Fig. 232 *a*. Evidently this could not be done unless the resultant of the three voltages, E_A, E_B, and E_C, which acts through the circuit 2, 1, 4, 3, 6, 5, 2, is zero; otherwise there would be a circulating current in this path. Suppose that the windings shown in Fig. 232 *a* are the same windings as in Fig. 231 *b*, the only difference being in the way they are connected together. The positive direction of the vectors is shown in Fig. 232 *a*. The resultant voltage might be found by combining algebraically the instantaneous voltages, of the three windings, as was done in the previous article for a Y-connected machine. The same result can, however, be obtained more easily by combining the vectors. The vector equation for the resultant voltage is $E_{25} = \oplus E_A \oplus E_B \oplus E_C$. Hence the vector diagram is as shown in Fig. 232 *c*. It is evident that the voltage E_{23} is the resultant of $+E_A$ and $+E_B$ and since they are 120 degrees apart, $E_{23} = E_A = E_B$. Also E_{23} is opposite E_C and is equal to it since $E_A = E_B = E_C$. Therefore, if the phase-voltages of the alternators are all equal, the resultant voltage E_{25} is zero, and there will be no current circulating in the closed circuit 2, 1, 4, 3, 6, 5, 2. Actually the voltages of the alternator phases are not exactly equal nor are they exactly a sine wave in form so that in practice a small amount of current will circulate through this closed circuit. When the windings of an alternator are connected in the manner shown in Fig. 232 *a* the machine is said to be delta (Δ) connected. It should be noted

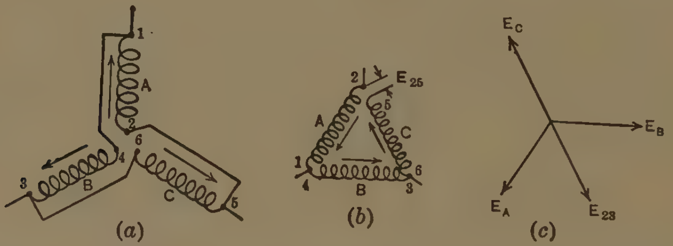


FIG. 232.—Voltage Relations in a Three-phase, Delta-connected Alternator.

that the connections were so made that the positive direction of each phase was counter-clockwise through the closed delta. The same result would be secured if the three phases acted in a clockwise direction, but, if one phase is reversed (Fig. 233 *a*), the resultant voltage is no longer zero but is double the phase voltage. This would produce a short-circuit in the delta. It is evident, from an examination of Fig. 234, that the voltage between terminals of a delta-connected alternator is the same as the phase-voltage. The current in a line wire (Fig. 234 *a*) is the resultant of the currents in the two phases connected to that line wire. Let the alternator be loaded in such a manner that equal currents flow in the three phases of the machine and that these currents are in phase with the voltage of the particular phase in which they flow (Fig. 234 *b*). Let the positive directions of these currents be as indicated by the arrows in Fig. 234 *a*. It has already been shown that there will be no current circulating around the delta. Hence, these phase-currents must flow into or out from the terminals of the machine. Let the positive direction of current in a line wire be indicated in the direction shown; that is, away from

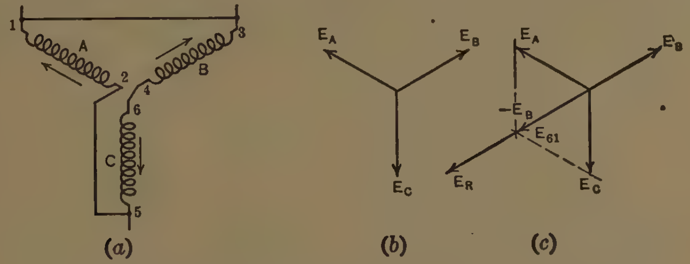


FIG. 233.—Effect of a Reversed Phase in a Delta System.

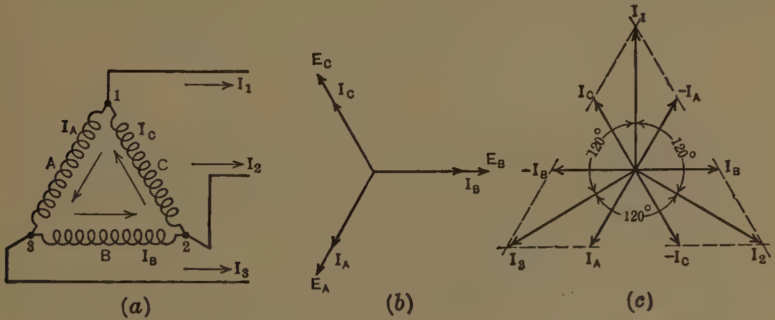


FIG. 234.—Current Relations in a Delta-connected, Three-wire System.

the alternator. Then the currents in the three line wires or the terminal currents are

$$I_1 = I_C \ominus I_A, \quad I_2 = I_B \ominus I_C, \quad I_3 = I_A \ominus I_B.$$

The vectors are combined in Fig. 234 *c*. Each of the line or terminal currents is therefore the resultant of two equal vectors, 60 degrees apart, since

$$I_A = I_B = I_C.$$

Therefore,

$$I_1 = 2I_A \cos 30° = \sqrt{3} I_A,$$

and similarly for I_2 and I_3.

The three line currents are each equal to $\sqrt{3}$ times the phase current, and these line currents are 120° apart. It should be noted, however, that the line currents are not in phase with the currents in the alternator windings as is shown in Fig. 234 *c*. If the currents I_A, I_B, and I_C were displaced from the corresponding phase voltages, by the same angle, the resultant line or terminal currents would still be equal and be $\sqrt{3}$ times a phase current. If, however, the phase angles were different or the three phase-currents were not equal, the line currents would not be equal. This is explained more completely in the next chapter.

Summarizing the conclusions of the preceding paragraphs, we have for a balanced delta-connected alternator or load:

$$E_l = E_p, \quad I_l = \sqrt{3} I_p,$$

where E_l = voltage between any two line wires;
E_p = phase voltage of one group of coils;
I_l = current in a line wire;
I_p = current in one phase.

255. Commercial polyphase systems can be classified as follows:

(*a*) Two-phase, four-wire.
(*b*) Two-phase, three-wire.
(*c*) Three-phase, three-wire.
(*d*) Three-phase, four-wire.

These systems may be supplied directly by polyphase alternators having a suitable arrangement of their phase windings, or they may be supplied through transformers. The only essential is a source of polyphase power having voltages with the proper phase relations.

For the **two-phase, four-wire system,** the arrangement of loads is as shown in Fig. 235. Motors are connected to both phases, using two single-phase transformers or a two-phase transformer, if it is necessary to step down the line voltage. In this system, four wires must be carried to each two-phase motor. Sometimes the low-voltage circuit leading to such a motor is made three-wire by interconnecting the transformer windings in the same way as for alternator windings. (See Fig. 230.) Lamps or single-phase motors may be connected to either phase. In some cases, the single-phase loads are evenly distributed on the two phases; in other cases, all the single-phase load is connected to one of the phases. Sometimes the midpoints of the two alternator or transformer windings which supply this system are connected together; in other cases, the two windings are electrically insulated from each other. **The two-phase, three-wire system** is produced by joining together an end of each of the windings as shown in Fig. 236. Polyphase motors require three wires, the common wire having 1.41 times the ampere capacity of the other two

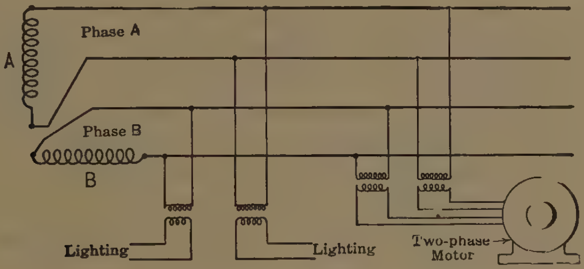


FIG. 235.—Two-phase, Four-wire System.

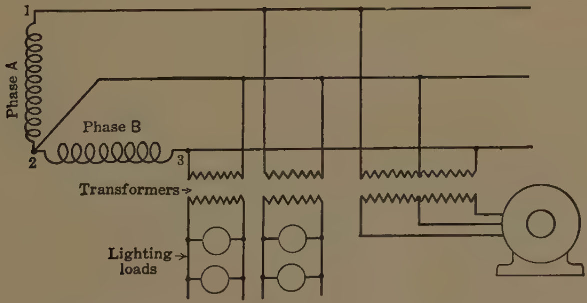


FIG. 236.—Two-phase, Three-wire System.

wires. (See Article 251.) Single-phase motors or lamps would be connected to either phase as shown.

The three-phase, three-wire system may be supplied from a three-phase alternator or transformer, with the windings connected either in delta or Y as was explained in Articles 253 and 254. Whether the source is delta- or Y-connected is immaterial as far as the transmission system is concerned, since the three line voltages are equal and 120 degrees apart in either case. Loads are connected as shown in Fig. 237. Three-phase motors would be connected by three wires to the three line wires, either directly or through transformers. If transformers are employed, the windings may be connected either Y or delta on either side according to the requirements of the particular case. The voltage rating of the windings of the individual transformers used to produce a given line voltage will not be the same for Y or delta connection. In general, for three-phase, three-wire systems the alternators are usually Y-connected and transformers delta-connected, although in very high-voltage systems Y connection of the high-voltage windings of the transformers is common. A discussion of the relative merits of these methods of connection is given in Article 281. The three-phase, three-wire system is commonly used for power transmission. **The three-phase, four-wire system** must be supplied from a three-phase alternator having its windings Y-connected; or from three transformer windings, which are Y-connected. The fourth wire is joined to the neutral point of the alternator or transformers and is called the "neutral wire." In this system, three-phase motors would be supplied from the three line wires (Fig. 238), and single-phase motors or lamps would be connected between one line wire and the neutral. This system is in common use for the distribution of lighting and power service in cities.

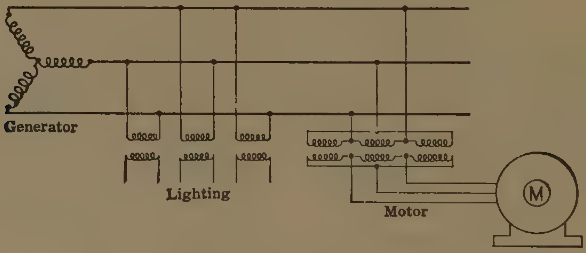


FIG. 237.—Three-phase, Three-wire System.

The two-phase, three-wire, and the three-phase, three-wire systems require about 15 per cent less copper[1] than a single-phase or a two-phase, four-wire system, assuming that the same voltage is to be used on the load in each case and disregarding the difference in the voltage drop in the several systems. On the basis of the same percentage line drop, the three-phase, three-wire, and the two-phase, three-wire systems, require about 25 per cent less copper than either the single-phase or the two-phase, four-wire systems. For equal voltages between line wires the four-wire, three-phase system would require more copper than the three-wire, but for a voltage *to neutral* equal to the voltage between line wires of a three-phase, three-wire system, the four-wire system requires only about one-third the copper of the three-wire system based on equal voltage loss. Polyphase systems are superior to the single-phase system in the following respects:

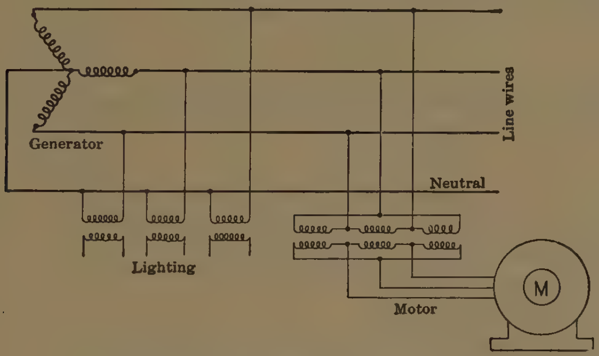


FIG. 238.—Three-phase, Four-wire System.

(*a*) The generators are cheaper because the entire surface of the armature can be effectively utilized.
(*b*) The generators have higher efficiency and better regulation.
(*c*) Polyphase motors are superior to single-phase as regards cost and operating characteristics.
(*d*) A considerable saving in copper is possible, as indicated above.

Nearly all the electric power produced in the United States is generated and transmitted in the form of alternating current and, for most of it, the three-phase system is used.

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[1] See table, Article 190.

PROBLEMS ON CHAPTER XXVII

Note: A vector diagram should be drawn for each problem.

1–27. A two-phase alternator has a phase voltage of 2300 and is connected like the diagram in Fig. 228*d*.

(*a*) What is the voltage between each terminal and the junction point between the two phases?

(*b*) Draw a vector diagram and calculate the voltages E_{13}, E_{32}, E_{24}, and E_{41}.

2–27. A two-phase alternator has a rated voltage of 250 volts across each phase. If the two windings are connected, as in Fig. 230*a*, calculate:

(*a*) The voltage between each outside wire and the common wire.

(*b*) The voltage between the two outside wires.

(*c*) If the connections of phase *B* are reversed, determine the voltages as required in (*a*) and (*b*).

3–27. The current in each phase of a generator connected as in Fig. 230*a* is 100 amperes, lagging 30 degrees behind the voltage for that phase. What would be the current in the common wire?

4–27. A two-phase, three-wire system like that shown in Fig. 230*a*, has phase voltages as follows: $E_A = 2200$ volts; $E_B = 2400$ volts. The phase voltages are 90 degrees apart.

(*a*) Calculate the resultant voltage E_{14}.

(*b*) What is the phase angle between E_{14} and E_B?

5–27. A two-phase, three-wire system like that shown in Fig. 230 has a current $I_A = 35$ amperes, lagging 20 degrees behind E_A, and a current $I_B = 50$ amperes, lagging 20 degrees behind E_B.

(*a*) Calculate the current I_C in the common wire.

(*b*) What is the phase position of I_C with respect to the voltage E_{14}? With respect to E_B?

6–27. Referring to Prob. 5–27, if the current I_A is in phase with E_A, other conditions remaining the same as in Prob. 5–27:

(*a*) Calculate the current I_C.

(*b*) What is the phase position of I_C with respect to E_{14} and E_B?

7–27. A three-phase generator has a rated voltage for each winding of 250 volts, with a current capacity of 100 amperes. If the windings are properly connected in Y with terminals numbered 1, 2, and 3:

(*a*) Calculate the voltage between 1 and 2, 2 and 3, 3 and 1.

(*b*) If coil 2 were reversed, calculate voltages between 1 and 2, 2 and 3, 3 and 1. Note: Draw complete vector diagrams.

8–27. The alternator of Prob. 7–27 is carrying a balanced load with three currents of 58 amperes each in phase respectively with the voltage E_{13}, E_{35} and E_{51}. (Fig. 231*b*.) Draw a vector diagram of these currents and the line and phase voltages and determine the phase angles between the currents and these voltages.

9–27. A three-phase generator has three windings each with a normal voltage rating of 1320 volts and a current rating of 157 amperes.

(*a*) If the windings were connected in delta, determine the terminal voltage and current per terminal for the machine.

(*b*) Determine these values if the windings were connected in Y.

10–27. If one phase of the alternator of Prob. 9–27*a* were reversed, calculate the voltage in the delta.

11–27. A delta-connected alternator has phase voltages of $E_A = 256$ volts $E_B = 263$ volts, and $E_C = 245$ volts. The three voltages are 120 degrees apart.

(*a*) Calculate the resultant voltage in the delta.

(*b*) If the impedance of each phase is 5 ohms, calculate the current which would circulate in the delta if it were closed.

CHAPTER XXVIII

POLYPHASE CIRCUITS

256. Two-phase, Four-wire System.—As shown in Article 249, Chapter XXVII, the two phases can be treated like two single-phase circuits. If the line voltages are E_A and E_B and the line currents I_A and I_B (Fig. 239) the power is

for phase A, $$P_A = E_A I_A \cos \theta;$$

for phase B, $$P_B = E_B I_B \cos \phi;$$

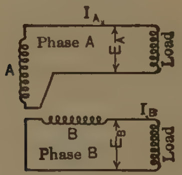


FIG. 239.—Power in a Two-phase, Four-wire System.

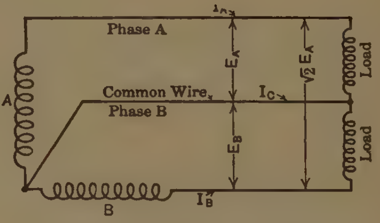


FIG. 240.—Power in a Two-phase, Three-wire System.

where θ and ϕ are the phase angles between current and voltage for each load. The total power is

$$P = P_A + P_B.$$

Where the load is balanced I_A would equal I_B and the power factor ($\cos \theta$) of both loads would be the same so that the total power in this case is

$$P = 2E_A I_A \cos \theta.$$

257. Two-phase, Three-wire System.—For this system the currents in the outside wires (Fig. 240) are determined by the load connected to the corresponding phase and are I_A and I_B in Fig. 240. The power for each phase is

for phase A, $$P_A = E_A I_A \cos \theta;$$

for phase B, $$P_B = E_B I_B \cos \phi;$$

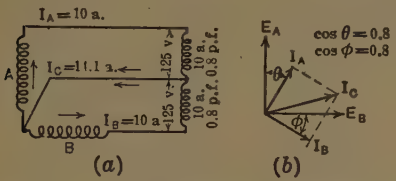
which is the same as for the four-wire system. (See Article 256.) The total power is

$$P = P_A + P_B$$

for a balanced load, $I_A = I_B$, and the total power is

$$P = 2E_A I_A \cos\theta,$$

which is the same as for the four-wire system. The current in the common wire is the vector sum of the currents in the outside wires. For a balanced load this current is $I_C = \sqrt{2}I_A$. (See Article 251.) Where there are several loads connected to the circuit, the resultant currents in the line wires are found by combining vectorially, the currents due to each load.



(*a*) (*b*)

FIG. 241.—Calculation of Power in a Balanced Two-phase System.

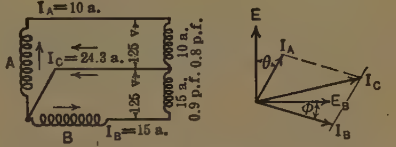


FIG. 242.—Calculation of Power in an Unbalanced Two-phase System.

Example 1.—A balanced two-phase, three-wire system has loads, as shown in Fig. 241 *a*. The power is

$$P_A = 125 \times 10 \times 0.8 = 1000 \text{ watts}$$
$$P_B = 125 \times 10 \times 0.8 = \underline{1000} \text{ watts}$$
$$\text{Total power } P = 2000 \text{ watts}$$

The total power is also

$$P = 2 \times 125 \times 10 \times 0.8 = 2000 \text{ watts.}$$
$$I_A = I_B = 10 \text{ amperes.}$$
$$I_C = \sqrt{2} \times 10 = 14.1 \text{ amperes.}$$

Example 2.—A two-phase, three-wire system has unbalanced loads, as shown in Fig. 242 *a*. The power is

$$P_A = 125 \times 10 \times 0.8 = 1000 \text{ watts}$$
$$P_B = 125 \times 15 \times 0.9 = \underline{1690} \text{ watts}$$
$$\text{Total power} = 2690 \text{ watts}$$

$I_A = 10$ amperes, $I_B = 15$ amperes, $I_C = I_A \oplus I_B$.

H	*V*
I_A, $10 \times \sin\theta = 6.0$	$10 \times \cos\theta = 8.0$
I_B, $15 \times \cos\phi = 13.5$	$15 \times \sin\phi = -6.5$
$H = 19.5$	$V = 1.5$

$$I_C = \sqrt{19.5^2 + 1.5^2} = 19.56 \text{ amperes.}$$

258. Three-phase, Three-wire System.—For a Y-connected, balanced load we have (Fig. 243):

$$I_l = I_1 = I_2 = I_3 \quad \text{and} \quad E_A = E_B = E_C.$$

By definition, the line voltage

$$E_l = E_{13} = E_{32} = E_{21}.$$

Hence

$$E_l = \sqrt{3}E_A. \quad \text{(See Article 253.)}$$

Let $\cos\theta$ be the power factor for each of the three loads; that is, the voltage E_A and the current I_1 are out of phase by the angle θ. The power in phase A is $P_A = E_A I_A \cos\theta$ and the apparent power is $E_A I_A$; similarly for the other two phases. Hence, the total power for a balanced system is

$$P = P_A + P_B + P_C$$

$$= \sqrt{3}E_l I_l \cos\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

and the total apparent power is $\sqrt{3}E_l I_l$. For an unbalanced system, the total power is the sum of the power in the three phases, but in this case the total power cannot be calculated by Equation (80), since the currents in the three line wires would no longer be equal.

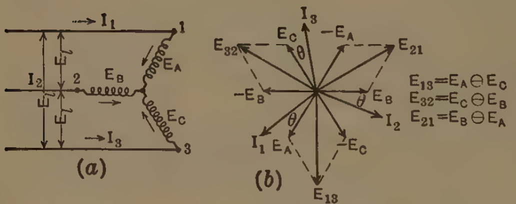


FIG. 243.—Power in a Three-phase, Y-connected System.

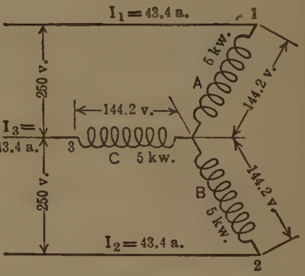


FIG. 244.—Calculation of Power in a Balanced, Y-connected System.

Example 1.—Three loads, each of 5 kw. at a power factor of 0.8 lagging, are Y-connected to a three-phase system (Fig. 244) which has a line voltage of 250 volts.

The phase voltage is

$$E_A = E_B = E_C = 250 \div \sqrt{3} = 144.2 \text{ volts.}$$

The line current in each wire is

$$I_1 = I_2 = I_3 = \frac{5000}{0.8 \times 144.2} = 43.4 \text{ amperes.}$$

The total power is

$$P = 3 \times 5 = 15 \text{ kw.}$$

Also

$$P = \sqrt{3} \times 250 \times 43.4 \times 0.8 = 15{,}000 \text{ watts.}$$

The apparent power is

$$\sqrt{3} \times 250 \times 43.4 = 18{,}750 \text{ volt-amperes.}$$

For a delta-connected, balanced load (Fig. 245) we have the following relations. (See Article 254.)

$$E_l = E_A = E_B = E_C \quad \text{and} \quad I_A = I_B = I_C,$$

$$I_l = \sqrt{3} I_A.$$

If each of the three loads have a power factor of cos θ, then the power for load A is

$$P_A = E_A I_A \cos \theta,$$

and, similarly, for B and C. The total power is

$$P = P_A + P_B + P_C$$

$$= \sqrt{3} E_l I_l \cos \theta,$$

which is the same expression for power as was obtained for the Y-connected system with *balanced* load. It should be noted that cos θ is the power factor of the load and θ is the phase angle between a *load current and the voltage across this load* or is the angle between I_A and E_A. The phase angle between E_l and I_l is not θ as will be seen by referring to Fig. 245 *b*. The apparent power for a balanced load is $\sqrt{3} E_l I_l$, which is the same as for a Y-connected load.

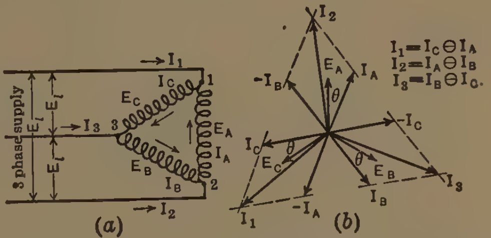


FIG. 245.—Power in a Three-phase, Delta-connected System.

Example 2.—Three loads of 5 kw. each at 0.8 power factor lagging are connected in delta on a 250-volt, three-phase system. The phase currents are

$$I_A = I_B = I_C = \frac{5000}{0.8 \times 250} = 25 \text{ amperes.}$$

The line currents are

$$I_1 = I_2 = I_3 = \sqrt{3} \times 25 = 43.4 \text{ amperes.}$$

The power is

$$P = 3 \times 5 = 15 \text{ kw.}$$

Also

$$P=\sqrt{3}\times 250\times 43.4\times 0.8=15{,}000 \text{ watts.}$$

The apparent power is

$$\sqrt{3}\times 250\times 43.4=18{,}750 \text{ volt-amperes.}$$

Compare with results in Example 1 of this article.

For an unbalanced load the currents in the three line wires will be different. In each case the current in a line wire is the vector difference of the currents taken by the two loads connected to that wire. (See Article 254.)

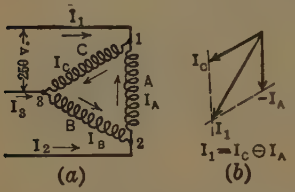


FIG. 246.—Calculation of an Unbalanced Three-phase System.

Example 3.—Three loads respectively 5 kw., 10 kw., and 15 kw. at 0.8 power factor lagging are connected in delta to a 250-volt, three-phase system (Fig. 246). Since the power factor of each load is the same, the phase currents are 120° apart; hence, the line currents can be found by taking the vector difference of the phase currents 120° apart or the vector sum of these currents 60° apart (Fig. 246 *b*).

The phase currents are

$$I_A=\frac{5000}{250\times 0.8}=25 \text{ amperes.}$$

$$I_B=\frac{10{,}000}{250\times 0.8}=50 \text{ amperes.}$$

$$I_C=\frac{15{,}000}{250\times 0.8}=75 \text{ amperes.}$$

The line currents are

$$I_1=I_C\ominus I_A$$

$$=\sqrt{25^2+75^2-2\times 25\times 75\times \cos 120^\circ}$$

$$=90.2 \text{ amperes.}$$

Similarly,

$$I_2=I_A\ominus I_B$$

$$=66.2 \text{ amperes.}$$

$$I_3=I_B\ominus I_C$$

$$=109 \text{ amperes.}$$

The total power is

$$P_e=5+10+15=30 \text{ kw.}$$

With several balanced loads connected to a three-phase system the total line current can be found by first determining the line current for each load and then combining these currents vectorially.

Example 4.—A balanced motor load aggregating 100 kw. at 0.75 power factor (lagging) is connected to a three-phase system in parallel with a balanced lighting load of 15 kw. on each phase. The line voltage is 250 volts.

For the motor load, the line current is

$$I_m = \frac{100{,}000}{\sqrt{3} \times 0.75 \times 250} = 308 \text{ amperes.}$$

For the lighting load the line current is

$$I_l = \frac{3 \times 15{,}000}{\sqrt{3} \times 250} = 104 \text{ amperes.}$$

Since the power factor is alike for each phase, the two line currents can be treated as if they were single-phase currents and combined vectorially. The current I_m therefore lags $\cos^{-1} 0.75 = 41.4°$ behind I_l.

H	V
I_m, $308 \times 0.75 = 231$	$308 \times 0.66 = 203$
I_l, $104 \times 1 \quad = 104$	$104 \times 0 \quad = 0$
$H = 335$	$V = 203$

The total line current is

$$I = \sqrt{335^2 + 203^2} = 392 \text{ amperes.}$$

The resultant power factor is

$$\cos \theta = \frac{335}{392} = 0.854.$$

The total power is, therefore,

$$P = \sqrt{3} \times 250 \times 392 \times 0.854 = 145{,}000 \text{ watts.}$$

Also it is

$$P = 100 + (3 \times 15) = 145 \text{ kw.}$$

The power factor of a balanced three-phase system is the same as the power factor of the loads connected to the different phases. Thus, in Example 2, the power factor of each load is 0.8 and, hence, the system power factor is also 0.8.

259. Three-phase, Four-wire System.—This requires a neutral connection which must be obtained from the neutral point on the generator or transformer windings, or by means of three balance-coils Y-connected to the system. The loads may be connected either Y or delta, but, in this article, we shall consider the former only since delta-connected loads would have no connection with the neutral and would be handled as described in the previous article.

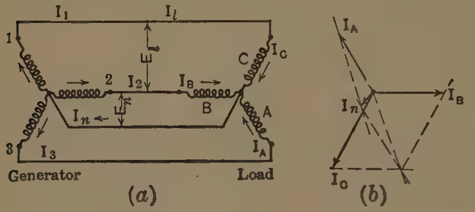
With Y-connected loads in a four-wire system (Fig. 247), the line current equals the current in the load connected to that line. Thus,

$$I_l = I_1 = I_c, \text{ etc.}$$

Also

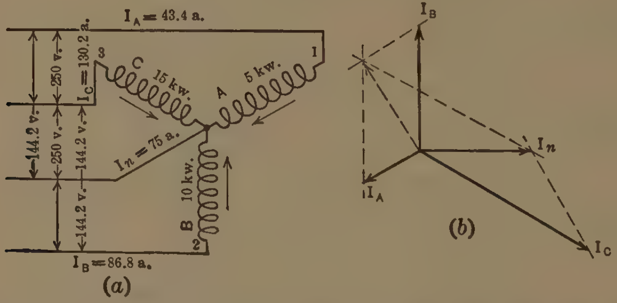
$$E_l = \sqrt{3}E_A.$$

The current in the neutral wire is the combined currents of the three phases. The assumed positive direction of the currents I_1, I_2, and I_3 are shown in Fig. 247. It will be seen that all of these are in the same direction in the neutral. Hence, these three currents should be added vectorially. If the load is balanced, i.e., the line currents and the power factor in each phase are alike, the neutral current will be the vector sum of three equal quantities, 120 degrees apart, and this will be zero. Hence, a three-phase, four-wire system will have no current in the neutral wire so long as the load is balanced. For unbalanced loads, however, there will be a neutral current.



(a) (b)

FIG. 247.—Power in a Three-phase, Four-wire System.



(a) (b)

FIG. 248.—Calculation of an Unbalanced, Three-phase, Four-wire System.

Example 1.—Three loads, respectively 5 kw., 10 kw., and 15 kw. at 0.8 power factor lagging are connected in Y on a three-phase, four-wire system with a line voltage of 250 (Fig. 248). The phase voltage is

$$E_A = E_B = E_C = 250 \div \sqrt{3} = 144.2 \text{ volts.}$$

The phase currents are

$$I_A=\frac{5000}{144.2\times0.8}=43.4 \text{ amperes.}$$

$$I_B=\frac{10,000}{144.2\times0.8}=86.8 \text{ amperes.}$$

$$I_C=\frac{15,000}{144.2\times0.8}=130.2 \text{ amperes.}$$

Since the power factor is the same in each phase, the neutral current is the vector sum of I_A, I_B, and I_C taken 120° apart (Fig. 248 *b*).

	H	V
I_A,	$43.4\times1.0=\ 43.4$	$43.0\times0\ =\ 0$
I_B,	$-\ 86.8\times0.5=-43.4$	$-\ 86.8\times0.866=-\ 75.3$
I_C,	$-130.2\times0.5=-65.1$	$130.2\times0.866=\ 112.6$
	$H=-65.1$	$V=\ 37.3$

$$I_n=\sqrt{65.1^2+37.3^2}=75 \text{ amperes.}$$

The power relations for a balanced load have already been given in Article 258. Obviously, the power in each phase is the same and the total power is

$$P=3P_A=3E_AI_A \cos \theta.$$

The total power is, therefore,

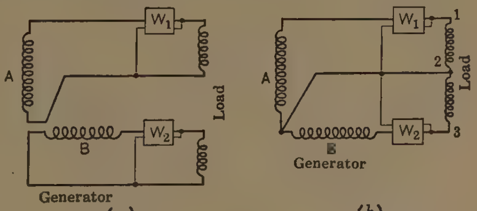
$$P=\sqrt{3}E_lI_l \cos \theta, \text{ as before.}$$

With unbalanced load, the total power is

$$P=P_A+P_B+P_C,$$

and must be found by calculating the power for each load.

260. Power Measurement, Two-phase Circuits.—The connections are given in Fig. 249, which shows a single-phase wattmeter in each phase. The total power for the circuit is the sum of the readings of the two wattmeters. This is true for either balanced or unbalanced loads and is also true for the three-wire system (Fig. 249 *b*), even if a load is connected across the outside wires 1–3. For a balanced load, in either a four-wire or three-wire system, the total power is twice the reading of the wattmeter in one phase, so that only one instrument



(*a*) (*b*)

FIG. 249.—Measurement of Power in Two-phase Systems.

need be used. Frequently, in practice, a two-phase wattmeter is used. This instrument consists of two single-phase elements, combined on one shaft. The connections of the two elements would be the same as shown in Fig. 249, and the instrument would give directly the sum of the power in the two phases.

261. Power Measurements, Three-phase Circuits.—The power in any three-phase system, either balanced or unbalanced, can be measured by means of three wattmeters connected as shown in Fig. 250 *a*. It is apparent that wattmeter W_1 reads the power in load A_1 ($P_A = E_A I_1 \cos\theta$) and similarly for the other loads. Each of these could be looked upon as independent single-phase loads so that the total power is the sum of the readings of the three meters. If the neutral point of the system is not available, the same result can be secured by joining the three ends of the voltage coils together to form a neutral provided the impedances of the three potential circuits are alike. Three wattmeters connected as shown in Fig. 250 *a* are necessary when measuring the power in a four-wire system.

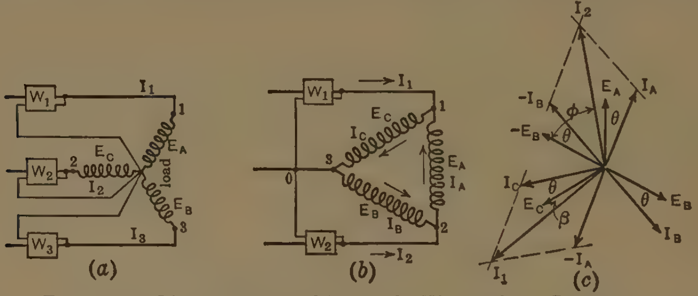


FIG. 250.—Measurement of Power in Three-phase Systems.

The power in a three-phase, three-wire system can be measured by two wattmeters connected as in Fig. 250 *b*. The algebraic sum of the readings of W_1 and W_2 gives the total power for the system, and this is true for a balanced or unbalanced load. The vector diagram for a balanced load having a lagging power factor of $\cos\theta$ is given in Fig. 250 *c*. The three line voltages $E_A = E_{21}$, $E_B = E_{32}$, and $E_C = E_{13}$ are drawn 120 degrees apart. The current in wattmeter W_1 is

$$I_1 = I_C \ominus I_A$$

and in W_2 is

$$I_2 = I_A \ominus I_B.$$

The currents I_A, I_B, and I_C are located on the vector diagram, lagging θ degrees behind the voltages which produce them; that is, voltages E_A, E_B, and E_C, respectively. The resultant currents I_1 and I_2 are then found by taking the vector difference of the phase-currents according to the vector equations above. The voltage terminals of the two wattmeters are connected with the proper polarity so that both W_1 and W_2 give a positive reading when cos θ is greater than 0.50. When this is done, the reading of W_1 is

$$P_1 = E_C I_1 \cos \beta,$$

where β is the angle between the current I_1 and E_C. (See Fig. 250 *c*.) The reading of W_2 is

$$P_2 = E_B I_2 \cos \phi,$$

where ϕ is the phase angle between the current I_2 and the voltage $-E_B$. The total power is the algebraic sum of the readings of W_1 and W_2, or

$$P = P_1 \pm P_2.$$

If the power factor is greater than 0.5, corresponding to $\theta = 60°$, the total power is $P_1 + P_2$. If the power factor becomes less than 0.5, the reading of wattmeter W_2 will reverse, *without any change of connections*, and the total power is $P_1 - P_2$. This can be seen by writing the expressions for P_1 and P_2 in terms of the power factor rather than for cos β and cos ϕ. An inspection of the vector diagram will show that

$$\beta = 30° - \theta \quad \text{and} \quad \phi = 30° + \theta.$$

The current in each wattmeter is the line current I_l which equals I_1 or I_2 since the load is balanced. Also the line voltage $E_l = E_C = E_B$. Hence the wattmeter readings become

$$P_1 = E_l I_l \cos (30° - \theta). \qquad (78)$$

$$P_2 = E_l I_l \cos (30° + \theta). \qquad (79)$$

If the power factor is unity, θ is zero and the two wattmeters each read one-half the total power. If the power factor is 0.5, $\theta = 60°$, P_2 reads zero and P_1 reads the total power. Between 0.5 and unity, P_1 reads higher than P_2 and the total power is $P_1 + P_2$. Below 0.5 power factor, θ is greater than 60 degrees, so that $(30 + \theta)$ is greater than 90 degrees and P_2 reads negative. Hence, the total power is $P_1 - P_2$. When it is not known if the power factor is greater or less than 0.50, a test must be made to determine whether the sum or the difference of the readings should be used. This can be done by moving the connection of the voltage coil of W_1 from point *o* on line 3 and placing it on line 2. The same test is applied to W_2 by moving the voltage lead from *o* to line 1. If either wattmeter reverses when thus tested, the power factor is less than 0.5.

PROBLEMS ON CHAPTER XXVIII

1–28. An alternator is rated at 2500 kv-a., 2400 volts, two-phase, four-wire

(*a*) What is the full-load terminal current?

(*b*) What horsepower load could this machine carry at 80-per cent power factor?

2–28. Referring to Prob. 3–27, calculate the total power when the machine is carrying the current specified.

3–28. Calculate the power for Prob. 5–27, assuming that $E_A = E_B = 2400$ volts.

4–28. Calculate the power for Prob. 6–27, assuming $E_A = E_B = 2400$ volts.

5–28. A two-phase, four-wire, 220-volt feeder carries a total load of 25 kw. at a power factor of 0.80. What is the current in each wire?

6–28. If a two-phase, three-wire system were used to carry the load in Prob. 5–28, what would be the current in each wire?

7–28. A certain two-phase motor is found to take 23 amperes in each wire. The voltage is 550 and the power input is found to be 18.5 kw. What is the power factor?

8–28. What would be the kilowatt input to a 35-hp., 440-volt, two-phase motor when carrying 30 hp.? (Use table, page 406.)

9–28. Given a 5000-kv-a., 11,000-volt, three-phase alternator:

(*a*) What is the rated full-load current per terminal?

(*b*) What load in kilowatts could this machine carry at 0.80 power factor?

(*c*) If the windings of this machine are Y-connected, what is the voltage and current per phase at rated output?

10–28. Determine the rated output of the alternator of Prob. 9–27 when the windings are Y-connected.

11–28. A three-phase, 250-volt feeder carries a load of 150 hp. in three-phase motors. Assuming an efficiency of 85 per cent and a power factor of 0.80:

(*a*) Calculate the power and the apparent power supplied by the feeder.

12–28. A 250-volt, three-phase feeder has a load of 15 kw. in incandescent lamps connected across each of the three phases.

(*a*) What is the current supplied to each of the three loads?

(*b*) What is the current in each wire of the feeder?

(*c*) What is the total power supplied by the feeder?

13–28. A three-phase generator is Y-connected. The voltage to neutral is 100 volts. A 50-ampere load, having a power factor of 0.707 is connected across each pair of terminals. Lagging power factor.

(*a*) Draw a *complete* vector diagram showing the phase voltages and terminal voltages.

(*b*) Using the terminal voltages from (*a*) draw another diagram showing the current in each part of the load and the current per terminal for the machine.

14–28. A three-phase feeder carries a balanced load of 3.5 kw. with a power factor of 0.65. The voltage is 115. What would be the current in each wire?

15–28. (*a*) What would be the kilowatt input to a 50-hp., 220-volt, three-phase induction motor when running at full load?

(*b*) What would it be at $\frac{3}{4}$ load. (Use table, page 406.)

16–28. Measurement of a three-phase, 440-volt feeder shows that there is a current of 55 amperes in each wire and that the power input is 31 kw. What is the power factor?

17–28. Three similar loads taking 100 amperes at 100 volts and a power factor of 1.0 are connected in delta on a three-phase circuit.

(*a*) What would be the current in the line wires?
(*b*) What would be the required voltage between line wires?
(*c*) What would be the total power in the three-phase circuit?

18–28. If the three loads in Prob. 17–28 are connected in Y on a three-phase circuit,

(*a*) What would be the current in each line wire?
(*b*) What would be the required voltage between line wires?
(*c*) What would be the total power in the three-phase circuit?

19–28. A three-phase generator is delta-connected. The voltage of each winding is 100 volts. Three loads, each of 50 amperes and 0.707 power factor lagging, are connected in delta and supplied by the generator.

Draw a vector diagram showing the terminal voltages of the generator and the currents in each part of the load, the line, and the generator windings.

20–28. Three loads of 100, 200 and 300 amperes respectively are connected in delta on a three-phase line. The power factor is unity.

(*a*) Draw a complete vector diagram showing the voltages between line wires and the current in each line wire and in each load.
(*b*) Determine the current in each line wire.

21–28. A 2300-volt, three-phase feeder supplies a balanced lighting load of 50 kw. total, at a power factor of 1 and a three-phase motor load of 150 kw. at 0.80 power factor lagging.

(*a*) What is the total power supplied by the feeder?
(*b*) What is the feeder current?
(*c*) What is the kv-a. for the feeder?
(*d*) What is the power factor of the entire load?

22–28. The alternator in Prob. 7–27 is Y-connected and has a load of 100 amperes connected between each terminal and the neutral.

(*a*) What is the current in each line wire and in the neutral?
(*b*) If coil 2 were reversed and the ampere load between each line wire and the neutral were kept the same, what would be the current in each line wire and in the neutral?

Draw complete vector diagrams.

23–28. If the circuit in Prob. 18–28 were provided with a neutral return,

(*a*) What would be the current in the neutral when the load is balanced?
(*b*) If the load connected between line 1 and neutral were disconnected, what would be the current in each wire?

24–28. A three-phase, four-wire system is used to operate 110-volt lamps connected between the neutral and the outside wires. The lamp load is evenly distributed on the three phases. It totals 3300 watts. The power factor is 1. What would be the current in each wire?

25–28. If a four-wire, three-phase system is used to drive 440-volt induction motors, what would be the voltage between the line wires and neutral?

26–28. A Y-connected alternator has a voltage of 200 volts to neutral. Loads are connected as follows:

Leg 1 to neutral 50 amperes, unity power factor.
Leg 2 to neutral 75 amperes, 0.9 power factor lagging.
Leg 3 to neutral 100 amperes, 0.8 power factor lagging.

(*a*) Draw a complete vector diagram.
(*b*) Calculate the neutral current.

27–28. Referring to the alternator of Prob. 22–28, what would be the readings of wattmeters connected as in Fig. 250*a* when the machine is carrying the load specified in Prob. 22–28? Assume unity power factor.

28–28. Referring to Prob. 26–28, what would be the readings of wattmeters connected as in Fig. 250*a* when the machine is carrying the load specified in Prob. 26–28?

29–28. A three-phase, 220-volt feeder carries 50 amperes in each wire. Two wattmeters properly connected in the circuit read 6330 watts and 2470 watts. (Reversed reading.)

(*a*) What is the total power supplied by the feeder?
(*b*) What is the kv-a. for the feeder?
(*c*) What is the power factor for the feeder?

30–28. A three-phase, 550-volt feeder has a load of 35 kw. of lamps on each of the three phases. The power is measured by two wattmeters.

(*a*) What is the total power supplied by the feeder?
(*b*) What would be the reading of each wattmeter?

31–28. A three-phase, 550-volt feeder has a load of 35 kv-a. on each phase, at a power factor of 0.9 lagging. The power is measured by two wattmeters.

(*a*) What is the total power supplied by the feeder?
(*b*) What would be the reading of each wattmeter?

32–28. If the feeder in Prob. 30–28 had the same kv-a. load as before at a power factor of 0.5 lagging,

(*a*) What is the total power?
(*b*) What would be the reading of each wattmeter?

33–28. If the feeder in Prob. 31–28 had the same kv-a. load on each phase as before at a power factor of 0.3 lagging,

(*a*) What is the total power?
(*b*) What would be the reading of each wattmeter?

PART IV

ALTERNATING CURRENT MACHINERY

CHAPTER XXIX

TRANSFORMERS

262. Ordinarily, a transformer is used to change the voltage or current value in an electrical circuit. Sometimes it is used merely to insulate two electrical circuits while still permitting an interchange of energy between them. The transformer makes long-distance power transmission economically possible. By this device the power may be transmitted at a high voltage and reduced, at the point where the power is to be used, to a value suitable for motors and other devices.

Transformers may be built for any voltage suitable for the other parts of the system. They have been built to deliver voltages as high as 220,000 volts for transmission systems and 1,500,000 volts for testing purposes. Ordinarily, transformers are designed to maintain practically constant terminal voltage at all loads, but there are transformers of the so-called constant-current type which are designed to maintain a constant current for all loads, the terminal voltage varying according to the change in load. These transformers are used in series lighting systems. (See Article 270.) For operating a.c. ammeters, wattmeters, and other measuring devices, current transformers are used. They produce in the meters a current smaller than the current to be measured, but proportional to it, so that this type of transformer performs a function similar to that of the shunt used on d.c. ammeters. The current-transformer also insulates the instrument from the circuit being measured. This is necessary for high-voltage circuits. (See Article 271.)

263. Construction.—The ordinary constant-potential transformer has two windings insulated electrically from each other but wound on a common magnetic circuit made of laminated sheet steel. The transfer of electrical energy from one winding to the other is accomplished magnetically by means of the flux in the coil, which links with both windings. The winding which receives the electrical energy is called the primary winding, and the other winding which receives energy inductively from the primary and delivers it to the load is called the secondary. In some cases a single winding is used for both primary and secondary; such a transformer is called an auto-transformer. If the primary receives a low voltage and the secondary delivers a higher voltage, the device is called a step-up transformer. A transformer which receives a high voltage and converts it to a lower voltage is called a step-down transformer.

264. Voltage Relations.—If an alternating voltage E_1 is impressed on a coil of wire having N_1 turns wound on a laminated iron core (Fig. 251), a current will flow, and the magnitude of this current will depend upon the impedance of the coil. Assuming that the resistance of the circuit is negligible, the current will be limited only by the counter e.m.f. of self-induction set up in the coil by the flux produced in the core, and the current i will adjust itself so that the magnetizing force $N_1 i$ is sufficient to produce enough flux to generate the required counter e.m.f. When the resistance of an inductive circuit is negligible, the current i would lead the e.m.f. of self-induction by 90 degrees (Fig. 251 *b*). The flux produced by this current would be in phase with the current and would have a sinusoidal wave form. Between points 1 and 2 on the curve of flux (Fig. 251 *c*) the total change of flux is 2 $\Phi_{max.}$, and this change occurs in one-half a cycle or $\frac{1}{2f}$ second. Then, according to Article 57, the average induced e.m.f. in one turn of the winding is

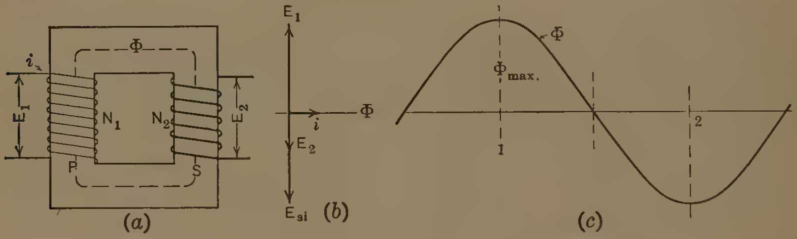


FIG. 251.—Voltage Relations for a Transformer.

$$E_{av.} = \left(\frac{2\Phi_{max.}}{\frac{1}{2f}}\right) 10^{-8} \text{ volts}$$

$$= 4\Phi_{max.} \times f \times 10^{-8} \text{ volts. } \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

But

$$E_{\text{av.}} = E_{\text{eff.}} \frac{0.636}{0.707}.$$

Hence

$$E_{\text{eff.}} = 4.44 \Phi_{\text{max.}} f \times 10^{-8} \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

Each turn of the coil is cut by the same flux; hence, the total effective voltage induced in a coil of N_1 turns is

$$E_{si} = 4.44 \Phi_{\text{max.}} f N_1 \times 10^{-8} \text{ volts.} \quad . \quad . \quad . \quad . \quad (82)$$

This voltage at each instant must be equal and opposite to the impressed voltage E_1 since E_{si} is the only voltage which limits the current in the coil. The impressed voltage E_1 is therefore 180° from E_{si} and is 90° ahead of the current i (Fig. 251 *b*).

If a second coil S is wound on the core, it will be cut by the flux passing through the core at that point. Assume that all the flux produced by the coil P passes through the iron core and cuts the coil S which has N_2 turns. A voltage E_2 will be produced by this flux, and the magnitude of this voltage, according to Equation (81), is

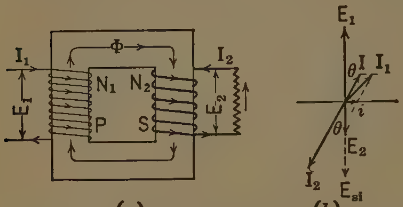
$$E_2 = 4.44 \Phi_{\text{max.}} f N_2 \times 10^{-8} \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Since the voltage E_2 is produced by the same flux which produces E_{si}, these two voltages are in phase (Fig. 251 *b*) and the secondary voltage E_2 is 180 degrees from the primary impressed voltage E_1. From Equations (82) and (83) we can obtain a relation between primary and secondary voltages; thus

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

That is, the ratio of primary to secondary voltage is the same as the ratio of turns for the two windings. In an ordinary transformer the flux cut by the secondary is not exactly the same as for the primary; also there is some voltage loss in the windings when carrying a load. Therefore, at full load the ratio of voltages is not exactly equal to the ratio of the turns of the two coils.

265. Current Relations.—When the secondary is without load (Fig. 251 *a*) the primary has a small "no-load" current i which supplies the no-load losses and produces the flux Φ. Since the circuit is highly inductive, this current lags about 70 to 80 degrees behind the primary voltage E_1. The magnitude of this current varies considerably in different transformers but may be from 6 to 10 per cent of the rated full-load current for average sizes. If the secondary circuit is closed (Fig. 252 *a*) a current I_2 will flow in this circuit, the amount of current being determined by the impedance of the circuit. This current I_2 must flow in such a direction as to oppose the flux Φ which produces it, according to Lenz's Law. The current will thus tend to reduce the flux Φ, which would reduce the counter e.m.f. in the primary and, hence, would allow more current to flow. This current, I_1, would flow in such a direction as to assist the production of the flux Φ (Fig. 252). The core of a transformer carrying a load on the secondary is thus subjected to three magnetizing forces as follows:



(*a*) (*b*)

FIG. 252.—Current Relations for a Transformer.

(*a*) N_2I_2 ampere-turns due to load current in secondary.
(*b*) N_1I_1 ampere-turns due to load current in primary.
(*c*) N_1i ampere-turns due to the magnetizing current in the primary.

The magnetomotive force (*a*) tends to weaken the flux Φ and (*b*) tends to strengthen this flux. These two magnetomotive forces are equal in amount and opposite in their effect, thus leaving the magnetomotive force N_1i to produce the flux Φ. The impedance of the transformer windings is relatively low so that only a small voltage is required to force full-load current through these windings. Hence, the counter e.m.f. produced in the primary at full load need be only slightly less than the counter e.m.f. at no load. This counter e.m.f. depends upon the flux Φ and, if this e.m.f. is practically constant, the flux Φ must be only slightly less at full load than at no load. It is usual to assume that Φ is constant at all loads in the case of a constant-potential transformer. Since $N_1I_1 = N_2I_2$ as stated above, the ratio of current is

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}. \qquad (85)$$

Hence the current is inversely as the number of turns. It might be thought that the primary load current I_1 would produce a large reactive drop in the primary winding since this winding is highly inductive. This would be the case if the current I_1 produced any flux, but it has already been shown[1] that the tendency of I_1 to produce a flux is exactly neutralized by the secondary current I_2, so that neither of these currents would produce an inductive drop in the windings. In an actual transformer we shall see (Article 269) that there is a slight inductive drop in both primary and secondary windings because the two magnetomotive forces are not entirely neutralized. The vector diagram for a transformer with load, neglecting the small current to supply the no-load losses and also neglecting the loss in voltage due to the load currents is shown in Fig. 252. If cos θ is the power factor of the load on the secondary, then I_2 will be out of phase with E_2 by θ degrees. The corresponding primary current calculated by the ratio of turns (Equation 85) will be 180 degrees from this. The total current in the primary would be the vector sum of i and I_1, but, since i is small and is nearly 90 degrees from I_1, the total current in the primary is nearly equal to I_1 and the power factor of the primary is nearly the same as the power factor of the secondary. In fact, when the transformer is loaded, it is customary to assume that the power factor in the primary is the same as the power factor of the load. Looked at in this way, the transformer may be considered as a device which transfers energy from one circuit to another, without altering the power factor, but by simply changing the voltage.

266. Induction Furnace. — The transformer principle is used to advantage where it is impracticable to supply the necessary current directly from the system as in the case of the induction furnace for melting metals. The primary winding (Fig. 253) is wound on an iron core which encloses a circular trough of refractory material containing the metal to be melted. This metal forms a closed secondary winding of a single turn. By suitably choosing the

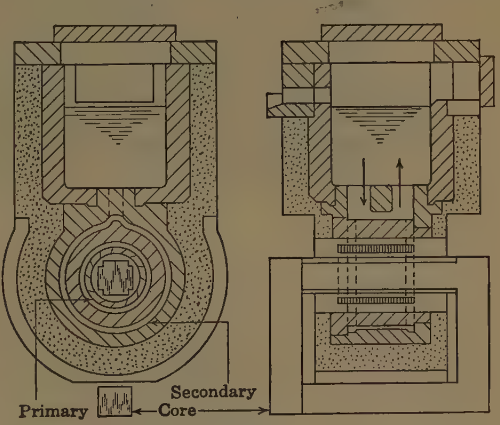


FIG. 253.—Sectional Views of a Repulsion-induction Furnace. Capacity, 75 kilowatt. *The General Electric Co.*

Circulation of the molten metal in the direction of the arrows is caused by the force action of the secondary current.

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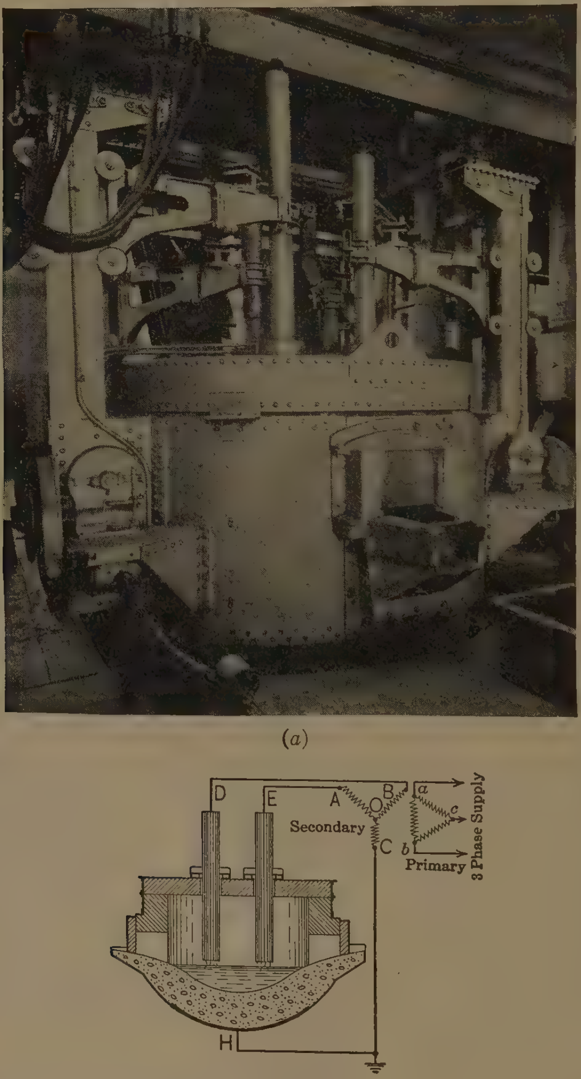
[1] Neglecting the effect of magnetic leakage, see Article 268.

number of turns on the primary, it is possible to induce a secondary voltage sufficient to produce a large secondary current which melts the


(a)

(b)

FIG. 254.—Electric Arc Furnace for Steel Making. Greaves-Etchells type, capacity 3 tons. Transformer rating, 1500 kv-a. *The Electric Furnace Construction Co.*

material. The amount of energy supplied to the secondary can be adjusted by varying the primary voltage.

267. Arc Furnace.—Another type of furnace, requiring a large current at a low voltage, is the arc furnace. Here, also, transformers are used to step down the supply voltage to the proper value. The secondary voltage ranges from 35 to 120 volts depending upon the kind of material melted in the furnace. Usually these furnaces are of the three-phase type rather than single-phase, in order to balance the load on the supply system. One type of steel furnace is shown in Fig. 254. Two terminals of the secondary are connected to the carbon electrodes (two in multiple) and the other terminal is connected to the bottom of the hearth, thus giving a three-phase arc. Regulation of the amount of energy supplied to the furnace is secured by raising or lowering the electrodes, either by hand or automatically.

268. Leakage Reactance.—In the preceding discussion, it was assumed that there was no leakage of magnetic flux; that is, the entire flux produced by the primary winding was assumed to link with and cut the secondary winding. This is never exactly true in any commercial transformer, and, in some designs such as the induction furnace, the limitations of design result in a large leakage of magnetic flux. The actual flux distribution in a transformer is illustrated in Fig. 255, which should be compared with Fig. 252, which shows a transformer with no leakage. The primary ampere-turns N_1I_1 are not entirely neutralized by the secondary, but these turns set up a flux Φ_1 which encircles the primary winding but does not pass through the secondary. Hence, this flux Φ_1 sets up a reactive voltage in the primary winding. Similarly, the current I_2 sets up a leakage flux Φ_2 which does not link with the primary, hence is not neutralized by the primary current, and this produces a reactive drop in the secondary winding. The effect of these leakage fluxes Φ_1 and Φ_2 is the same as if reactances X_1 and X_2 were introduced in the primary and secondary circuits of the ideal transformer (Fig. 252). The diagram for this is shown in Fig. 255 *b*. In addition, the windings have conductor resistances R_1 and R_2, respectively. The vector diagram for an actual transformer, taking into account the loss in voltage due to the resistance of the windings and the reactance due to magnetic leakage, is shown in Fig. 256. A portion of the impressed primary voltage E_p is consumed in overcoming the drop due to the primary resistance R_1

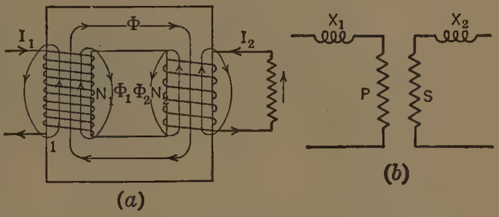


FIG. 255.—Leakage Flux in a Transformer.

and the reactance X_1. This leaves E_1, which must be balanced by the counter e.m.f. of self-induction E_{si} of the primary due to the flux Φ which links with both coils. This flux induces a voltage E_2 in the secondary, according to Equation (83). A portion of this voltage E_2 is lost, due to the secondary resistance R_2 and the leakage reactance X_2, leaving a secondary terminal voltage of E_s. The no-load current of the transformer i_0 is made up of a magnetizing component i and an energy component i_1 which supplies the core losses of the transformer. The total primary current I_p is the vector sum of i_0 and I_1, the latter being the primary current corresponding to the secondary load current I_2 and calculated by the ratio of turns according to Equation (85). The current I_1 varies directly as the secondary load current I_2, but i_0 is practically constant at all loads and amounts to only a few per cent of the full-load current. Consequently, the total primary current I_p is nearly proportional to I_2, and they are nearly 180 degrees apart. The resistance and reactive drops in primary and secondary are only a few per cent of the terminal voltages so that E_p and E_s are nearly constant at all loads and are nearly 180 degrees apart.

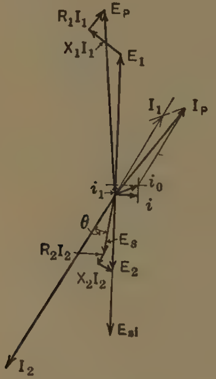
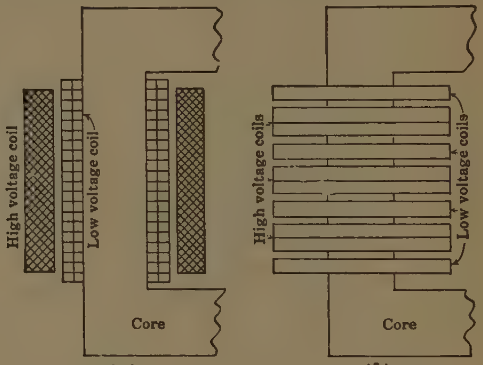


FIG. 256.—Vector Diagram of a Transformer.

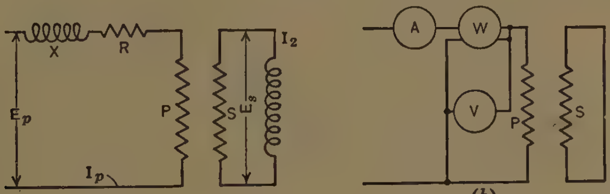


(a) (b)

FIG. 257.—Arrangement of Transformer Coils.

In order that the reactive drop shall be kept at a minimum, thus securing good voltage regulation, it is necessary to reduce the leakage flux as much as possible. This is done by placing the primary and secondary windings as close to each other as is consistent with maintaining the necessary insulation between them. As this leakage would be very high if the primary and secondary windings were on opposite sides of the core, as shown in Fig. 255, this arrangement is never used for a transformer designed to deliver a constant potential at all loads. Instead, the primary and secondary windings are divided into sections which are placed close together, as illustrated in Fig. 257.

269. The voltage regulation of a transformer is the change in secondary voltage from full load to no load, assuming that the primary impressed voltage is maintained constant. The per cent regulation is based on the full-load voltage. For ordinary constant-potential transformers the voltage regulation is usually only slightly greater than the resistance drop. For unity power factor it is about 2.5 per cent for small transformers and 1.1 per cent for large sizes. The per cent regulation at either a lagging or a leading power factor is larger than at unity power factor. With a lagging power factor, the voltage at full load is lower than at no load, while with a leading power factor the full-load voltage is in general higher than the no-load voltage. High voltage transformers have poorer regulation than low voltage transformers of the same rating because of the greater separation between primary and secondary coils.



(*a*) (*b*)

FIG. 258.—Transformer Impedance Measurement.

Transformer regulation can be calculated as if all the resistance and reactance were in one side of the circuit of an ideal transformer having no resistance or reactance drop. Thus X, in Fig. 258, represents the total equivalent reactance and R the total equivalent resistance of both primary and secondary windings of the actual transformer. If I_p is the primary current of the transformer at a given load, then XI_p is the total equivalent reactance drop and RI_p the total equivalent resistance drop in the transformer, due to the actual reactance and resistance of the two windings. The values of X and R which give the same drop as actually occurs in the transformer can easily be determined by experiment (Fig. 258 *b*). One side of the transformer is short-circuited and a low voltage applied to the other winding sufficient to pass full-load current through the winding. When this is done there will be full-load current flowing through the winding which is short-circuited. The total voltage induced in the S winding will be consumed by the secondary resistance and reactance, and this will require only a small flux in the core since this voltage will be only a few per cent of the normal voltage. The flux in the core will set up a counter e.m.f. of self-induction in the primary winding which must be balanced by the impressed voltage. The magnitude of this voltage is determined by the ratio of turns (Equation 84). In addition, to force full-load current through the primary winding requires a voltage to overcome the primary resistance, and reactance and this voltage must be combined vectorially with that required for the secondary. The reading of the voltmeter gives the total voltage required.

Example 1.—A 20-kv-a. transformer, having full-load voltages of 2200 and 220 volts, has the secondary or 220-volt side short-circuited (Fig. 258 *b*) and the primary voltage adjusted so that full-load current flows in the primary or 2200-volt circuit. The reading of the ammeter is 9.1 amperes and the voltmeter 100 volts, which is the total impedance volts for the transformer. The wattmeter reads 268 watts which is practically all copper loss since the impressed voltage is only 100 volts and, therefore, the flux Φ is so small as to make the core loss negligible. The wattmeter reads the loss in both primary and secondary; hence, the equivalent resistance of the transformer is

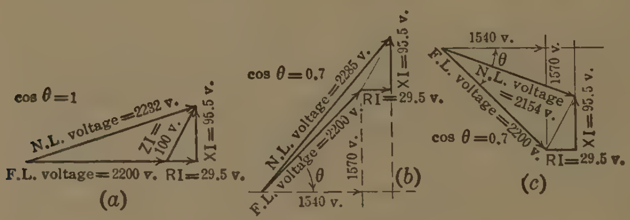


FIG. 259.—Transformer Regulation. (*a*) At unity power factor; (*b*) At a lagging power factor; (*c*) At a leading power factor.

$$R=\frac{P}{I^2}=\frac{268}{9.1^2}=3.24 \text{ ohms.}$$

The equivalent resistance drop is, therefore,

$$E_R=IR=9.1\times 3.24=29.5 \text{ volts.}$$

The equivalent reactance drop is

$$XI=\sqrt{(ZI)^2-(RI)^2}=\sqrt{100^2-29.5^2}=95.5 \text{ volts.}$$

The regulation can be calculated by the same method that was used for the transmission line (Article 232). For unity power factor the vector diagram is as shown in Fig. 259 *a*. The no-load voltage is 2232 volts, and the regulation is

$$\frac{2232-2200}{2200}\times 100=1.45 \text{ per cent.}$$

For a lagging power factor of 0.7, the regulation is calculated as shown in Fig. 259 *b* and is

$$\frac{2285-2200}{2200}\times 100 = 3.9 \text{ per cent.}$$

For a leading power factor of 0.7, the regulation is calculated as in Fig. 259 *c* and is

$$\frac{2154-2200}{2200}\times 100 = -2.1 \text{ per cent.}$$

270. The constant-current transformer is used to supply a series system, requiring a constant current, from a constant-potential source. Series systems are commonly used for street lighting, and for this system the voltage must change as the load changes. In the constant-current transformer the design is such as to give large leakage reactance whereas, for a constant potential transformer, the leakage reactance is made as small as possible. One type of constant-current transformer is shown in Fig. 260. The primary winding is stationary and is supplied from a constant potential source usually of about 2300 volts, single-phase. The secondary winding is movable vertically. When the two coils are close together the leakage reactance is small and the secondary voltage is a maximum. The value of this secondary voltage depends upon the number of lamps for which the transformer is designed. It seldom exceeds 5000 volts. The current is usually from 4 to 6 amperes. If the secondary is separated from the primary the leakage flux increases, and the secondary voltage decreases even when the primary voltage and secondary current are kept constant. The currents in the primary and secondary coils flow in opposite directions; hence, the two coils tend to repel each other. The secondary coil is balanced by counterweights so that, when the desired current is flowing in the secondary winding, the secondary coil will be stationary. If some of the lamps in the series circuit were extinguished, the secondary current would tend to increase, but this increase of current would increase the repulsion between the coils, and the secondary would move away from the primary. This would increase the leakage and reduce the secondary voltage, which would tend to reduce the secondary current. The secondary coil would continue to move

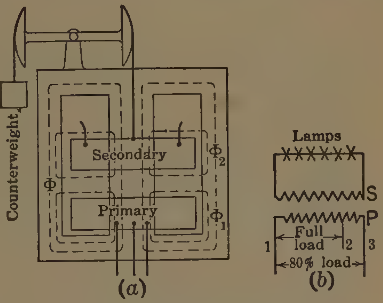


FIG. 260.—Diagram of a Constant-current Transformer.

away until the normal current was again flowing when equilibrium would be restored and the secondary coil would remain stationary. The secondary voltage would then be reduced as required for a smaller number of lamps. Constant-current transformers are very commonly used for supplying series lighting systems instead of series generators which were formerly used for this purpose.

271. The Current Transformer.—In d.c. circuits ammeters are usually provided with separate shunts so that only a small fraction of the load current passes through the meter. The use of shunts for a.c. circuits is in general not satisfactory because of inductive effects, so that a current transformer is used when the current to be measured is too large to pass through the meter or when it is desired to insulate the instrument from a high-voltage line. The primary winding of the current transformer consists of one or more turns of wire placed directly in the circuit to be measured (Fig. 261). The current in this winding is, therefore, proportional to the load current. The secondary winding has a larger number of turns and is connected to the ammeter or other measuring device which is designed to operate at any suitable current, usually about 5 amperes for full scale deflection. The current transformer is designed to operate at a very low-flux density in the core, and the instrument circuit is arranged to have a low impedance. In this way, it is possible to obtain in the meter circuit a current which is closely proportional to the load current. Usually the scale of the meter is marked to indicate the actual current flowing in the load circuit. In the case of the transformer in Fig. 261, the meter would give full scale deflection with 1000 amperes in the line and 5 amperes in the meter. The full scale point on the meter would be marked 1000 amperes. Current transformers have only a low voltage in the secondary, the value being sufficient to force the required current through the instrument windings. If the secondary circuit is opened while the primary circuit is carrying a heavy load, the demagnetizing effect of the secondary current no longer exists, and the flux in the core increases greatly. This induces a high voltage in the secondary circuit, which might puncture the insulation or give an unpleasant or possibly a dangerous shock to anyone coming in contact with it. For this reason, a current transformer should always have its secondary winding short-circuited if the meter is to be removed.

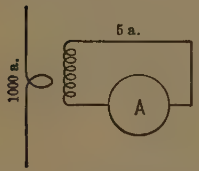


Fig. 261.—Diagram of a Current Transformer.

272. Transformer Losses and Efficiency.—The losses in a transformer are

(*a*) Primary copper loss $= I_p^2 R_1$;
(*b*) Secondary copper loss $= I_s^2 R_2$:
(*c*) Core loss, consisting of
    (1) Hysteresis loss,
    (2) Eddy-current loss.

There is no energy loss due to the primary and secondary reactances, although these cause a voltage loss.

The hysteresis and eddy-current losses are due to the alternating flux in the core and depend upon the frequency and the flux density. The hysteresis loss is given by the equation

$$P_h = K_h f B^{1.6}_{\text{max.}}, \quad \dots \dots \quad (86)$$

where K_h is a constant depending upon the grade of steel used.[2] The eddy-current loss is

$$P_e = K_e f^2 B^2_{\text{max.}}, \quad \dots \dots \quad (87)$$

where K_e is a constant depending upon the thickness of the laminations and the kind of steel used.[2]

In a constant potential transformer the flux is practically constant for all loads (see Article 265); hence, the core losses are constant and can easily be determined by measuring the power input with no load on the secondary. The small primary copper loss at no load can be neglected. The copper loss varies as the square of the load current, and, since the terminal voltage is practically constant, the copper loss can be taken as proportional to the square of the load.

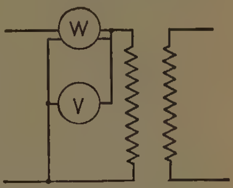


FIG. 262.—Measurement of Core Loss.

The efficiency of a transformer is

$$\frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{losses}}.$$

Example 1.—Referring to Example 1, Article 269, the transformer was tested with the secondary open-circuited and normal voltage of 2200 volts applied to the primary. The power input as measured by the wattmeter (Fig. 262) was found to be 164 watts, which is the core loss, since the primary current flowing at no load is so small that the primary copper loss at no load can be neglected. The full-load copper loss is given by the short-circuit test (Example 1, Article 269, and Fig. 258). Hence, the full-load efficiency at unity power factor is

$$\text{Per cent efficiency} = \frac{20{,}000}{20{,}000 + 164 + 268} \times 100 = 97.89 \text{ per cent.}$$

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[2] These are empirical equations, based on experimental tests and are sufficiently correct for the densities usually employed in practice. The exponents of $B_{\text{max.}}$ vary somewhat with the density.

The copper loss at half load is

$$268(\tfrac{1}{2})^2 = 67 \text{ watts.}$$

The core loss is the same as at full load or 164 watts. The efficiency at half load, unity power factor is

$$\text{Per cent efficiency} = \frac{10{,}000}{10{,}000 + 164 + 67} = 97.74 \text{ per cent.}$$

It is usual to calculate the efficiency at unity power factor. At any other power factor the efficiency would be based on kilowatt input and kilowatt output and would, therefore, be lower than at unity.

273. The all-day efficiency of a transformer may be defined as the ratio of the total energy delivered by the transformer to the total energy supplied to the transformer during twenty-four hours. Usually the energy would be expressed in kilowatt-hours. The all-day efficiency is important where the transformer is connected to the supply for the entire twenty-four hours. The input will be the output plus the core loss for twenty-four hours plus the copper loss for the time during which the transformer delivers energy. All of these quantities should be calculated in kilowatt-hours.

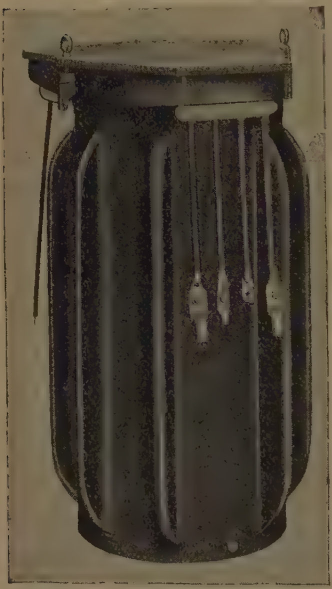


FIG. 263.—Distribution Type of Transformer. Oil-insulated, self-cooling. 50 kv-a. for 2300/115 volts. *The General Electric Co.*

274. Commercial Types of Transformers.—For general power or lighting service, transformers are classified according to the method used for dissipating the heat caused by the losses. **The oil-insulated, self-cooling type** is immersed in oil in a cast-iron or sheet-steel tank and depends for cooling upon the radiation from the surface of the tank. For moderate sizes, tanks having a smooth surface are employed (Fig. 263). For larger sizes, corrugated tanks are used to increase the radiating surface and, for very large units, tanks with external radiators must be used (Fig. 264). By the use of radiators (Fig. 264 *b*) it is possible to design self-cooling units of very large capacity, the present limit in the polyphase type being about 15,000 kv-a. The oil, in which the transformer is immersed, serves the double purpose of carrying the heat from the core and windings to the surface of the tank, and also of furnishing a portion of the insulation between primary and secondary windings. Large-capacity, self-cooling transformers are expensive and bulky and are not so commonly used as the **oil-insulated, water-cooled type** (Fig. 265). This type of transformer is immersed in oil in a sheet-steel tank which has, near the top, a coil of copper or brass pipe through which water is circulated.

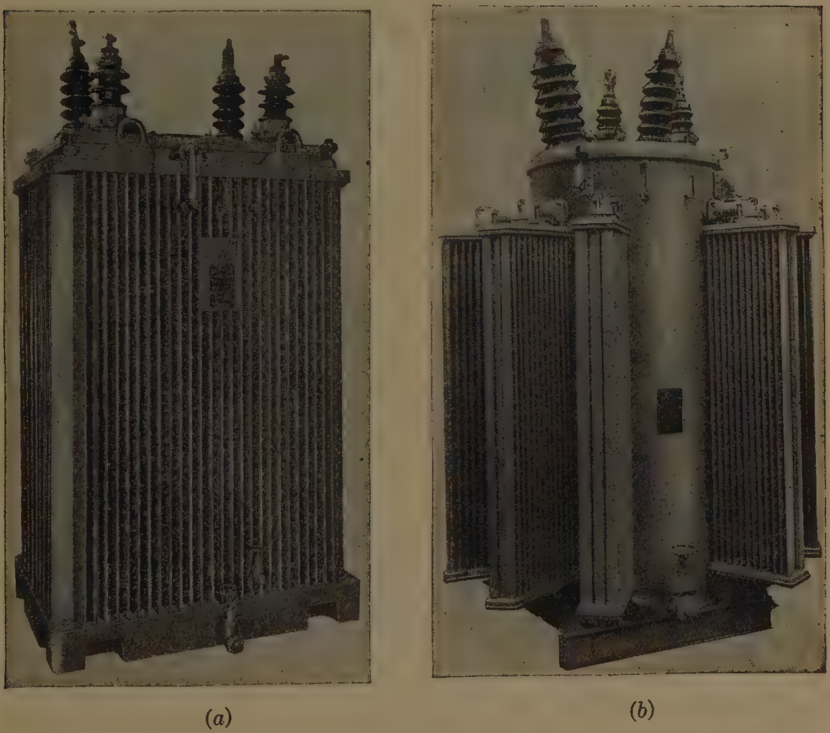
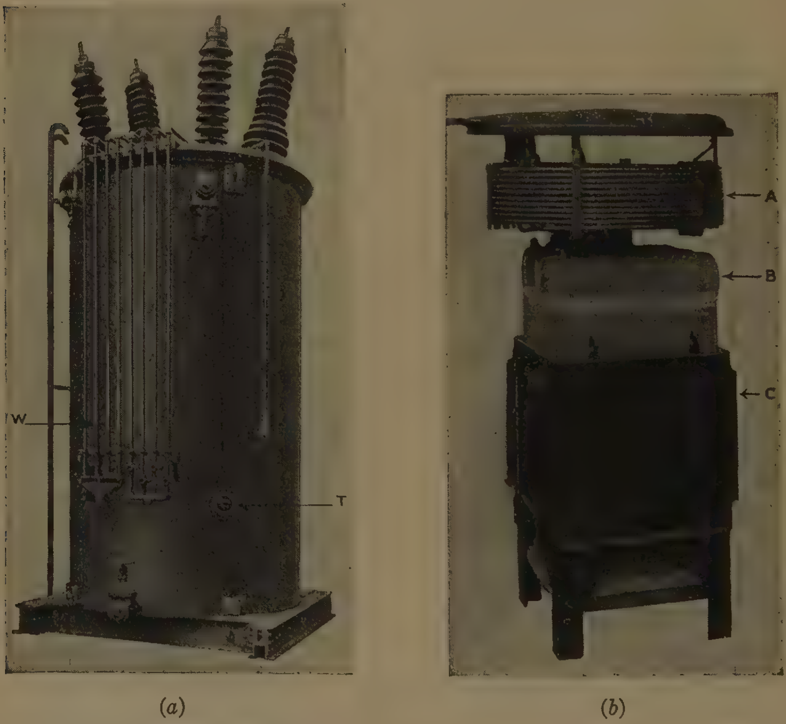

(*a*) (*b*)

FIG. 264.—Oil-insulated, Self-cooling Transformers. (*a*) Corrugated sheet-steel sides cast into top and base; (*b*) Boiler-iron tank with radiators. *The Westinghouse Electric & Mfg. Co.*

This cools the oil and carries off the heat developed in the transformer, due to the losses. A water-cooled transformer can be built in any size that is commercially desirable, the present maximum size being about 22,000 kv-a. Ordinarily water-cooled transformers would not be used in small sizes because of the expense of the necessary water connections. In some cases, transformers are cooled by circulating the hot oil through an external cooling device, but this method of cooling is not extensively used and is open to the objection that it is more difficult to prevent moisture and other impurities from getting into the oil. The method has an advantage where very hard water must be used for cooling, since this water, if circulated through coils in the transformer, would form a scale which it would be difficult to remove. **The air-blast transformer** is cooled by circulating air through the windings and core by means of a fan. The transformer is contained in a thin sheet-steel case.



(*a*) (*b*)

FIG. 265.—Oil-insulated, Water-cooled Transformers. (*a*) Complete transformer in boiler-iron tank; (*b*) Shell type transformer removed from tank. *A*—cooling coils. *B*—windings. *C*—core. *T*—temperature indicator. *W*—water connections. *The Westinghouse Electric & Mfg. Co.*

The oil-insulated, self-cooling transformer is particularly suitable for relatively small capacities or for locations where cooling water is expensive or scarce. It is, however, heavier and about 35 per cent more expensive than the water-cooled transformer[3] in large sizes.

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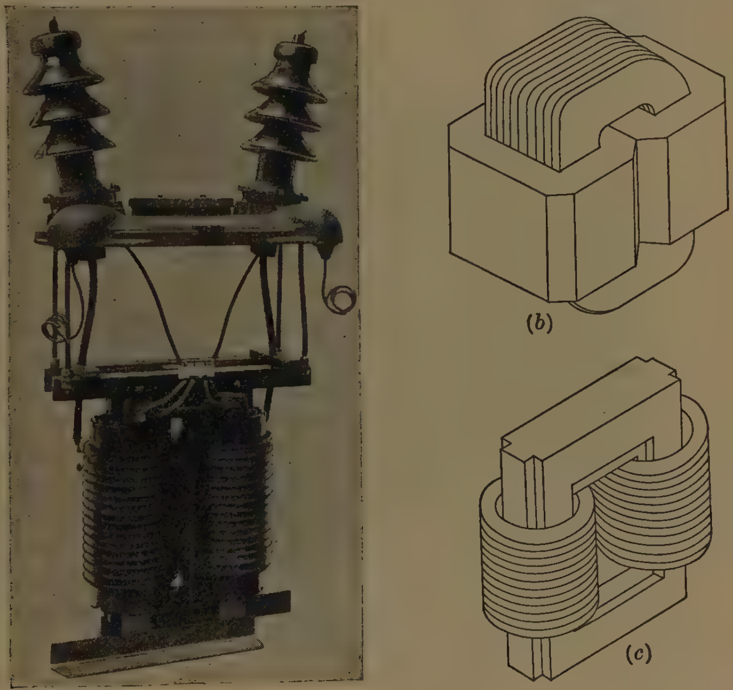
[3] Not including cost of water connections.

Either type of transformer can be designed for any voltage which is required in practice. The air-blast transformer is lighter than the water-cooled type, but it cannot be designed for high voltages, the maximum standard voltage for this type being about 25,000 volts. For moderate voltages, where a supply of clean dry air is available, it has advantages because of the lower cost. Air-blast transformers have been very extensively used for supplying power to synchronous converters in large substations where the complication of the blower outfit is not a serious matter.



(a)

Fig. 266.—Transformer Construction. (a) Small high-voltage core-type transformer removed from tank; (b) Shell-type construction; (c) Core-type construction. *The Westinghouse Electric & Mfg. Co.*

Power transformers are divided into two classes, shell-type and core-type. With the shell-type of construction (Fig. 266 *b*) the coils are assembled in one group on a core having a central leg. With the core-type, the coils are divided into two groups which are placed on opposite legs of a rectangular core (Fig. 266 *c*). In general, the core-type of construction is best for small transformers, particularly if they are designed for high voltage, while the shell-type of construction is used for large transformers. Figures 265 *b* and 267 show shell-type transformers.

For supplying polyphase loads, groups of single-phase transformers have been very commonly used. These are connected either Y or delta, depending on requirements. (See Article 279.) The tendency in modern practice is, however, to use polyphase transformers especially for large-size units, in which case, for a three-phase system, there would be three primary and three secondary windings arranged on a common core and contained in a single case (Fig. 267). The polyphase transformer is lighter, occupies less space, and costs about 20 per cent less than three single-phase transformers of the same aggregate rating. There is the disadvantage, however, that in case of the burn-out of one of the windings of the polyphase transformers, the entire unit would be out of service until repairs could be made; whereas, with three single-phase units, if one fails, the other two could still operate at reduced capacity (see Article 280) or a spare unit of one-third the rating of the bank could be connected in service after only a short delay and the service quickly restored. Modern

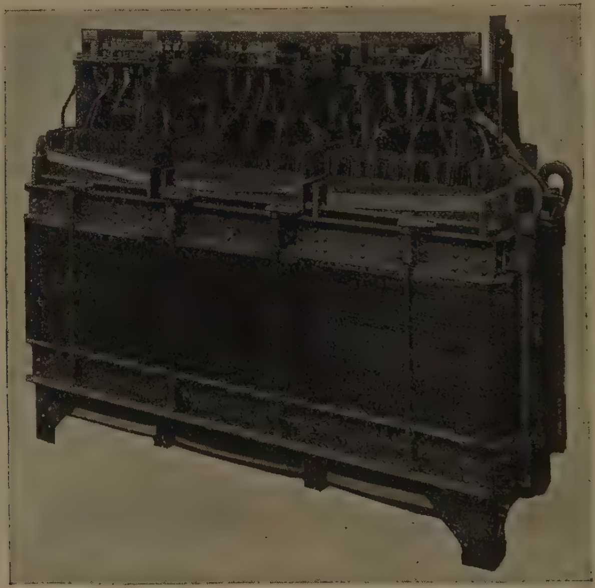


FIG. 267.—Three-phase Transformer Removed from Tank. Oil-insulated, water-cooled, shell-type. Rating, 12,000 kv-a., 88,000/2300 volts, 50 cycles. *The Westinghouse Electric & Mfg. Co.*

polyphase transformers are, however, so reliable that the chances of failure are rather small and, therefore, polyphase units are extensively used at the present time.

It is comparatively easy to design oil-insulated transformers so they can be made water-tight and they are, therefore, frequently installed outdoors without any protection from the weather, with a resulting large saving in the size of building required for a substation. In high-voltage transformers, the oil is depended upon to furnish a considerable proportion of the insulation between primary and secondary. For this reason, care is taken that the oil shall not absorb too much moisture since, if water is present to the extent of two parts in 100,000, or about three drops of water in eight quarts of oil, the oil becomes unfit for use. The oil used in high-voltage transformers is therefore treated periodically by means of a centrifugal separator or a filter to remove any moisture that may have accumulated, and thus prevent an insulation breakdown.

275. Rating and Performance Guarantees.—The copper loss of a transformer depends upon the current, and the core loss upon the voltage and frequency. Therefore, the rating of a transformer is specified in kilovolt-amperes based on a specified terminal voltage and definite frequency. It is standard practice to guarantee that the temperature rise will not exceed 55° C. when the transformer is delivering rated kilovolt-amperes continuously at rated voltage and frequency. No overload guarantee is specified. This temperature rise is based on surrounding air at 40° C. for air-blast and self-cooling transformers and for entering water at 25° C. for water-cooled transformers. If the temperature of the surrounding air or the cooling water differs materially from the standard specified, the temperature rise will be different.[4] If the water supply for an oil-insulated, water-cooled transformer fails, the transformer will rapidly overheat unless the load is reduced.

276. Effect of Variation of Voltage or Frequency.—If a transformer were operated on a voltage higher than normal, but at rated current output, the kilovolt-ampere output would be increased proportionally, but the copper losses would remain the same. The flux density would, however, be increased by an increase of voltage [Equation (82)], and hence the core loss would be increased approximately as the square of the voltage, according to Equations (86) and (87). The magnetizing current would also increase very rapidly. Standard transformers are designed to operate satisfactorily at rated output and a voltage not more than 10 per cent above normal. A

[4] See Standardization Rules of American Institute of Electrical Engineers.

higher voltage would result in overheating the core and might cause failure of the insulation due to the excessive voltage in the windings. Operation at voltages below normal would cause no overheating provided the rated current were not exceeded, but the kilovolt-ampere capacity would be correspondingly reduced. Ordinarily the primary winding of a power transformer is provided with several taps so that the rated secondary voltage can be secured even if the primary voltage varies (see Fig. 268). Thus, if the voltage were 2300, connection would be made to *1* and *6*; if the voltage were 2130, connection would be made to leads *1* and *3*. In either case, the normal voltage of 460 volts would be delivered by the secondary.

If the frequency is varied, and the voltage kept normal, the core losses are changed. An increase in frequency would cause a proportional decrease in the flux density, according to Equation (82). Therefore, according to Equations (86) and (87), the hysteresis loss would decrease and the eddy current loss would not change with an increase in frequency. The total core loss would, therefore, decrease if the frequency were increased. A 25-cycle transformer could be operated successfully on 60 cycles, with a lower core loss, but a 60-cycle transformer, if operated on 25 cycles, would have an excessive core loss and would overheat.

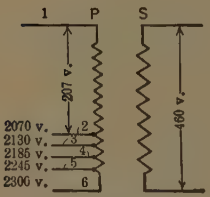


FIG. 268.—Transformer Taps.

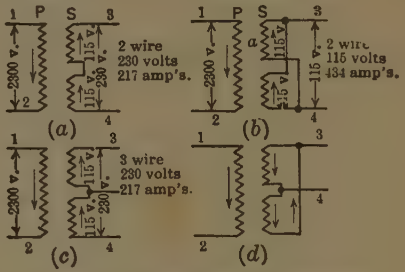


FIG. 269.—Single-phase Transformer Connections.

277. Single-phase Transformer Connections.—Single-phase transformers used by supply companies for distribution purposes ordinarily have two secondary windings and a single primary winding. By this means, it is possible to supply service at several voltages by changing the secondary connections (Fig. 269). When several windings are connected together, it is necessary that the relative polarity of the different coils be known so that the windings may be properly connected in series or in multiple. This is determined, in a diagram, by assigning a polarity to the primary and indicating opposite polarities on all the secondary windings. If two windings are to be connected in series, their voltages must add (Fig. 269 *a*); otherwise, they would neutralize each other, and the voltage between terminals (*3* and *4*) would be zero. Similarly, unless the voltages of coils in parallel are equal and opposed to each other (Fig. 269 *b*), there would be a short-circuit as shown in Fig. 269 *d*. In order that the rated output of a transformer may be secured, the various windings should all operate at normal voltage and all be connected into the circuit. Thus, a 115-volt, two-wire system could be supplied from coil *a* alone (Fig. 269), but the current output would be one-half of the transformer rating.

278. Transformer Connections for a Two-phase System.—For two-phase motors or other polyphase loads, two single-phase transformers of equal size would be connected as shown in Fig. 235. For a four-wire system, single-phase loads would be taken from each of the two phases. For the three-wire, two-phase system, Fig. 236, single-phase loads may be connected between the common wire and either *1* or *3*, but, in some cases, it is found desirable to connect between *1* and *3*, using the correct transformer voltage to suit the different line voltage.

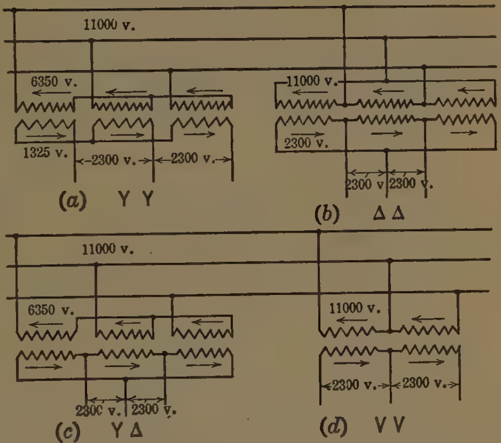


FIG. 270.—Three-phase Transformer Connections.

279. Transformer Connections for a Three-phase System.—For supplying a three-phase load from a three-phase line, single-phase transformers may be connected in several ways, as shown in Fig. 270. If correct three-phase voltages are to be secured on the secondary, the relative polarities of the secondary windings must be known and the connections properly made. Thus, in a Y-connection, the arrows representing secondary polarity should all point away from or towards the neutral; in a delta connection, these arrows should all head in the same direction through the delta. The actual voltage of primary and secondary windings of the transformers will depend upon the kind of connection and the line voltages. These are given in Fig. 270 for a case where it is necessary to step down from a transmission voltage of 11,000 volts, three-phase to a distribution voltage of 2300 volts, three-phase. If transformer windings are Y-connected, the voltage of each winding would be the line voltage divided by $\sqrt{3}$. For delta-connected windings, the transformer voltage would be the same as the line voltage. It is common practice to use three-phase transformers for three-phase lines; and, when this is done, all the connections between phases are made inside the transformer case so that only three primary and three secondary leads are brought outside the case for connection to the system.

280. The V or Open Delta Connection is shown in Fig. 271. By this arrangement, it is possible to supply a three-phase system through two single-phase transformers. In a delta system, the voltage of any phase at any instant is equal and opposite to the sum of the voltages of the other two phases (Article 254). If one of the transformers, *A* in Fig. 271, were removed, transformers *B* and *C* would still maintain the correct voltage and phase relations on the secondary of the system so that three-phase power could still be supplied. Referring to Fig. 271, let I_l be the current in each line wire and E_l the voltage between wires. For delta-connected transformers, each would carry $\frac{I_l}{\sqrt{3}}$ amperes at E_l volts. If one transformer is disconnected, making a V-connection, it is obvious that the remaining two transformers would be heavily overloaded unless the load were reduced, because the transformer current would be I_l instead of $\frac{I_l}{\sqrt{3}}$. With V-connection, the line current must not exceed the rated current of a transformer if it is not to be overloaded. Hence, referring to Fig. 271 *a*, the line current must be $\frac{I_l}{\sqrt{3}}$ and the total load which the two transformers could carry without being overloaded would be

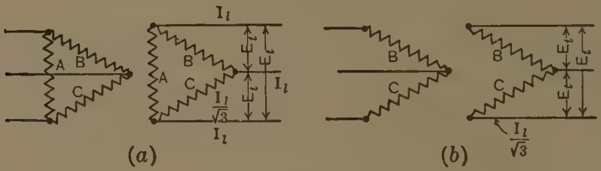


FIG. 271.—Open Delta Connection.

$$P=\sqrt{3}\times\frac{I_l}{\sqrt{3}}\times E_l\div 1000=I_lE_l \text{ kv-a.}$$

But the rating of the transformers is

$$2\times\frac{I_l}{\sqrt{3}}\times E_l\div 1000 \text{ kv-a.}$$

Hence, the load which transformers can carry, when connected V, is

$$I_lE_l\div\sqrt{3}I_lE_l=\frac{1}{\sqrt{3}} \quad \text{or} \quad 57.6 \text{ per cent}$$

of the load which the three transformers could carry when connected delta. Two transformers when connected V, therefore, must operate at only 0.576÷0.667 or 86.5 per cent of their rating if they are not to be overloaded.

Example 1.—Suppose that three 100-kv-a., 2300-volt single-phase transformers are connected in delta to supply a distributing system. The ampere rating of each transformer is 100,000÷2300=43.5 amperes. The line current when each transformer is carrying full load would be $\sqrt{3}\times 43.5=75.3$ amperes. Hence, the total load which could be carried by the transformers would be

$$\sqrt{3}\times 75.3\times 2300=300{,}000 \text{ volt-amperes} \quad \text{or} \quad 300 \text{ kv-a.}$$

If one transformer were disconnected, the remaining two, connected in V, could carry a current of 43.5 amperes without being overloaded, and this would be the line current. The total load which could be carried by these two transformers, is therefore,

$$\sqrt{3}\times 43.5\times 2300=173{,}000 \text{ volt-amperes} \quad \text{or} \quad 173 \text{ kv-a.}$$

Therefore, the two transformers aggregating 200 kv-a. capacity, when connected V, can carry only 173 kv-a. or $\frac{173}{200}\times 100=86.5$ per cent of their rating, without being overloaded.

281. Advantages and Disadvantages of the Y and Delta Connections.—With a delta-connected system if one transformer develops trouble, it can be cut out of circuit, and the three-phase service can be continued by means of the two remaining transformers connected V. The load must, however, be reduced so that it will not exceed 86.5 per cent of the kilovolt-ampere rating of the two transformers. (See Article 280.) If a transformer in a Y-connected bank breaks down it is not possible to operate with two transformers, so that service cannot be restored until a spare transformer has been substituted. For this reason it is generally customary to use the delta connection for distribution systems supplying individual customers.

A transformer intended for delta-connection must be designed for full-line voltage; whereas, each transformer in a Y-connected system must be designed for only $\frac{1}{\sqrt{3}}$ or 57.7 per cent of the line voltage. For very high voltage transmission lines, transformers for Y connection would be cheaper, because they would not have to be insulated for as high a voltage, provided the neutral was grounded. For this reason Y connection of transformers is used in many high-voltage systems. It is common practice on such systems to connect the high-voltage side of the transformers in Y and the low-voltage side in delta.

282. Transformation from Two-phase to Three-phase.—It is impossible to transform from either two-phase or three-phase to single-phase by means of static transformers and still have a balanced load on the polyphase system. It is, however, possible to convert from one polyphase system to another and to have balanced loads. This is accomplished by means of two transformers, using the Scott or T-connection (Fig. 272). One winding of each transformer is connected to the two-phase system, these windings being designed for equal voltages E_2. The other winding of each transformer is designed to produce the voltage E_3 for the three-phase system. A tap at the center of one of these windings is connected to a tap on the other winding which is so located as to give a voltage between this tap 0 and lead 1 of 86.5 per cent of the three-phase voltage E_3. When so connected, the voltages E_{12}, E_{23}, and E_{31} will be equal and 120 degrees apart, thus giving a three-phase system.

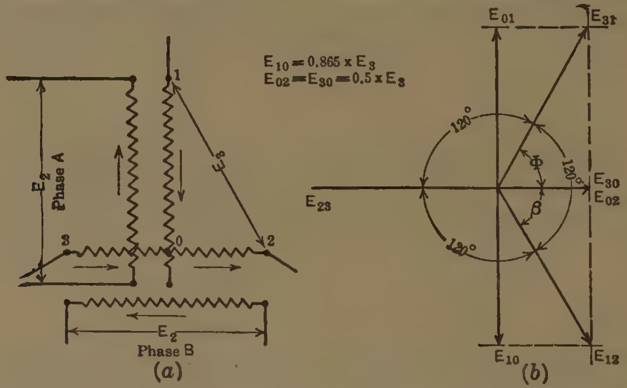


FIG. 272.—Transformation from Two-phase to Three-phase.

This can be proved as follows:

The voltage E_{12} is the resultant of the voltages E_{10} and E_{02} which are 90° apart since they are supplied from the two-phase system. This is also true of E_{31}. Then

$$E_{12} = E_{10} \oplus E_{02};$$
$$E_{23} = E_{20} \oplus E_{03};$$
$$E_{31} = E_{30} \oplus E_{01}.$$

The vector diagram is shown in Fig. 272 *b*. Let E_{10} be $\frac{\sqrt{3}}{2}$ or 86.5 per cent of the three-phase line voltage E_3, and E_{02} be $0.5E_3$. Then numerically the voltage

$$E_{12}=\sqrt{(0.865E_3)^2+(0.5E_3)^2}=E_3;$$

$$E_{23}=E_3;$$

$$E_{31}=\sqrt{(0.865E_3)^2+(0.5E_3)^2}=E_3.$$

Referring to Fig. 271 *b*:

$$\tan\phi=\frac{E_{01}}{E_{02}}=\frac{\sqrt{3}}{2}E_3\div\frac{E_3}{2}=\sqrt{3}.$$

Hence $\phi=60°$ and similarly $\beta=60°$.

Therefore, the three voltages E_{12}, E_{23}, and E_{31} are 120° apart and we have a true three-phase system.

The Scott connection can be used to convert from two-phase to three-phase or the reverse, and if the load is balanced on one side of the transformer bank, it will also be balanced on the other.

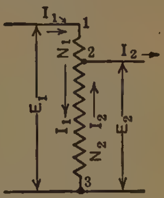


FIG. 273.—Auto-transformer.

283. The auto-transformer has only one winding, a portion of which is used for both primary and secondary. If a single winding (Fig. 273) on an iron core is connected across a source of voltage E_1, then the fall of potential along the winding is proportional to the number of turns, and any voltage less than E_1 can be secured by connecting to the proper points on the winding. Let N_1 and N_2 be the number of turns between points 1–3 and 2–3, respectively. Then

$$\frac{E_1}{E_2}=\frac{N_1}{N_2}=\frac{I_2}{I_1}.$$

When the primary current I_1 flows in the direction of the arrow, the secondary current I_2 will flow in the opposite direction (Article 265). Hence, for the portion of the winding between points 2 and 3 the current is the *difference* of I_1 and I_2. When the transformer ratio is small, E_1 and E_2 are nearly equal; hence, I_1 and I_2 are also nearly equal, so that the portion of the winding between 2 and 3, which carries the difference of the currents, could be made of small cross-section since it would have to carry only a small current. Under these circumstances, the auto-transformer is very much cheaper than the two-coil transformer of the same kilovolt-ampere rating. The auto-transformer has the disadvantage, however, that the primary and secondary circuits are electrically connected and, therefore, it could not safely be used for stepping down from a high voltage, for example, 2300, to a voltage suitable for lamps or motors. The auto-transformer is, however, extensively used for reducing the line voltage in starting induction motors. (See Article 361.)

284. Feeder Regulators.—It is usually necessary to vary the voltage of a.c. distribution feeders in order that a constant voltage may be maintained at the lamps or motors under varying load conditions. Special types of transformers, called feeder regulators, are used for this purpose.

If the feeder current is not too large, a switch-type of regulator can be used. This has a primary winding designed to be connected across the line and a secondary winding with taps which are connected to contacts so that more or less of the secondary winding can be connected in series with the feeder, through the switch *S*. This switch is usually moved automatically by a motor which is controlled by a voltage relay adjusted to give the required variation in feeder voltage.

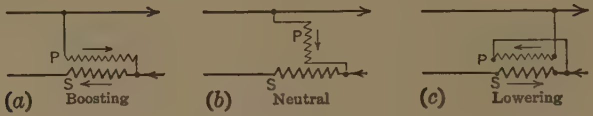


FIG. 274.—Diagram of an Induction Regulator.

The induction regulator is used where the feeder currents are too large to be conveniently handled by a switch. The primary winding is mounted in slots in a cylindrical, laminated steel core which can be rotated on its axis. This primary winding is connected across the line. The secondary winding, which is in series with the line, is placed on a stationary core surrounding the primary (Fig. 274). When the primary and secondary coils have a position shown diagrammatically in Fig. 274 *a* the voltage induced in the secondary is added to the supply voltage. When the primary coil is rotated until it is at right angles to the secondary (Fig. 274 *b*) there is no voltage induced in the secondary, and the feeder voltage is the same as the supply voltage. The primary can also be rotated until the voltage of the secondary is opposed to the supply voltage, thus reducing the feeder voltage when required. By turning the secondary the desired amount, any voltage within the range of the regulator can be secured. These feeder regulators are made either single-phase or polyphase as required. They are sometimes called induction regulators and are not only used for varying a feeder voltage but also for adjusting the voltage supplied to synchronous converters. (See Article 334.) They can be made automatic by means of a voltage relay which controls a small motor arranged to move the primary winding as required.

PROBLEMS ON CHAPTER XXIX

1–29. A 60-cycle transformer has 1500 turns on its primary coil which is connected to a 2200-volt supply. The secondary has 150 turns.

(*a*) What would be the maximum value of the flux in the core?

(*b*) What would be the voltage at the terminals of the secondary winding?

(*c*) If a single turn of wire were placed around the core what would be the voltage induced in this wire?

2–29. Referring to Prob. 1–29, assume that there is a drop at the secondary terminals of 2 per cent at full load.

(*a*) Assuming the primary voltage held constant, how would the number of secondary turns have to be changed to produce 220 volts on the secondary at full load?

(*b*) Calculate the number of secondary turns to meet the requirements of (*a*).

3–29. A 7.5 kv-a., 2300/230 volt, 60-cycle transformer has 1560 turns in the high-tension winding. The area of the core is 9 sq. in.

(*a*) How many turns are there on the low-tension winding?

(*b*) What is the maximum value of the flux in the core?

(*c*) Find the ampere turns per inch required to set up this flux. (Use curve *a*, Fig. 30.)

4–29. The length of the magnetic circuit for the transformer of Prob. 3–29 is 25 inches.

(*a*) What would be the effective value of the magnetizing current required to produce the required flux, assuming a sine wave?

(*b*) What percentage is this of full-load current?

5–29. A 60-cycle, 2300/230-volt transformer is to have a density in the core of 62,000 lines per square inch. Calculate the area of the core, assuming that the effective area of iron is 0.9 of gross area of core. Assume that there are 1500 turns on the high-tension winding.

6–29. Referring to the transformer of Prob. 5–29, calculate the core area if the transformer were to be designed for a 25-cycle circuit with a density of 75,000 lines per square inch.

7–29. The secondary of the transformer in Prob. 1–29 has a load of 10 kw. with a power factor of unity.

(*a*) What is the secondary current?

(*b*) What is the primary current? (Neglect no-load current.)

(*c*) What is the ratio of transformation?

(*d*) What is the power supplied to the primary, neglecting the losses in the transformer?

8–29. If the transformer in Prob. 1–29 carried a load of 5 kw. in the secondary:

(*a*) What would be the load supplied to the primary?

(*b*) What would be the current in the secondary winding?

(*c*) What would be the current in the primary winding?

9–29. Referring to the transformer of Prob. 7–29:

(*a*) What would be the ampere-turns in the primary and secondary windings of the transformer when carrying the load specified in Prob. 7–29?

(*b*) What would be the primary and secondary ampere-turns for the load specified in Prob. 8–29?

10–29. If the transformer of Prob. 1–29 is designed to deliver a rated output of 10 kv-a., calculate the size of wire for primary and secondary allowing 1000 circular mils per ampere.

11–29. A 2200/220-volt transformer carries a load on the secondary (220-volt) side consisting of 5 kw. of incandescent lamps and 4 kw. of arc lamps having a lagging power factor of 0.70.

(*a*) What would be the current in the primary?

(*b*) What would be the power supplied to the primary of the transformer, neglecting the losses?

(*c*) What would be the kv-a. supplied to the primary?

(*d*) What would be the power factor of the primary load, and of secondary load?

12–29. For the transformer in Prob. 1–29:

(*a*) If the cross section of the core were made twice as great, how many turns would be required on primary and secondary to maintain the same flux density at the same voltage?

(*b*) If the transformer were operated at 25 cycles with the number of turns and voltages specified in Prob. 1–29, what would be the total flux?

(*c*) At what voltage should the transformer be operated at 25 cycles to give the same total flux as in Prob. 1–29?

13–29. A certain transformer has a secondary voltage of 460 volts when carrying full load and 465 volts when the load is thrown off and the primary voltage is held constant. What is the percentage regulation?

14–29. A 75 kv-a., 2500/320-volt, 25-cycle transformer has a primary copper loss of 428 watts and a secondary copper loss of 512 watts. A short-circuit test with full-load current flowing in the windings gives a voltage across the 2500-volt side of 45 volts.

(*a*) What is the percentage regulation at unity power factor?

(*b*) What is the regulation at 0.7 power factor lagging?

(*c*) What is the regulation at 0.7 power factor leading?

15–29. A 2500-volt, 75 kv-a., 60-cycle transformer has a normal hysteresis loss of 800 watts and an eddy-current loss of 670 watts. The full-load copper loss is 940 watts.

(*a*) What would be the full-load efficiency?

(*b*) If the transformer were operated at normal full-load current and a voltage 20 per cent higher than normal, what would be the value of the hysteresis loss? The eddy-current loss? The copper loss?

(*c*) What would the full-load efficiency be for conditions as in (*b*)?

16–29. If the transformer of Prob. 15–29 were operated at normal voltage and full load, at a frequency of 25 cycles,

(*a*) What would be the value of the hysteresis loss? The eddy-current loss? The copper loss?

(*b*) What would the full-load efficiency be?

17–29. A 20 kv-a. transformer is rated at 2200/220 volts at 60 cycles. A test shows that the core loss is 160 watts and the full-load copper loss is 280 watts. Neglect the magnetizing current. The power factor is unity.

(*a*) What is the efficiency at full load?
(*b*) What is the efficiency at $\frac{3}{4}$ load?
(*c*) What is the efficiency at $\frac{1}{2}$ load?
(*d*) What is the efficiency at $\frac{1}{4}$ load?

18–29. Referring to the transformer of Prob. 17–29, what would be the efficiency at the loads specified if the power factor were 0.80?

19–29. The transformer in Prob. 17–29 is connected continuously to the supply circuit and carries each day the following loads: 20 kw., 4 hours; 15 kw., 2 hours; 10 kw., 3 hours. What is the all-day efficiency?

20–29. Referring to Prob. 4–29, the core loss is 62 watts.

(*a*) Calculate the no-load current of the transformer.
(*b*) Calculate the power factor at no load.
(*c*) What is the phase angle for the no-load current?

21–29. Three 1100/220-volt transformers are to be operated from 1100-volt, three-phase mains, and supply a three-phase circuit.

(*a*) If the transformers are connected delta-delta, what is primary and secondary voltage across each transformer and between the primary and the secondary mains?
(*b*) If the transformers are connected delta-Y, calculate the voltage as in (*a*).
(*c*) If transformers are connected in Y-delta, calculate voltages as in (*a*).
(*d*) If transformers are connected in Y-Y, calculate voltages as in (*a*).

22–29. With the loads specified in Prob. 21–28, calculate:
(*a*) Size of transformers to supply lighting (use three).
(*b*) Size of transformers to supply power.

23–29. The lighting (Prob. 22–29) is to be supplied as 220–110-volt, three-wire, balanced on the three phases. Power is supplied at 440 volts, three-phase.

(*a*) Specify voltage ratings of the transformers, assuming delta-connection.
(*b*) Draw diagram of connections showing primary and secondary mains and transformers.

24–29. The power load, specified in Prob. 21–28, is to be carried by three, single-phase transformers connected in delta. (See also Probs. 22–29 and 23–29.)

(*a*) What is the line current? What is the phase current in each transformer?
(*b*) If one of the transformers were disconnected and the total load remained the same, calculate the percentage overload on the remaining transformers.

25–29. What would be the kv-a. rating of each of two transformers operated on open delta if they were to carry the power load specified in Prob. 21–28, without exceeding their rated current capacity?

26–29. It is desired to purchase three transformers of sufficient size so that if one burns out, the other two can carry the total power load specified in Prob. 21–28 with a current overload not exceeding 25 per cent. What should be the kv-a. rating of each transformer?

27–29. (*a*) If the transformers cost $6.00 per kv-a., compare the investment, on the basis of the arrangement of Prob. 26–29, with the investment if four transformers were used to carry the load, one transformer being held as a spare.

(*b*) Compare the advantages and disadvantages of the two schemes.

28–29. Two similar transformers rated at 2300/230 volts are to be used to change from two-phase 2300 volts to 230 volts, three-phase.

(*a*) Draw a diagram of the connections for primary and secondary, and calculate the positions of the taps.

(*b*) If the three-phase system carries a balanced load of 500 kv-a. at 0.80 power factor lagging, calculate the currents in the transformer windings, both primary and secondary. Neglect no-load current.

29–29. An auto-transformer is to be built for changing from 2200 to 2300 volts. The load on the 2300-volt side is 100 amperes.

(*a*) What is the total kv-a. supplied to the 2200-volt side?

(*b*) What is the current in the 2200-volt portion of the winding? (Neglect magnetizing current.)

(*c*) What is the current in the remainder of the winding?

(*d*) What is the kv-a. supplied by the transformer to the 2300-volt circuit?

CHAPTER XXX

INDUCTION MOTORS

285. More than 95 per cent of all the electrical energy produced in the United States is generated as alternating current and most of this energy is used in this form. The principal exceptions are the use of direct current for electric traction, some chemical processes, general power and lighting supply in some large cities where the Edison system is used, and special industrial applications where d.c. motors must be used. Therefore we find a.c. motors used more commonly than d.c. motors for general power purposes. Alternating current motors are usually of the polyphase induction motor type, because these machines have no commutator, and, in many instances, no slip-rings or sliding contacts, and are, therefore, one of the simplest types of motors known. Because of this simplicity induction motors will, in general, withstand more severe operating conditions than will d.c. motors.

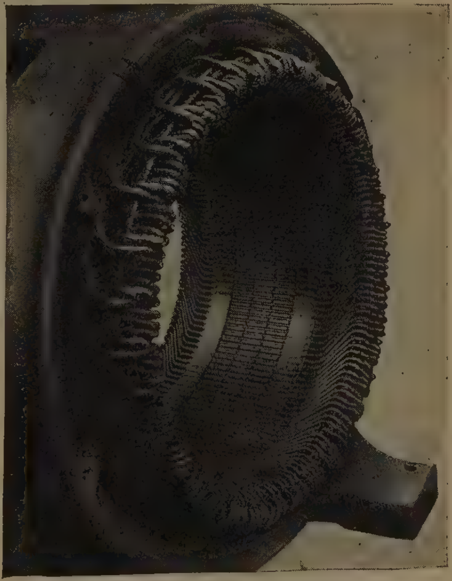


FIG. 275.—Induction Motor Stator. *The Westinghouse Electric & Mfg. Co.*

286. Construction. — The stationary portion of the polyphase induction motor, called the stator or primary, consists of a laminated steel ring with slots in its inside circumference. The winding which is put in the stator slots is of the distributed drum type and has the same general appearance as a d.c. armature winding. The coils forming this stator winding are arranged in two or three groups in order to produce a two- or three-phase winding similar to an alternator winding. For a three-phase motor, the three groups of coils are connected either Y or delta, depending upon which arrangement gives the better design. A three-phase stator is shown in Fig. 275. The stator winding is connected directly to the power supply. Inside the stator is a rotor or secondary which consists of a laminated cylinder with slots on its surface somewhat like the armature core of a d.c. dynamo. These slots contain the rotor winding which may be of either the squirrel-cage or phase-wound type. The squirrel-cage rotor has a winding consisting of bare copper bars filling the slots. These bars are connected together at each end by rings of copper, brass, or other alloy (Fig. 276). The joint between these bars and the end rings is commonly made by electrically welding the two together, although in some machines the rotor bars and end rings are all cast in one piece. No insulation is required on the rotor windings of squirrel-cage motors because the voltage in the conductors is very small. The phase-wound or slip-ring type of rotor has coils forming a drum winding like that of a direct-current armature. The coils are connected to form a polyphase winding like that of an alternator. While it is possible to use either a two- or three-phase arrangement of the rotor conductors, it is customary to use the three-phase arrangement for either a two- or three-phase stator. In most phase-wound rotors the three leads are connected to three slip-rings so that an external circuit may be connected to the rotor winding (Fig. 277). A motor of this type is commonly called a slip-ring induction motor. Small size motors are sometimes provided with phase-wound rotors

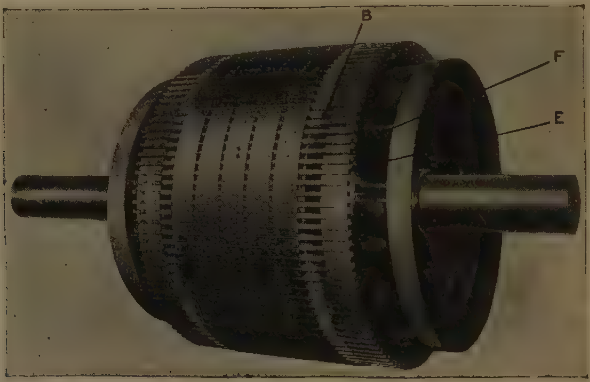


Fig. 276.—Squirrel-cage Rotor. *B*—bar winding. *E*—end rings. *F*—fan for cooling winding. *The Westinghouse Electric & Mfg. Co.*

which have no slip-rings, the rotor winding being connected to a resistance and switch which is mounted on the rotor core and revolves with it. For either a squirrel-cage or phase-wound type of motor, there is no *electrical* connection between the stator and the rotor windings. The energy taken from the supply is delivered to the stator and is introduced into the rotor circuit by induction, through magnetic action, in much the same manner as in the static transformer. The air gap, or clearance between stator and rotor core, is made smaller than for d.c. machines and is, in fact, determined by the requirements for mechanical clearance, allowing for reasonable bearing wear. A small air gap is essential for good performance characteristics.

287. The Revolving Field.—In Fig. 278 is shown a stator winding consisting of two coils, A and B, set at right angles to each other. If direct-current were passed through winding A alone, a two-pole field would be produced with its axis horizontal along the line *gh*. If alternating current were passed through winding A alone, there would be a two-pole alternating field along line *gh*. Similarly, with direct-current in winding B alone, a two-pole field would be formed along the line *ef* or at right angles to the field produced by winding A. With alternating-current in B alone, there would be a two-pole alternating field along the line *ef*. If direct-current were passed through both windings at the same time, each would tend to produce a magnetic field, but they would combine, giving a resultant two-pole field crossing the air gap. The position of this resultant field would depend upon the relative strength of the current in the two windings. If the currents were equal in both windings, the resultant field, which would be two-pole, would be at an angle 45 degrees with the axis of the coils (Fig. 278 *b*). If the current in A were larger than in B, the resultant field would be less than 45 degrees with respect to the horizontal axis *gh*; whereas, if B were stronger, the angle would be greater than 45 degrees. Whatever the values of direct-current in

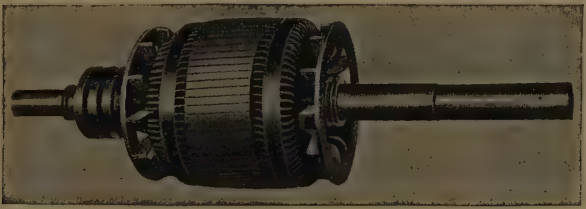


FIG. 277.—Phase-wound Rotor with Slip-rings. *The Westinghouse Electric & Mfg. Co.*

these windings, the resultant field would be two-pole, would be stationary, and could be located by means of a compass or iron filings. If these two windings were connected to a two-phase constant-potential supply, the currents which would flow in the two windings would be 90 electrical degrees apart as represented by the curves (Fig. 278 *c*). These currents would produce a magnetic field which would pass through the rotor and stator core. At the point in a cycle represented by position 1 (Fig. 278 *d*) the current in winding *A* would be a maximum and in *B* would be zero. Hence, the field pro-

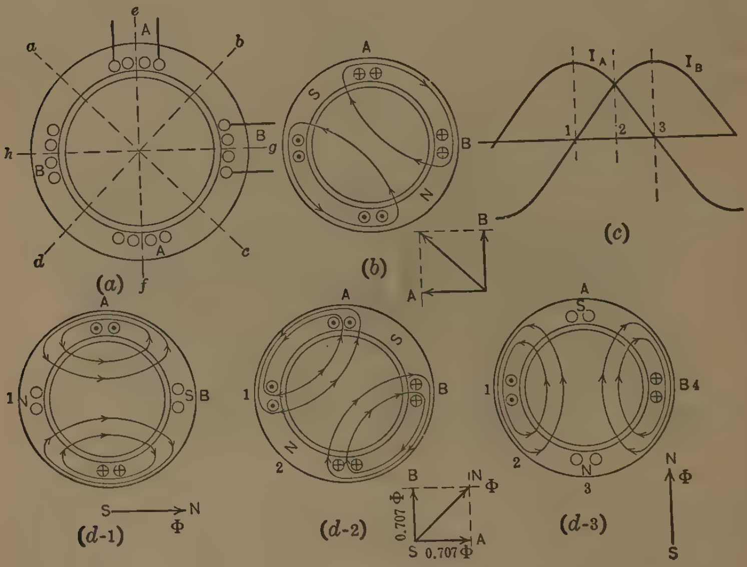


FIG. 278.—Production of a Revolving Field.

duced by the current in *A* would be located horizontally and would be a maximum, because I_A is a maximum. At position (3) in the cycle the current in winding *A* would be zero and that in *B* a maximum; therefore, the field would be a maximum in a vertical direction. At point (2) in the cycle there would be current in both phases which is equal to $I_{\max.}\sin 45°$ or $0.707 I_{\max.}$ Hence, the flux produced by each coil would be $0.707\Phi_{\max.}$ The flux produced by *A* would be horizontal and that produced by *B* vertical, and these two fields would combine to give a resultant flux at 45 degrees with the horizontal (Fig. 278 *d*–2). This flux would be

$$\sqrt{2\times 0.707^2}\times\Phi_{\max.}=\Phi_{\max.}$$

Hence, the strength of the resultant field which passes across the air gap would be the same whether produced by winding A, winding B, or by the two combined. By applying the same reasoning it can be shown that after one-half a cycle or point (4) the direction of the resulting field would be horizontal and opposite to the position (1); that is, the resulting flux in the air gap would have traveled from position (1) (Fig. 278) to point (4) during one-half cycle. For the next half cycle, it would travel from (4) back to (1) in a counter-clockwise direction. Hence, with a two-phase winding arranged as shown, the resulting magnetic field is of *constant* strength and moves one complete revolution around the stator core in one cycle. These windings, therefore, produce what is called a revolving field. For the particular arrangement of windings shown in Fig. 278, this resulting field *has only two poles.* The speed of this field would depend upon the frequency; thus, for a 60-cycle supply, the flux would move around at the rate of 3600 r.p.m. It should be noted that no part of the stator, either core or windings, rotates, but the flux produced by these windings travels around the air gap between rotor and stator. The field produced by these windings is the same as would be produced if the rotor were surrounded by a bipolar magnetic field produced by two poles excited by direct current and these two pole pieces were caused to rotate mechanically (Fig. 279). It should be noted that each of the two windings which were connected to the two-phase supply was arranged to produce a two-pole field when either is acting alone (Fig. 278). When they are both connected to the supply at the same time, the resultant field still has two poles. A two-phase motor thus arranged would be called a two-pole motor because that is the number of poles for the revolving field. A motor may be arranged to produce more than two poles by so connecting the conductors in each phase of the winding that they produce the required number of poles. Thus, if the coils of phase A (Fig. 278 a) are arranged to span 90 mechanical degrees instead of 180 this phase, acting alone, would produce four magnetic poles. Winding B would be similarly arranged and so located that the center line of winding B would be 90 electrical

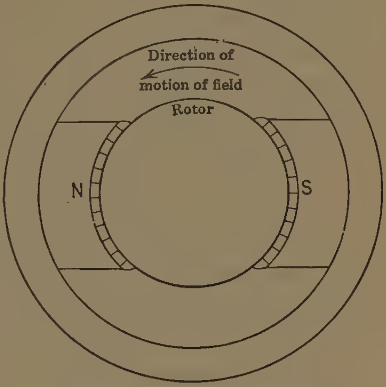


FIG. 279.—Principle of the Rotating Field.

degrees from winding *A*. If these windings were connected to a two-phase supply, a resultant field of constant strength would be produced the same as for Fig. 278, but this field would have four poles instead of two. The speed of a two-pole field on a 60-cycle supply is 3600 r.p.m.; hence, the speed of the four-pole field would be half that or 1800 r.p.m. Thus, by changing the windings, any number of poles can be produced, and the speed of the revolving field correspondingly reduced.

If the connections of one phase of the two-phase machine of Fig. 278 were reversed, this would reverse the relative polarity of the field produced by this phase. The strength of the revolving field would not be changed, but the resulting field would rotate in the opposite direction. This is the method employed to reverse the direction of rotation of an induction motor.

If a rotating field is to be produced by means of a three-phase supply, three groups of conductors are required on the stator, and these would be connected either Y or delta as is done with an alternator. Each of the three windings would be so arranged that, when acting separately, they would produce the required number of poles. When all three are properly connected to a three-phase supply, a revolving field of constant strength is produced in the same manner as for a two-phase winding. The number of poles of this revolving field would be the same as the number of poles produced by each of the three windings acting independently as was the case for the two-phase system. The strength of this revolving field would be 1.5 times the maximum strength of field produced by a single winding. In order to reverse the direction of rotation of a field produced by a three-phase winding, any two of the three leads are interchanged. A 60-cycle, three-phase motor wound for two poles per phase would be called a two-pole motor and would run at 3600 r.p.m. the same as a two-pole, two-phase motor.

288. Torque.—If a rotor, which is blocked so that it cannot turn, be acted upon by a revolving field produced by either a two- or three-phase supply, this revolving field will cut the rotor conductors at a speed which depends upon the number of poles produced by the stator winding. The revolving field will, therefore, induce an alternating e.m.f. in the rotor winding of a frequency which is the same as the frequency of the supply connected to the stator.

Example 1.—A two-pole field produced by a 60-cycle supply would revolve at 3600 r.p.m. and would induce in the rotor voltages having a frequency of 60 cycles. A four-pole, 60-cycle field would revolve at 1800 r.p.m. but would have to travel only one-half as far to produce one cycle; hence the rotor frequency would still be 60 cycles.

The flux density in the air gap due to the revolving field is not uniform, but is higher at the center of each pole of flux than at the two edges. Hence, the e.m.f. induced in a rotor conductor is the highest when this conductor is in line with the center of the revolving flux, line *xy* in Fig. 280 *a*. This e.m.f. induced in the rotor of a squirrel-cage motor causes currents to flow in the rotor conductors since they are short-circuited at each end. The strength of this current depends upon the induced e.m.f. and the resistance and reactance of the rotor winding. The rotor current will, however, lag behind the e.m.f. producing it, due to the inductive reactance of the rotor. Therefore, at a particular instant, when rotor conductors 1 and 2 (Fig. 280) have a maximum value of e.m.f. induced in them, the conductors which have the maximum current are not the same but are, for example, bars 3 and 4 displaced θ degrees behind bars 1 and 2. The axes of maximum current *mn* and of maximum e.m.f., *xy* move at the same speed as the revolving field, and, therefore, the angle θ between these two axes remains constant and is the same as the phase angle between the e.m.f. and current in the rotor winding. Due to this rotor current, there are forces acting on the rotor conductors. Referring to Fig. 280*b*, the rotor conductors between *q–o* and *r–p* produce forces tending to turn the rotor clockwise; whereas, the remainder of the conductors, between *p–q* and *o–r* tend to turn the rotor counter-clockwise. The latter groups of conductors produce a stronger force than the former so that the resultant turning effect would tend to make the rotor revolve in the same direction as the rotating field. If the resistance of the rotor winding were increased the angle of lag θ would be small and the axes of current (*mn*) and e.m.f. (*xy*) would nearly coincide (Fig. 280 *c*). In this case, for a given strength of current, the resulting force tending to turn the rotor in the direction of the

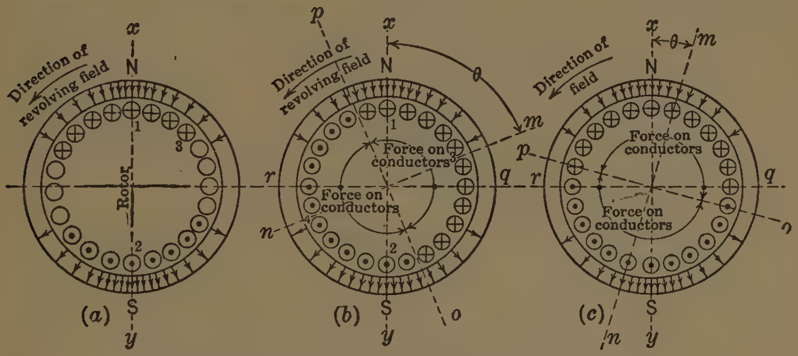


FIG. 280.—Effect of Rotor Power Factor upon Torque.

rotating field would be greater, since the opposing force is less. Hence, anything which will reduce the angle θ will produce a greater torque for a given rotor current.

If the rotor is free to revolve, it will turn in the same direction as the rotating field and tend to reach the same speed. As the rotor speed increases, however, the rotor conductors are cut by the revolving field at a slower rate, and, hence, the induced e.m.f. in the rotor becomes less as the rotor speed increases. In fact, if the rotor should turn at exactly the same speed as the revolving field, it would not cut the rotor conductors at all, there would be no induced e.m.f., no rotor current, and, hence, no turning torque. In other words, an induction motor must always run at a speed less than that of the revolving field, even at no load, since a small torque is required to overcome the friction and other losses in the rotor. As the external load on the motor is increased, more rotor current is required to produce the necessary torque and this must be obtained by slowing down the rotor so that a higher rotor e.m.f. will be induced, thus causing a larger rotor current. In the ordinary squirrel-cage motor, a change of rotor speed of about 5 per cent is sufficient to produce the necessary rotor current to carry full load.

It has already been stated that, at standstill, the rotor frequency is the same as the frequency of the supply. If the rotor were to operate at exactly the speed of the revolving field, the frequency in the rotor would be zero. For any speed between these limits, the frequency is less than the supply frequency, becoming lower as the rotor approaches the speed of the revolving field.

Example 2.—The field of a four-pole, 60-cycle induction motor would turn at 1800 r.p.m. At standstill, the rotor frequency would be 60 cycles. At no load the rotor speed would be about 1795 r.p.m. and the rotor frequency would be $\frac{5}{1800}\times 60 = 0.17$ cycle per second. At full load, the speed would be about 1700 r.p.m. and the rotor frequency would be $\frac{100}{1800}\times 60 = 3.3$ cycles per second.

289. Synchronous Speed and Slip.—The speed of the revolving field is called the synchronous speed of the motor. It has already been shown (Article 288) that the synchronous speed depends upon the frequency of the supply and the number of poles for which the stator is wound. The slip is the difference between the synchronous speed and the actual speed of the rotor and is usually expressed in per cent, which is calculated as follows:

$$\text{Per cent slip } s = \frac{\text{synchronous speed} - \text{rotor speed}}{\text{synchronous speed}} \times 100. \quad (88)$$

Example 1.—A four-pole, 60-cycle induction motor runs at 1795 r.p.m. at no load and 1700 r.p.m. at full load. The no-load slip is

$$s=\frac{1800-1795}{1800}\times 100=0.27 \text{ per cent}$$

and the full-load slip

$$s_1=\frac{1800-1700}{1800}\times 100=5.6 \text{ per cent.}$$

The rotor frequency increases with the slip and is proportional to it. (See Article 288.) Hence, rotor frequency = stator frequency × per cent slip.

290. Starting Conditions.—When the rotor is stationary and its resistance is unchanged, the impedance of the rotor circuit is approximately constant for different values of the rotor current. Therefore, the current in the rotor will be proportional to the e.m.f. induced in the rotor. This e.m.f. is, however, proportional to the strength of the rotating field. But this magnetic field is approximately proportional to the voltage applied to the stator, hence, we can say that the rotor current is approximately proportional to the stator voltage. In fact, when the rotor is stationary, its windings can be considered as acting like the short-circuited secondary of a transformer. The rotor current reacts on the revolving field tending to weaken it and thus allow more current to flow in the stator circuit to balance the rotor current, just as the primary current of a transformer increases as the secondary current increases. Hence, the starting current taken by the motor is proportional to the voltage impressed on the stator. There is magnetic leakage in the induction motor air gap as in a transformer; therefore, the strength of the revolving field decreases somewhat as the rotor and stator currents increase. For a well-designed motor with small air gap, the field is, however, approximately constant at all loads.

Example 1.—A 200-hp., 25-cycle three-phase motor takes a starting current of 8.25 times full-load current when normal voltage is impressed on the stator. Calculate the starting current for 80 per cent of normal voltage.

$$I=0.80\times 8.25=6.6 \text{ times full-load current.}$$

We have seen that the torque produced by the rotor current depends on the strength of the revolving field, the rotor current, and the phase angle between rotor voltage and current (Article 288). With the rotor stationary, the phase angle does not change; hence, the starting torque depends upon the product of rotor current and strength of revolving field. But the strength of this field is proportional to the stator voltage and the rotor current is also proportional to the stator voltage; hence, the starting torque is proportional to the square of the voltage applied to the stator.

Example 2.—The motor of Example 1 produces a starting torque 1.53 times full-load running torque with normal voltage applied to the stator. Calculate the starting torque when 80 per cent of normal voltage is applied. The starting torque is

$$T=(0.80)^2\times 1.53=0.98 \text{ times full-load torque.}$$

291. Running Conditions.—The performance of a squirrel-cage induction motor having a low-resistance rotor is shown in Fig. 281. It will be seen that the slip at full load is small, being only 3.4 per cent, and, furthermore, the slip between no load and full load is proportional to the torque. Hence, the motor speed for any value, up to full load, can be easily calculated if the full-load speed is known.

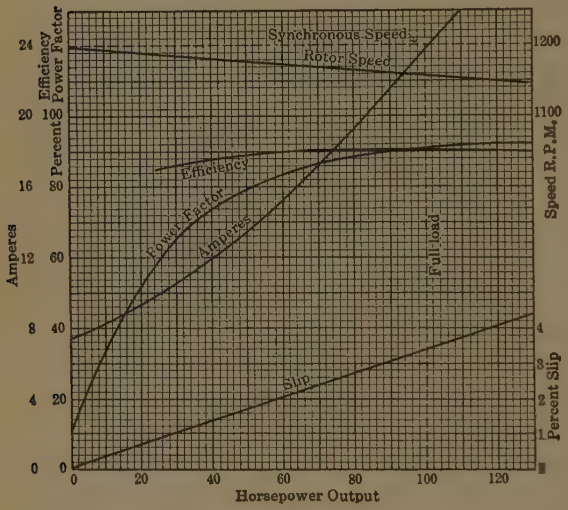


FIG. 281.—Performance of a Squirrel-cage Motor. 100-hp., 2200-volt, three-phase, 60 cycles.

Example 1.—The full-load slip of the motor in Fig. 281 is 3.4 per cent. Hence the full-load speed would be 1159 r.p.m. The slip at half load would be one-half that at full load or $0.5\times 3.4=1.7$ per cent. The half-load speed would therefore be 1179 r.p.m.

The reason for this change in speed will now be considered. In Article 288 it was shown that if the rotor turned at exactly synchronous speed there would be no induced e.m.f., no rotor current, and, hence, no torque. Therefore, even at no load there must be enough slip to produce the small rotor current required to drive the rotor. The resistance of the rotor circuit is small, and, at no load, the slip is also very small; hence, the frequency of the rotor current is very low and the rotor reactance is negligible. Therefore, only a small rotor e.m.f. is required to force the no-load current through the rotor, and this can be secured by a small slip. If the load were increased, this small current would not produce sufficient driving torque; hence, the rotor would slow down. When this happens, the revolving field cuts the rotor conductors at a higher rate, and a larger e.m.f. is induced in the rotor. This produces a larger rotor current and gives a greater torque. Therefore, when additional load is applied to an induction motor it must slow down until the increased rate of cutting, due to the greater slip, produces sufficient rotor current to give the required torque. When this speed is reached the motor will continue to operate at a steady speed until the load changes. With a low resistance, squirrel-cage rotor, a small decrease in speed is sufficient to produce enough current to carry full load; thus, the rotor frequency and also the rotor reactance is small even at full load. Therefore, the rotor impedance is nearly constant, and the slip is proportional to the load from no load to full load. This is shown in Fig. 282 where the torque curve is a straight line from no load to full load. As the rotor current increases, due to increased load, there is a corresponding increase in stator current. (See Article 290.) This is shown in curve (*b*), Fig. 282.

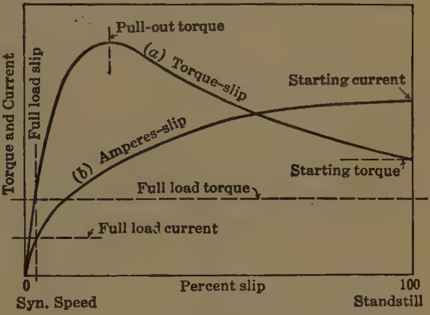


FIG. 282.—Torque and Current Curves for a Squirrel-cage Motor.

For values greater than full load the slip increases faster than the torque (Fig. 282), until finally a maximum value of torque is reached at a slip of 20 per cent for this particular motor. If an induction motor were loaded beyond this maximum torque value, it could no longer carry the load but would slow down and stop. For this reason the maximum torque is sometimes called the " pull out " or " breakdown " torque of the motor. This curvature of the torque curve is due to magnetic leakage at the air gap and to the lower rotor power factor which exists at large values of the slip because of the high rotor frequency. The point where the torque curve cuts the ordinate for 100 per cent slip gives the starting torque for the motor with full voltage applied to the stator. The motor of Fig. 282 gives a pull-out torque 3 times full-load torque at 20 per cent slip corresponding to a speed of 480 r.p.m. The starting torque is 1.5 times full-load torque, and the corresponding starting current is 4.5 times full-load current.

If the rotor resistance of the motor of Fig. 282 were doubled the same current (13.5 amperes) would be required to produce full-load torque, but in order to force this current through the rotor circuit, double the voltage would be required. This must be produced by doubling the slip. This would, therefore, double the rotor frequency and since the resistance is doubled, the power factor of the rotor circuit remains the same and the torque is the same as before (Fig. 283). Hence, for a given torque, doubling the rotor resistance will double the *slip*. This applies to any point on the torque curve. An increase of rotor resistance does not affect the maximum torque but shifts it nearer to the standstill point. In Fig. 284 is shown torque curves for various rotor resistances. There is a certain value of rotor resistance which will produce the maximum torque at starting (curve *d*, Fig. 284).

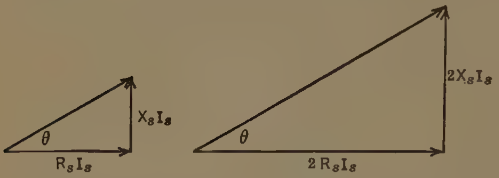


Fig. 283.—Effect of Increasing Rotor Resistance.

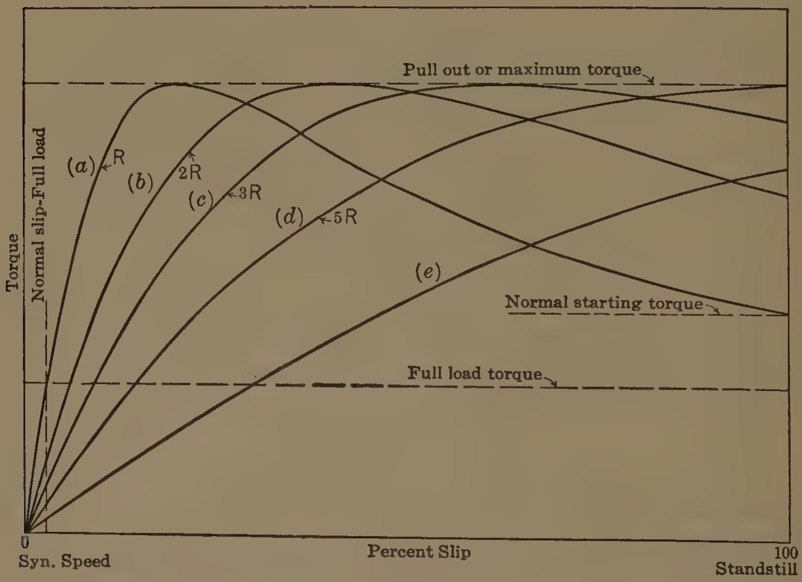


Fig. 284.—Effect of Change of Rotor Resistance upon the Torque Curve.

If a still higher resistance were inserted (curve *e*) the starting torque would be less than the maximum. If the rotor circuit is opened, there would be no current and, hence, no torque.

The reason why a greater starting torque can be secured by inserting resistance in the rotor of an induction motor can be seen by referring again to Article 288 and Fig. 280. For a given rotor current the starting torque can be increased if the phase angle θ of the rotor circuit is decreased. The rotor circuit has a fixed reactance since the frequency is constant; therefore, by increasing the rotor resistance the current and voltage will be more nearly in phase. Thus, Fig. 285 *a* represents the starting performance for a squirrel-cage rotor of low resistance, where the phase angle θ is large and therefore the torque is low. By inserting a higher resistance in the rotor the phase angle becomes less (Fig. 285 *b*) and, therefore, the starting torque is increased. To obtain a high starting torque for a squirrel-cage rotor the end rings are made of higher resistance, while, for a slip-ring motor, an external resistance is inserted. In either case it is possible to produce maximum torque at starting and to secure a starting torque equal to full-load torque with a starting current only slightly higher than full-load current; whereas, a low-resistance type of squirrel-cage machine requires about 4.5 times full-load current for full-load torque at starting. (See Articles 292 and 293.)

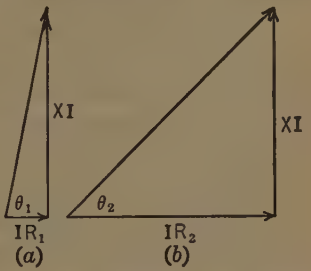


FIG. 285.—Effect on Rotor Power Factor of Changing Resistance in Rotor.

Example 3.—Referring to the motor in Fig. 281, what would be the full-load speed if the rotor resistance were doubled?

The slip with double rotor resistance is

$$3.3 \times 2 = 6.6 \text{ per cent.}$$

Hence the full-load speed would be

$$1200 - (1200 \times 0.066) = 1121 \text{ r.p.m.}$$

The speed regulation of an induction motor is expressed as the per cent change in speed from no load to full load. Since the no-load speed of an induction motor having a squirrel-cage rotor is practically equal to the synchronous speed the regulation of such a motor can be calculated as follows:

$$\text{Per cent speed regulation} = \frac{\text{Synchronous speed} - \text{full-load speed}}{\text{Synchronous speed}} \times 100.$$

Example 4.—A six-pole, 25-cycle induction motor has a full-load speed of 480 r.p.m. The speed regulation is

$$\frac{500-480}{500} \times 100 = 4 \text{ per cent.}$$

292. Performance Characteristics of Squirrel-cage Motors.—The standard motor of this type made for general power applications is essentially a constant speed device giving a speed regulation about the same as the d.c. shunt motor. The speed regulation varies somewhat with the size of motor but is from 1.5 to 3 per cent for large machines and 5 per cent for small machines. The maximum or pull-out torque is not less than 2 times full-load torque.[1]

At starting, the squirrel-cage motor will deliver from 1.5 to 1.1 times full-load torque with full voltage applied, the higher figure being for two-pole motors and the lower for fourteen or more poles. The starting current with full voltage applied is from 4 to 10 times full-load current, the lower figure being for a motor with a small number of poles. The ordinary squirrel-cage motor with low-resistance rotor is, therefore, not well suited for use where a large starting torque is required. A starting torque not greater than full-load running torque could, however, be secured without an excessive motor current. Thus, a two- or four-pole squirrel-cage motor would deliver full-load torque at starting with an applied voltage of $\sqrt{\frac{1.0}{1.5}} = 0.815$ or about 82 per cent of full voltage and would require a current about $0.82 \times 4 = 3.3$ times full-load current. The performance of motors with a larger number of poles is not so good.

The starting torque may be increased by increasing the rotor resistance (see Articles 288 and 291), and any value of starting torque may thus be obtained up to the maximum torque of the motor. High-torque squirrel-cage motors are manufactured in this manner, and have a higher rotor resistance than the standard motor. However, the curves of Fig. 284 show that with high rotor resistance the speed regulation is poor.

Example 1.—Referring to Fig. 284, the speed regulation with normal resistance is 3 per cent. By doubling the rotor resistance the starting torque could be increased from 103 pound-feet to 137.4 pound-feet, but the speed regulation would be 6 per cent.

A squirrel-cage motor designed to give high-starting torque by means of a high-resistance rotor would, therefore, show large speed variations with changes in load. The performance of such a machine is, in fact, very much like that of a d.c. shunt motor with resistance in the armature circuit. The use of high resistance in the rotor also increases the rotor copper losses; hence, the efficiency of the motor is decreased. In general, the use of a high-resistance squirrel-cage rotor is not satisfactory although it is successfully employed in some applications.[1] Where high-starting torque with good efficiency and good speed regulation is required, the slip-ring type would be used instead of the squirrel-cage motor.

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[1] Requirements of The Electric Power Club, an association of the electric motor manufacturers.

293. Performance Characteristics of the Slip-ring Type of Induction Motor.—The disadvantages of a high rotor resistance can be overcome if the high resistance is used only for starting and is cut out when the motor is running. This is accomplished by using the slip-ring type of rotor.[2] Since the additional resistance in the rotor is used only in starting, the running characteristics of the slip-ring motor are much the same as those of the squirrel-cage motor. The speed regulation of slip-ring motors with the starting resistance cut out is approximately 2 to 8 per cent. By inserting the resistance in the rotor it is possible to secure the maximum torque at starting. (See Article 291.) Full-load torque at starting can be secured with a slip-ring motor with about 1.15 times full-load current; whereas, according to Article 292, a squirrel-cage motor requires about 3 times full-load current to produce full-load torque when starting. For this reason, a slip-ring motor is used where heavy starting torque is required or where it is necessary to limit the starting current to a minimum.

There is now on the market a so-called automatic starting induction motor which has two rotor windings, one, a high resistance which is used when starting and another, of low resistance used when running. The low-resistance winding is open-circuited until a predetermined fraction of normal speed is reached, when a centrifugal switch cuts in this winding in such a way that it forms the practical equivalent of a squirrel-cage winding of low resistance. The high-resistance winding is of the squirrel-cage type and is located in the same slots with the other winding. Both windings carry current while the machine is operating at normal speed, but the low-resistance winding carries a large proportion of the total rotor current so that the running performance resembles that of a squirrel-cage motor with low-resistance rotor, giving high efficiency, good speed regulation, and high power factor. By the use of two rotor windings in this way the desirable running performance of a squirrel-cage motor can be combined with the large starting torque possibilities of a slip-ring motor. The motor is thrown directly on the line at starting, but, owing to the high-resistance rotor, it gives a starting torque about 2.5 times full-load running torque with a starting current not more than 3 times full-load current.

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[1] See Chapter XXXIX, Applications of A.C. Motors.

[2] See Article 286 for constructional details.

294. Efficiency and Power Factor of Polyphase Induction Motors.—The losses in an induction motor include rotor and stator copper loss, stator core loss, friction and windage. The rotor core loss is very small for the usual constant-speed motor, because the rotor frequency is low. The stator core loss is nearly constant for all loads as long as the impressed voltage and frequency remain constant. The copper losses vary approximately as the square of the load. The friction and windage losses are constant for all loads. The efficiency of induction motors at full load is high, ranging from about 85 per cent for small machines to 92 for large motors. (See table, page 406.) The efficiencies at different loads are shown in the curve Fig. 281. The power factor depends to a considerable extent upon the length of air gap between rotor and stator and for this reason the gap is made as small as mechanical clearance will allow to obtain a high power factor. At full load the power factor is about 0.80 to 0.93, depending upon the size and number of poles of the motor. The power factor and efficiency are about the same for either a slip-ring or a squirrel-cage motor. The power factor of an induction motor is very low for light loads (see Fig. 281) and, for this reason, it is desirable to so choose the size of motors that they will be well loaded in order that the power factor of the system supplying them may be kept as high as possible. When starting, squirrel-cage motors have a power factor of about 30 to 40 per cent. Slip-ring motors using starting resistance would have a power factor at starting about 60 to 65 per cent. Whether starting or running, the power factor produced by an induction motor is lagging; i.e., the motor current lags behind the e.m.f. and, hence, is equivalent to an inductive reactance.

The losses in an induction motor are:

(*a*) Stator copper loss.
(*b*) Rotor copper loss.
(*c*) Stator core loss.
(*d*) Rotor core loss.
(*e*) Friction and windage.

The stator copper loss is proportional to the square of the stator current and therefore is approximately proportional to the square of the load. The rotor copper loss in a motor with low-resistance winding is small at no load and is also approximately proportional to the square of the load. The stator flux is nearly constant at all loads and, therefore, the stator core loss is constant. The rotor core loss at no load is small because the frequency is low. It increases with increased load. The friction and windage losses depend upon the speed; therefore, when the slip is small, they are approximately constant at all loads. The core losses, friction and windage losses are therefore nearly constant for all loads for induction motors having a small slip.

The efficiency of an induction motor can be determined by a method resembling that used for determining the efficiency of a transformer. (See Article 272.) This method, although somewhat approximate, gives results which are satisfactory for ordinary industrial testing. It is especially valuable where a determination must be made of the power required to drive a machine.

Example 1.—A three-phase, 440-volt, 60-cycle, eight-pole squirrel-cage induction motor is used to determine the load taken by a machine when running under specified conditions. When driving the load, the following readings were obtained:

Test A. Input = 25,200 watts. Stator current = 38.4 amperes.

The load was then disconnected and the motor operated without load at normal voltage with the following results:

Test B. Input = 1475 watts. Stator current = 14 amperes.

The rotor was then blocked and a reduced voltage applied to the stator sufficient to give a stator current about the same as in Test A. The results were:

Test C. Input = 3380 watts. Stator current = 40 amperes. Voltage between stator terminals = 110 volts.

Since the rotor is stationary in Test C, the motor acts like a transformer with secondary short-circuited. The friction and windage losses are zero and the core losses are small because the impressed voltage is low (110 volts in this example). Hence the power input during Test C is nearly all copper loss in stator and rotor. The copper loss in any three-phase machine, whether Y- or delta-connected is $\frac{3}{2}I^2R$, where R is the resistance measured between terminals.[1] Hence the equivalent resistance R of stator and rotor, measured in the stator circuit can be computed from the losses found in Test C.

$$3380 = \tfrac{3}{2}I_1^2R = \tfrac{3}{2} \times 40^2 \times R$$

or

$$R = 1.408 \text{ ohms.}$$

This value of R is the equivalent resistance of rotor and stator measured in the stator circuit. In Test B the total copper loss is

$$\tfrac{3}{2}I^2R = \tfrac{3}{2} \times 14^2 \times 1.408 = 413 \text{ watts.}$$

The friction, windage, and core loss at normal voltage is therefore

$$1475 - 413 = 1062 \text{ watts.}$$

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[1] The proof of this is given in Article 312, Chapter XXXI.

These losses can be assumed constant at all loads. In Test A the copper losses for stator and rotor are

$$\frac{3}{2}I^2R=\frac{3}{2}\times 38.4^2\times 1.408=3120 \text{ watts.}$$

The output is

$$25{,}200-3120-1062=21{,}018 \text{ watts.}$$

The power required to drive the load is, therefore,

$$\frac{21{,}018}{746}=28.3 \text{ hp.}$$

The power factor, when loaded as in Test A, is

$$\frac{25{,}200}{\sqrt{3}\times 38.4\times 440}=0.862 \text{ or } 86.2 \text{ per cent.}$$

The efficiency is

$$\frac{21{,}018}{25{,}200}\times 100=83.5 \text{ per cent.}$$

The output, power factor, and efficiency at any other load could be obtained by repeating Test A at the desired load value.

The approximate power factor and efficiency of squirrel-cage motors is given in the tabulation below.

Performance of Squirrel-cage Induction Motors

Horsepower	Efficiency, Per Cent			Power Factor, Per Cent		
Load	½	¾	Full	½	¾	Full
2	82	84.5	85	58	71	78
5	84	86.5	87	77	80	87
10	88	89	90	78	84	89
50	88	90	90.5	80	85	90
100	90	91	91.5	80	87	90.5
200	90.5	91.5	91.5	81	88	91
300	91	91.5	91.5	83	89	92

The disadvantage of a lagging power factor, which is characteristic of the usual design of polyphase induction motor, has been overcome in the Fynn-Weichsel motor which has recently been placed on the market. This machine combines the characteristics of a synchronous[1] motor and an induction motor. The stator and rotor punchings resemble those of the ordinary induction motor. There are two stator windings, one called the starting winding, corresponds to the secondary of the usual wound rotor motor and is connected, when starting, to an external resistance which is short-circuited after the motor is running. The second stator winding, called the operating winding, is displaced 90 electrical degrees from the first winding and is connected to brushes bearing on a commutator attached to the rotor. This stator winding carries the exciting current, which is furnished by a rotor winding connected to the commutator and corresponds to the field winding of a synchronous motor. The rotor also has two windings. One of these is a plain polyphase winding which corresponds to the primary of the ordinary induction motor and is connected to the supply circuit through slip rings. The other rotor winding, which is in the same slots, is connected to the commutator to furnish exciting current to the operating winding of the stator. The voltage on the commutator is very low and, since it carries only the exciting current of the motor, there is no trouble from sparking. The machine starts as a slip-ring induction motor with a starting torque of 1.5 times full-load torque with from 1.5 to 2.0 times full-load current. When the motor reaches synchronous speed, it becomes a self-excited, synchronous-induction motor. Over the normal load range, that is up to 150 per cent of normal load, the motor operates as a synchronous motor at an absolutely constant speed with a leading power-factor. If the load exceeds 150 per cent of normal, the motor falls out of step and operates as an induction motor with a lagging power-factor. As soon as the overload is removed, however, the motor pulls into synchronism again. By using these motors on the same system with ordinary induction motors, the lagging component of the latter can be neutralized and the total power factor of the system be made nearly unity.

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[1] See Chapter XXXII for description of synchronous motors.

295. Speed adjustment of polyphase induction motors may be effected by changing the rotor resistance. It was shown in Article 291 that, for a given torque, the speed was decreased by increasing the rotor resistance. This is shown by the curves in Fig. 286. Speed adjustment in this way is accomplished by using a slip-ring type of motor, with external resistance connected in the rotor circuit. The amount of this resistance is adjusted by means of a suitable switch or controller. It should be noted that, with a certain amount of resistance in the rotor circuit as, for example, curve R_2 (Fig. 286), the speed will vary widely as the torque changes. Thus with this particular setting, the speed is 170 r.p.m. when full-load torque is required, while, at half-load, the speed will rise to 340 r.p.m. and, if the load is all thrown off, the speed will be nearly synchronous or 500 r.p.m. Therefore, with this method of speed control, a change in load usually requires a change in rotor resistance which is accomplished by setting the controller on a different notch. When a motor is adjusted in this manner, the resistance must be large enough to carry the load without overheating, and, in general, this requires a larger rheostat than would be used simply for starting purposes; in which case the resistance would be left in circuit only for a short time. This method of speed reduction not only gives poor speed regulation but it also wastes a large amount of power in the rheostat. The amount of this loss is proportional to the slip so that for a reduction to half the normal speed, approximately one-half of the input to the rotor would be wasted, while the input to the motor, assuming the same torque, would be the same. Hence, for constant torque, the horsepower output would be about one-half, but the horsepower input would be approximately the same in each case. The motor would therefore be operating very inefficiently at the reduced speed.

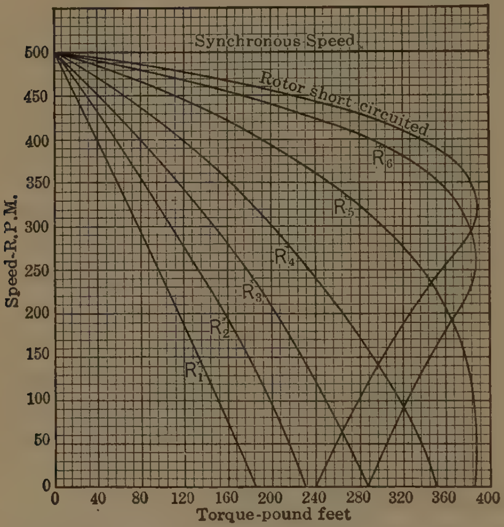

Fig. 286.—Speed Torque Curves for a Slip-ring Induction Motor with Different Rotor Resistance. 15-hp., 440-volt, three-phase, 50 cycles.

The speed of either a squirrel-cage or a slip-ring motor can be changed by varying the number of poles. It is possible, by means of a special arrangement of the windings on the stator, to make changes of this sort so that a limited amount of speed adjustment can be secured. The change in number of poles is accomplished by changes in the connections of the separate parts of the stator winding, and these changes are effected by means of multipolar switches. When the speed is changed in this way either a squirrel-cage or wound-rotor may be used, and the speed regulation will be good for each setting of the pole-changing switch. Owing to the complication of the stator winding, however, the number of speed changes which can be secured in this way is limited, and the speeds must of course have a fixed relationship. It is not feasible to secure more than two different speeds in this way, except for small motors. If the pole-changing method is used with a slip-ring motor, intermediate speeds can be secured by inserting rotor resistance. The speed of induction motors can also be changed by varying the frequency of the supply. This method, however, is not suitable for general power purposes, as a power supply of only a single fixed frequency is generally available. Where, however, the motor and its driving generator constitute an independent unit as in electrically propelled battleships, variation of the motor speed can be secured by varying the speed of the steam turbine which drives it.

A consideration of the three methods just described will show that frequency variation cannot be generally applied in ordinary industrial applications, and the pole-changing method is limited to a few fixed speeds, thus leaving the rotor-resistance method as the only one which would give sufficient flexibility for the usual requirements. This has the disadvantage of low efficiency and poor speed regulation so that for large motors such a plan would be undesirable. The use of very large induction motors for driving the main rolls in steel mills is becoming increasingly common and, for such purposes, wide speed adjustment of a very flexible nature is required. Control by rotor resistance would, however, be out of the question because of the poor speed regulation and the large losses in the rotor resistance. For such applications, a special regulating set is provided. It was shown in Article 291 that the speed of an induction motor must decrease if the rotor resistance is increased, because there would be a larger drop in the rotor circuit for a given current and hence a greater induced rotor e.m.f. is required to overcome this drop. If, however, instead of an increased rotor drop, a counter e.m.f. is introduced in the rotor circuit, then, in order to force the required current through the rotor circuit, the slip must be increased, or in other words, the speed must decrease. By introducing this additional counter e.m.f., therefore, the speed can be reduced without any change in rotor resistance and hence there is no increased I^2R loss in the rotor. Of the several schemes for doing this, the Sherbius[1] method is most commonly used in this country. The main induction motor is of the slip-ring type and has its rotor circuit connected to the rotating part of a special regulating machine (Fig. 287). The stator or field of this machine resembles the stator of an induction motor although the arrangement of the stator winding is different. The stator winding is connected through autotransformers (Fig. 287) to the rotor of the main motor, and hence a revolving field is produced having a speed corresponding to the rotor frequency. The strength of this field can be adjusted by changing the exciting voltage by means of the autotransformers. The rotor or armature of the machine has a winding connected to a commutator and resembles an ordinary d.c. dynamo armature. Currents from the rotor of the main motor, *M*, are led into the rotor of machine *B* by means of brushes spaced 120 electrical degrees apart. In the rotor of this machine is produced an alternating e.m.f. having the same frequency as the rotor and opposed to the e.m.f. generated in the rotor of the main motor *M*. If the contacts on the auto-transformer are set at points 0_1, 0_2, 0_3, the voltage on the field of machine *B* is zero; hence, its rotor has no e.m.f. induced in it and it acts as a low impedance in the rotor circuit of the main motor; therefore, the motor would run at its maximum speed. As the contacts are moved along the taps of the auto-transformer, more and more voltage is impressed on the stator of the regulating machine *B*, which would produce a field in this machine and generate a counter e.m.f. in the rotor circuit of the main motor. Therefore, the speed of the main motor would have to decrease in order to generate an e.m.f. in the rotor circuit sufficient to produce the current necessary for the required torque. To do this, the rotor slip must increase and the rotor speed must decrease. Furthermore, the counter e.m.f., introduced into the rotor circuit by the regulating machine *B*, is nearly independent of the rotor current; hence, for a given adjustment of the autotransformer voltage, the speed of the main motor will be practically the same regardless of the load. In other words, the speed regulation is much better than where resistance is used in the rotor circuit. The regulating machine is driven by an induction motor connected to the stator side of the main motor. This regulat-

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[1] For more complete information regarding large adjustable-speed induction motor outfits, see "Some Methods of Obtaining Adjustable Speed with Electrically Driven Rolling Mills," by K. A. Pauly, *General Electric Review*, May, 1921. Also "Theory and Power Factor Control of Large Induction Motors by Neutralized Polyphase Alternating Current Machines," by J. I. Hull, *General Electric Review*, July, 1920.

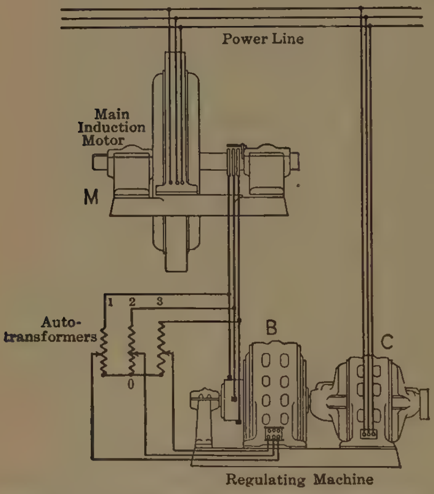

FIG. 287.—Sherbius Method of Regulating Speed of an Induction Motor.

ing machine introduces a counter e.m.f. in the rotor circuit, and this represents a certain amount of power which is not lost in heat, as is the case with a regulating resistance, but is fed back into the supply through the machine C, which really acts as a generator of the non-synchronous or induction type.[1] As a consequence, the efficiency of this method of speed adjustment is much higher than that using rotor resistance. By the addition of a machine called an ohmic drop exciter, it is possible to operate the main motor at speeds above as well as below synchronous speed.

Another scheme for speed regulation, the Kraemer, uses a synchronous converter instead of a special induction motor for returning a part of the power in the rotor circuit to the system. The principle of operation is the same as described, but the set is not as flexible in its operation.

296. The Brush-shifting Polyphase Induction Motor.—The Sherbius and Kraemer methods of adjusting the speed of an induction motor are limited to large size installations because of the complication and expense of the additional apparatus required. The brush-shifting motor is, however, adapted to smaller sizes and is less complicated. The motor is built like a polyphase, slip-ring type of induction motor with the primary or input winding on the rotor and the secondary on the stator. In the same rotor slots with the primary winding is a second winding called a regulating winding. This is built like a d.c. armature winding and has conductors connected to a commutator. The stator has a phase-wound secondary with the two ends of each phase brought out separately. There are two brush yokes located at opposite ends of the commutator and so arranged that the brushes carried by one yoke can be set in line with the brushes carried by the other yoke or they can be separated by shifting the yokes. One end of each phase of the secondary is connected to a brush on one of the yokes, and the other end, to a corresponding brush on the other yoke (Fig. 288). The circuit through the secondary is completed through the commutator. When the brushes carried by the two yokes are in line, the secondary is short-circuited by the commutator bars, and the motor operates like an ordinary induction motor. When the rotor is turning, a voltage is induced in the regulating winding on the rotor. If the yokes are shifted so that the brushes are no longer in line, an e.m.f. is introduced in the secondary winding. This e.m.f. is due to the voltages between brushes a_1–b_1, a_2–b_2, and a_3–b_3. The amount of this voltage will increase with the separation between the brushes connected to the two ends of the same coil, such as a_1–b_1, etc. It was shown in the previous article that the speed of an induction motor could be decreased by introducing a counter e.m.f. in the secondary circuit. The voltage produced by this regulating winding is of the same frequency as that of the secondary winding and, therefore, the effect of shifting the brushes, as described, is to introduce a counter e.m.f. into the secondary circuit and thereby vary the speed. When the brushes are set in a particular position the machine has a speed regulation resembling that of a shunt motor. The machine can be adjusted for speeds above or below synchronism by shifting the brushes in the proper direction. The performance of this machine resembles that of the Sherbius outfit described in Article 295. The brush-shifting motor cannot be made in as large sizes as the Sherbius type of control, but brush-shifting motors of 600-hp. capacity are now in use in steel mills. There is a standard line of these motors now on the market ranging in size from 5 to 50 hp. They are used to drive adjustable speed machine tools.

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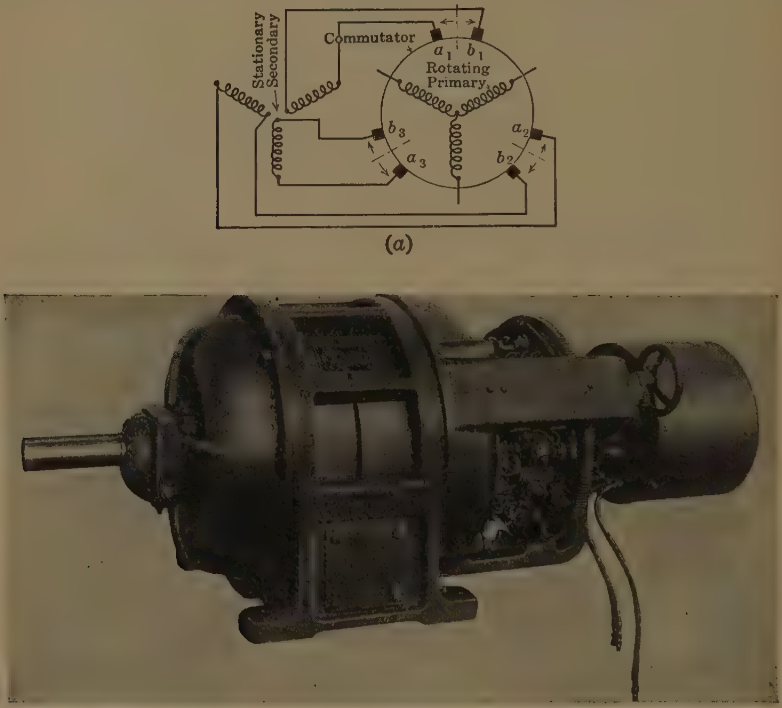
[1] The induction motor used as a generator is explained in Article 299.



(a)

(b)

FIG. 288.—Brush-shifting Polyphase Motor. *The General Electric Co.*

297. Effect of Varying the Voltage or Frequency of an Induction Motor.—The induction motor, either squirrel-cage or slip-ring type which is sold for general power applications, is designed to operate at rated frequency and voltage. It sometimes happens, however, that the supply system may not deliver power at exactly rated value so that the effect of variation of voltage or frequency is important.

If a voltage lower than normal is applied to an induction motor the torque is reduced proportionally to the square of the voltage. (Article 290.) Therefore, if this voltage is much below normal, the motor may become stalled on an overload. The maximum torque of a motor at normal voltage is about 2 times full-load torque. At reduced voltage the maximum torque would be

For 90 per cent voltage $T_{\text{max.}} = (0.90)^2 \times 2 = 0.81 \times 2 = 1.62.$

For 80 per cent voltage $T_{\text{max.}} = (0.80)^2 \times 2 = 0.64 \times 2 = 1.28.$

Hence, 20 per cent drop in voltage would result in the motor becoming stalled if the load happened to rise momentarily to 1.3 times full load. At the same time the motor would take an excessive current. A 10 per cent drop in voltage would, however, probably not cause any trouble. If an attempt is made to carry the same horsepower load at reduced voltage, the stator and rotor currents would be greater than normal; hence, the copper losses would be higher than normal, and the motor would operate at a higher temperature. The core losses would, however, be decreased because the strength of the rotating field would be less, and this would tend to offset the increased copper loss. If a motor is operated above normal voltage, the core losses are increased approximately as the square of the voltage and the copper losses are decreased, since the motor would deliver full load for less current input. An excessive increase in voltage would cause the core to become so hot as to endanger the windings. It is customary for manufacturers to guarantee that their induction motors will operate satisfactorily if the voltage is not more than 10 per cent above or below normal. The effect of a reduced voltage is to cause a greater slip while an increased voltage produces less slip. The slip is, in fact, approximately inversely proportional to the square of the voltage although a slight decrease in voltage produces a greater effect than the same percentage increase above normal voltage.

If the frequency of the supply is changed, the synchronous speed of an induction motor is changed correspondingly; therefore, the speed would vary approximately as the frequency. If the voltage is maintained normal and the frequency decreased, the maximum and starting torques are both increased proportionally. The core loss is also increased, and the copper loss remains approximately the same at rated horsepower output. The machine would, therefore, run hotter because of the greater total losses and the decreased cooling effect due to the lower speed. It is customary for manufacturers to guarantee satisfactory operation at rated load if the frequency is not more than 10 per cent above or below normal. A motor can be operated to produce the *same torque* at different frequencies if the impressed voltage is varied proportionally. Thus a 550-volt, 60-cycle motor could be operated on a 25-cycle supply if the applied voltage were lowered to $\frac{25}{60} \times 550 = 229$ volts. The speed would, of course, be proportional to the frequency and hence the horsepower would also be increased proportionally since the torque would remain the same. The speed regulation would be better at the higher frequency since the revolutions slip would be approximately the same at either frequency. An induction motor will operate satisfactorily when both frequency and voltage are different from normal provided the resultant change due to both is not more than 10 per cent.

298. Effect of Opening One Phase of a Polyphase Induction Motor.—If one of the line wires supplying a polyphase induction motor were opened, for any cause, such as a defective fuse or broken wire, the motor would not start because there would no longer be a rotating field. If the motor is started by pulling on the belt or by other means and is brought up to about one-fifth normal speed, it will then generally come up to normal speed. If, however, one line wire becomes open while the motor is running, the motor will continue to run and act like a single-phase induction motor. When operating single phase in this manner, it is possible to carry about 70 per cent of the rated load with about the same slip and temperature rise as when operating normally as a polyphase motor and carrying rated load. The pull-out or maximum torque, when running single phase, will be from one-third to one-half less than normal, the motor with the small slip having the greater pull-out. The performance characteristics stated apply to squirrel-cage rotors; the slip-ring type, in general, gives poorer single-phase operation. A slip-ring motor supplied with polyphase current on its stator, but with one lead of the rotor circuit open, will not start, but, if the lead is opened while the motor is running, the motor will continue to operate with a maximum torque about 30 to 40 per cent of normal. The motor should not, however, be operated with a rotor lead open as it would vibrate badly.

299. The Induction Generator.—We have seen in Article 288 that when the rotor of an induction motor revolves at a speed less than that of the revolving field, the e.m.f. induced in the rotor conductors produces a current which gives a motor action tending to turn the rotor in the same direction as that of the rotating field. As the speed of the rotor approaches that of the revolving field the rate of cutting decreases, and, hence, the induced rotor e.m.f. and the resulting current decreases. If the rotor were turned at exactly the same speed as the rotating field, there would be no cutting of the rotor conductors and, hence, no e.m.f. induced in the rotor. If the rotor speed were increased so that it turned in the same direction as the rotating field but at a higher speed, there would again be a cutting of the rotor by this revolving field, but the direction of the induced e.m.f. would be the reverse of that which occurs when the rotor runs below synchronous speed. Hence, the rotor current resulting would be in the opposite direction, and, since at less than synchronous speed, this current produced a force tending to turn the rotor in the same direction as the field (motor action) there would now be a force tending to slow down the rotor (generator action). Hence, power must be supplied from an external source to keep the rotor of an induction motor turning above synchronous speed, and the machine is called an induction generator. The power delivered by an induction generator depends upon the slip, and full load as a generator would be produced at approximately the same slip as would exist when the machine delivers full load as a motor. Thus, a squirrel-cage induction motor, when acting as a motor, might run at about 1150 r.p.m. and as a generator would be fully loaded at about 1250 r.p.m. The induction motor will not act as a generator except when it is run in parallel with an ordinary synchronous type of alternator because the revolving field of the induction machine must be excited from the alternator. The generator action just described is made use of in producing a braking action in hoists and in electric locomotives which are sometimes equipped with polyphase motors. When the hoist is lowering a load, or the locomotive is descending a grade, the motors remain connected to the line. As soon as the speed becomes higher than synchronism, the motor becomes an induction generator and returns power to the system. The lowering speed would be only slightly higher than the hoisting speed provided the rotor resistance were made a minimum. In fact, the percentage increase above synchronous speed, when the machine is delivering rated current as a generator, would be approximately the same as the percentage slip when running at full load as a motor. If an induction motor, having its stator excited by direct current, were rotated by mechanical means, the rotor would generate alternating current but the machine would no longer be called an induction generator. The power thus produced in the rotor could be consumed by a resistance and the machine could thus be made to produce a braking action. This principle is made use of very commonly in large induction-motor-driven mine hoists to control the speed in lowering a load, instead of depending upon friction brakes.

PROBLEMS ON CHAPTER XXX

1–30. A three-phase, 60-cycle circuit is applied to the stator of an induction motor producing a rotating field. The motor has six poles for each phase.

(*a*) What is the speed of the field in r.p.m.?

(*b*) If 25 cycles were used instead of 60 cycles, what would be the speed of the field?

2–30. A three-phase, 60-cycle, 8-pole induction motor has a rotor speed at no load of 895 r.p.m.

(*a*) What is the slip in per cent?

(*b*) At full load the speed of rotor is 850 r.p.m. What is the slip in per cent?

(*c*) What would be the rotor frequency at no load and at full load?

3–30. The following data were obtained from the nameplate of an induction motor: full-load speed 870 r.p.m., frequency 60 cycles.

(*a*) What would be the approximate no-load speed?

(*b*) How many poles does the motor have?

4–30. A 20-hp., 220-volt, three-phase, squirrel-cage induction motor with normal voltage applied to the stator, has a starting torque 1.8 times the full-load running torque.

(*a*) What would be the torque if 110 volts were applied for starting?

(*b*) What voltage should be applied to give full-load starting torque?

5–30. The motor in Prob. 4–30 requires 3.5 times full-load current when starting with full-load torque. Normal full-load current is 60 amperes.

(*a*) What would be the current when full voltage is applied at standstill?

(*b*) What would be the required current to produce a starting torque equal to three-fourths full-load torque?

(*c*) If the current at starting is limited to full-load current, what would be the torque in terms of full-load running torque?

6–30. A 50-hp., 440-volt, 25-cycle, three-phase induction motor has a power factor of 0.93 and an efficiency of 0.90 at full load. Calculate the full-load current.

7–30. Calculate the current at full load for a two-phase, 4-wire motor of the same size and characteristics as in Prob. 6–30.

8–30. When the motor in Prob. 6–30 is running at no load the current is 16 amperes, and the power input is 1100 watts.

(*a*) Calculate the power factor at no load.

(*b*) What percentage is the no-load current of the full-load current?

9–30. The maximum torque of the motor in Prob. 4–30 is 3.0 times full-load torque.

If sufficient resistance were inserted in the motor circuit to give this maximum value of torque at starting, with full voltage applied to the motor, (*a*) Calculate the starting torque with 110 volts applied to the stator.

(*b*) Calculate the voltage which must be applied to give full-load torque at starting.

10–30. The motor in Prob. 4–30 has 6 poles and runs at 1160 r.p.m. under full load.

(*a*) What is the percentage slip?

(*b*) What would be the speeds at three-quarter full-load torque and at half full-load torque?

(*c*) If the rotor resistance were made 4 times as great, what would the full-load speed become?

(*d*) What would be the speed regulation with the resistance as in (*c*)?

11–30. Referring to the motor of Prob. 3–30, what would be the speed at half load?

12–30. A 50-hp., 440-volt, three-phase, 25-cycle, 6-pole squirrel-cage induction motor has a full-load speed of 475 r.p.m. It is used to determine the horsepower required to drive a certain machine by means of the following tests:

Test A. Input 30,550 watts, stator current, 44.3 amperes. The load was then disconnected and the following test made:

Test B. Input, 1160 watts, stator current, 16.3 amperes. The motor was then blocked and a reduced voltage applied to the stator with the following results:

Test C. Input, 2100 watts, 44.3 amperes, 52.5 volts.

(*a*) What is the power required to drive the load?

(*b*) What is the motor efficiency at this load?

(*c*) What is the power factor?

CHAPTER XXXI

ALTERNATING-CURRENT GENERATORS

300. Types of Alternators.—Since a.c. generators require no commutator, either the armature or the field may be the rotating member. The **revolving armature type** of alternator may have a winding arranged much like that of a d.c. machine; in fact, alternating current may be secured from an ordinary d.c. armature by taps on the winding. (See Article 328.) Connection with this revolving armature winding is made by brushes bearing on insulated rings, called collector rings or slip-rings. The stationary field of this type of alternator is much the same as that of a d.c. generator. The necessary d.c. excitation is usually obtained by means of a small commutator connected to the armature winding, the same as for a d.c. generator.

With the **revolving-field type** of machine, connection can be made directly to the stationary armature winding without the use of collector rings, and this is an advantage, particularly where the machine generates a high voltage, because of the difficulty of properly insulating a moving high-voltage winding. In this type of machine, the revolving field is excited by a low-voltage d.c. source through two slip-rings and brushes. The voltage used for field excitation is usually 125 or 250 volts, and such a winding is comparatively easy to insulate. This is particularly true of steam-turbine-driven alternators, which operate at very high speeds, and which have large mechanical strains in the revolving member. These machines are made with revolving fields because of the difficulty which would be encountered in maintaining proper insulation and mechanical balance in a high-voltage revolving armature. Even for slow-speed machines there are advantages in the use of the revolving field, so that alternators are, in general, built with revolving field and stationary armature, except in very small sizes. The d.c. excitation required by a revolving-field alternator is usually supplied by a d.c. generator, called an exciter. (See Article 304.)

301. Revolving Field Construction.—The field or rotor for a slow-speed alternator is shown in Fig. 289. The field poles, are bolted to a cast-iron spider, which also supports the two collector rings, through which the exciting current is led from the external source. The field winding consists of coils surrounding each pole. For small machines the coils are wound with round wire, but, for larger machines, rectangular copper strap, wound on edge, is used. The pole faces are sometimes provided with a cage damper winding, which consists of a series of copper bars embedded in the pole face, with the ends short-circuited by two rings. This cage damper is used to improve the parallel operation of alternators. Cage dampers are particularly necessary when alternators, driven by internal combustion engines, are operated in parallel but are generally not required for machines driven by reciprocating steam engines or water wheels.[1] Alternators are not provided with a compound field winding, but, instead, the necessary voltage adjustment to meet varying load requirements is made by means of a separate device called a voltage regulator, which varies automatically the field excitation in order to maintain the required voltage with varying load. (See Article 314.)

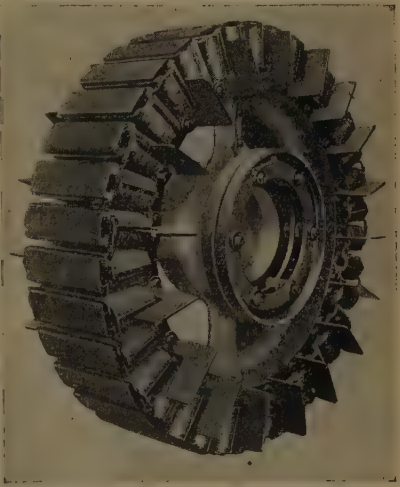


FIG. 289.—Revolving Field for a Slow-speed Alternator. *The Westinghouse Electric & Mfg. Co.*

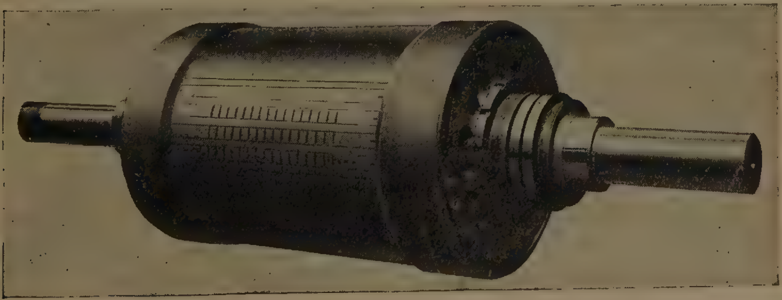


FIG. 290.—Revolving Field for a Turbo-alternator. *The Westinghouse Electric & Mfg. Co.*

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[1] An explanation of the action of these cage dampers is given in the Chapter XXXII, Synchronous Motors, see Article 320.

Slow-speed machines, driven by reciprocating engines or water wheels, require a large number of poles in order to obtain commercial frequencies. The machine illustrated in Fig. 289 is typical. Steam-turbine-driven alternators operate at much higher speeds and generally have a revolving field with two or four poles (Fig. 290).

302. Armature Construction.—The armature of the revolving-field alternator consists of a laminated steel ring having slots on its inner circumference in which the armature conductors are placed. The construction of this type of alternator armature is much like that of the stator of an induction motor. In fact, an induction motor could be converted to an alternator by replacing the rotor with a field structure similar to that shown in Fig. 289. The armature for a slow-speed alternator is shown in Fig. 291. The winding consists of coils, which span a distance about equal to the pole pitch and which are connected together in groups to form either a two-phase or a three-phase winding. If a single-phase supply were required, usually a three-phase machine would be employed and the load connected to two of the three terminals. The armature of a turbo-alternator is shown in Fig. 292. Because of the high speeds (1500, 1800, or 3600

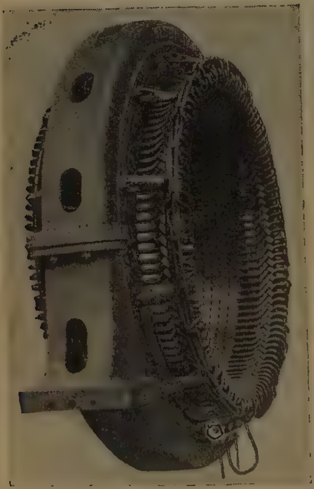

FIG. 291.—Armature for a Slow-speed Alternator. *The Westinghouse Electric & Mfg. Co.*

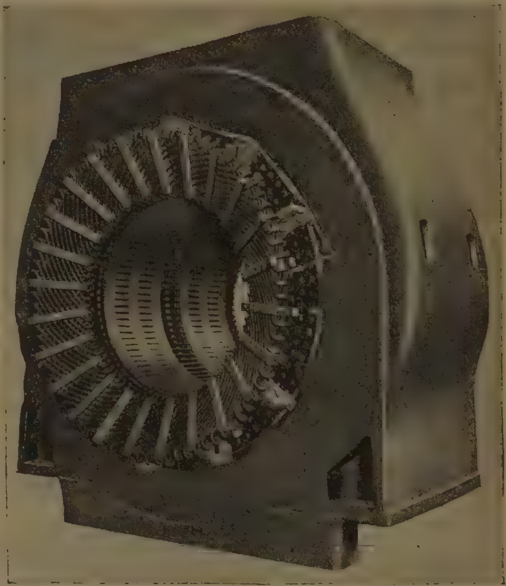

FIG. 292.—Armature of a Turbo-alternator. *The Westinghouse Electric & Mfg. Co.*

r.p.m.), the revolving field must be of relatively small diameter and, hence, the armature of a turbo-alternator is much smaller than that of a corresponding size of engine-driven machine. The ventilation of the latter is not difficult, but for the turbo-alternator, the necessary cooling air must be circulated by a fan, and the armature is enclosed so that the air circulation may be controlled. The fan may be either separately driven or mounted directly on the alternator shaft. Steam turbines are more efficient than reciprocating steam engines, except in small sizes so that turbo-alternators operating at high speed are much more common than slow-speed, engine-type alternators. For water-power stations, the most economical speed is determined by the hydraulic head, and this usually requires an engine type of alternator.

303. Method of Specifying Alternator Rating.—The rated output of an alternator is usually expressed in kilovolt-amperes and not kilowatts because the rating is limited by the heating, which depends upon the voltage and current delivered by the machine and is not dependent directly upon the power developed by the machine. The rating is calculated from the line or terminal voltage and the current. If E_l = voltage between terminals and I_l = current in each terminal or line wire, the rating is:

For single-phase machines $E_l I_l \div 1000$ kv-a.

For two-phase machines $2E_l I_l \div 1000$ kv-a.

For three-phase machines $\sqrt{3}E_l I_l \div 1000$ kv-a.

The rating, expressed in kilovolt-amperes, is always based on a specified terminal voltage and power factor. It is necessary to specify the power factor because an alternator which will deliver rated kilovolt-amperes at unity power factor and normal voltage may not be able to maintain this voltage at rated kilovolt-ampere load and a lagging power factor. (See Article 310.) It is customary to base ratings on a power factor of 0.80 lagging, as this meets the usual power requirements. A machine rated in this manner could be depended upon to deliver its rated load at any power factor not lower than 0.80.

304. Excitation of Alternators.—As was stated at the beginning of this chapter, all alternators must have a magnetic field produced by a winding excited by direct current.[2] This is true for either the revolving-field or revolving-armature type. The necessary exciting current is taken from a d.c. source at a potential of 125 or 250 volts as a general rule. This source is usually one or more separate d.c. generators called exciters. These exciters are frequently independently driven either by an a.c. motor or an engine or water wheel, depending upon the requirements of the installation. In other cases, the exciter is direct connected or belted to the alternator itself. Sometimes a storage battery is used as a source of excitation.

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[2] The induction generator (Article 299) does not require d.c. excitation, although it generates an alternating current. The a.c. generators referred to in this chapter are known as the synchronous type and must always have a d.c. field.

Exciters may be either shunt or compound wound. Shunt exciters are preferred for individual exciters supplying only one alternator, or when operating in parallel with a storage battery. Compound exciters are favored when several exciters are operated in parallel without a storage battery. Shunt exciters are somewhat more sensitive than the compound when used with voltage regulators (see Article 314), but either type can be used successfully.

305. E.M.F. of an Alternator.—The e.m.f. generated in an alternator armature depends upon the rate of cutting of the field flux, the same as for a d.c. dynamo.

Let Φ = flux per pole;
N = number of conductors in one phase of the winding;
n = speed in revolutions per second;
P = number of poles;
f = frequency in cycles per second.

Assume that the flux density in the air gap is sinusoidal. Then the total flux cut by one conductor in one revolution is ΦP lines, and the time taken to cut this flux is $\frac{1}{n}$ seconds. Hence the rate of cutting is $\Phi P \div \frac{1}{n}$. The average voltage induced is therefore $e_{\text{av.}} = \Phi P n \times 10^{-8}$ volts. But since the flux distribution is sinusoidal, the e.m.f. induced will also be sinusoidal. But $e_{\text{eff.}} = e_{\text{av.}} \times \frac{0.707}{0.637}$. Hence the effective voltage induced in one conductor is

$$e_{\text{eff.}} = 1.11 \Phi P n \times 10^{-8} \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

But since the frequency $f = \frac{Pn}{2}$, Equation (89) can be expressed in terms of frequency instead of number of poles and speed.

$$e_{\text{eff.}} = 2.22 \Phi f \times 10^{-8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

It has already been shown in Chapter XXVII that the conductors in an alternator armature are arranged in one or more groups according as the machine is designed to produce single-, two-, or three-phase voltages. Considering the total number of conductors N in one group or phase, the total voltage produced by one phase will be the combined voltage of all the conductors in this phase. Thus, referring to the four-pole three-phase generator shown in Fig. 293, the conductors are distributed in twenty-four slots, there being eight slots (two per pole) for each phase. At the instant shown, the poles are opposite slots 1, 7, 13, and 19, and therefore, the voltages induced in these four conductors have their maximum values at the same instant so that by properly connecting these conductors in series the total voltage produced by the conductors in these four slots is four times the voltage of one conductor. After the field has turned an angle of θ degrees, slots 2, 8, 14, and 20 are opposite the center of the pole, and, hence, the voltages in these slots can be combined in the same way. But, if the first group of conductors is connected in series with the second group, the total voltage for all eight conductors will not be eight times that of one conductor because these two groups have their voltages displaced by a phase angle of θ degrees. Therefore, the resultant voltage must be found vectorially in the usual manner. Furthermore, if a coil did not span the exact distance between pole centers as it does in Fig. 293, the e.m.f.'s in the two sides of a coil would also be less than the numerical sum of the voltages of the two sides as they would be out of phase. The total voltage produced by one phase of the armature winding is therefore

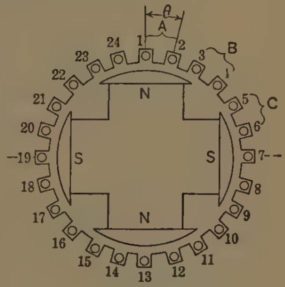

FIG. 293.—Phase Constant.

$$E=2.22K_bK_p\Phi N_pf\times10^{-8} \text{ volts}, \quad . \quad . \quad . \quad . \quad (91)$$

where K_b is a constant which takes account of the distribution of the winding in two or more slots per pole per phase and K_p takes account of fractional pitch of the coils. If there is only one slot per pole for each phase $K_b=1$, for more than one slot per pole per phase K_b is less than one. For full-pitch coils $K_p=1$. Usually the number of slots and arrangement of coils is such as to produce a terminal voltage at no load which very closely approximates a sine wave. If it differs appreciably from the sine wave form, the constant 2.22 in Equation (91) will be changed. This constant, which is called the form factor, is the ratio of effective to average volts. The frequency of the e.m.f. generated by an alternator depends upon the number of poles and the speed. This has been covered in Article 205.

306. Voltage Regulation.—If an alternator is loaded, it will be found that the terminal voltage varies with the load. The amount of variation depends upon the design of the machine and also upon the power factor of the load. With a load having a lagging power factor the drop in voltage with increased load is greater than for unity power factor (Fig. 294). With leading power factor, the voltage tends to rise instead of drop. The reasons for a change of terminal voltage are:

(*a*) Drop due to resistance of armature.
(*b*) Drop due to reactance of armature.
(*c*) Effect of armature current upon the magnetic field. (Armature reaction.)

As the resistance of an alternator armature is small, it has very little effect upon the terminal voltage. Since the armature conductors are placed in slots in the iron core, the armature current produces an appreciable magnetic field around these conductors; hence, there will be a reactive voltage induced in the armature winding. The value of the armature reactance is several times as much as the armature resistance. The magnetic field produced by the armature current reacts upon the main field produced by the exciter. At unity power factor, this effect is a minimum; at lagging power factor, the armature current tends to demagnetize the main field, while, with leading power factor, the armature current tends to increase the main field strength. (See Article 311.) This armature reaction is the principal cause for variation of the alternator voltage. The effect of this armature reaction is very much the same as if additional reactance were introduced into the armature circuit and for purposes of calculation causes (*b*) and (*c*) are combined and called the reactance of the machine. This equivalent reactance is found by first determining the impedance of the armature in a manner described in the next article.

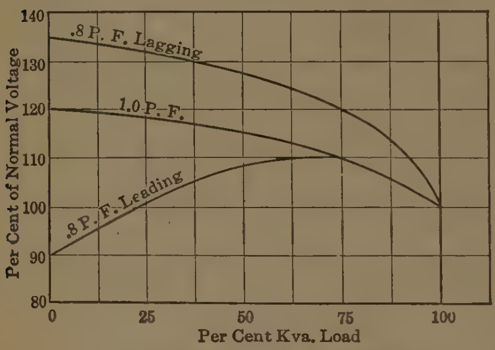

FIG. 294.—Alternator Regulation.

307. Experimental Determination of Alternator Impedance.—If alternating current from an external source were passed through the armature of an alternator, with the field not excited nor turning, the e.m.f. observed at the terminals could be used to calculate the impedance of the armature. This impedance would include the armature resistance and reactance (*a*) and (*b*) of tabulation in Article 306, but would not give an indication of the armature reaction (*c*) of this tabulation. If, however, the alternator armature terminals were short-circuited through an ammeter and enough excitation applied to the field to produce full-load armature current when the machine is running at normal speed, the effect of armature reaction could be determined. After the field strength has been adjusted to give the required armature current, the short-circuit is removed and the voltage between armature terminals is observed while the field current is held constant. This voltage, divided by the current which flowed in the armature when short-circuited, is called the synchronous impedance since it is determined from readings taken while the machine is running at synchronous speed.

Example 1.—A 2100-kv-a., 6600-volt, 25-cycle three-phase alternator is operated at normal speed with the armature short-circuited and sufficient field current to circulate full-load current in the armature. The field current is kept the same, and the armature circuit opened. The terminal voltage then is 2900 volts. The current per terminal at full load is

$$I_t=\frac{2100\times1000}{6600\times\sqrt{3}}=184 \text{ amperes.}$$

The synchronous impedance between terminals is

$$Z_t=\frac{2900}{184}=15.75 \text{ ohms.}$$

For a Y-connected machine the impedance per phase would be

$$Z_p=\frac{2900}{\sqrt{3}\times184}=9.1 \text{ ohms.}$$

The impedance for other values of armature current can be taken in the same manner. The resistance of the armature may be determined by measurement with direct current in the usual way and the synchronous reactance then calculated.

Example 2.—Referring to the alternator of Example 1, which is Y-connected, measurement of resistance between terminals gave an average value of 0.103 ohm. Hence the resistance per phase is $0.103\div2=0.0515$ ohm. The reactance per phase is

$$X_p=\sqrt{9.1^2-0.0515^2}=9.1+\text{ohms.}$$

Hence the resistance is negligible in its effect, and the reactance practically equals the impedance.

The synchronous reactance thus found includes the true reactance of the armature winding and also the effect of armature reaction. By using this value of synchronous reactance, the approximate voltage regulation of an alternator can be determined by treating it like a series circuit having resistance and reactance. (See Article 232.)

Example 3.—Referring to the results of Examples 1 and 2, the impedance drop per phase at full load is

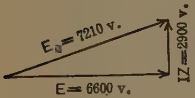

FIG. 295.—Regulation at Unity Power Factor.

$$e_p = 9.1 \times 184 = 1675 \text{ volts},$$

and the equivalent drop between terminals, since the machine is Y-connected, would be

$$e_t = \sqrt{3} \times 1675 = 2900 \text{ volts}.$$

Since the resistance is negligible, this voltage is 90° ahead of the current. The no-load voltage is then

$$E_p = \sqrt{6600^2 + 2900^2} = 7210 \text{ volts}$$

(Fig. 295.) The regulation for unity power factor is

$$\frac{7210 - 6600}{6600} \times 100 = 9.2 \text{ per cent.}$$

308. Effect of Power Factor upon Regulation.—It has been pointed out in Article 306 that the power factor of the load has a marked effect upon the voltage regulation of an alternator. In fact, the machine acts very much like a transmission line or a transformer in this respect. The principal cause for a change in voltage is the effect of armature reaction, and this depends to a large extent upon the power factor.

Whenever a revolving-field type of alternator is carrying a balanced polyphase load the armature current produces a magnetic field which resembles the revolving flux of an induction motor. This revolving flux, produced by the alternator armature current, has the same number of poles as the d.c. or exciting field and therefore the armature field moves synchronously with the d.c. field. The position of this armature field with respect to the exciting field is determined by the power factor.

309. Regulation at Unity Power Factor.—Consider a two-phase alternator with a bipolar revolving field as shown in Fig. 296. At the particular instant shown, the conductors in the plane 1–3 (phase *A*) will have a maximum voltage induced in them, while those at 2–4 (phase *B*) will have zero voltage. The direction of the flux produced by the direct-current winding and the resulting induced e.m.f. in phase *A* are shown in Fig. 296 *a*. If the load connected to the alternator has unity power factor, the armature current is in phase with the terminal voltage and is also in phase with the induced voltage if the drop due to the armature reactance be neglected. Hence, for the position of the field magnet shown in Fig. 296 *a* the current in phase *A* would be a maximum and in phase *B* would be zero. The magnetic flux produced by the armature current would, for this position of the field magnet, take the position shown in Fig. 296 *b*, or 90 degrees from the main or exciting field. Similarly, when the field magnet has moved one-fourth a revolution to the line 2–4 the armature flux would be along the line 1–3; that is, still 90 degrees from the exciting field.

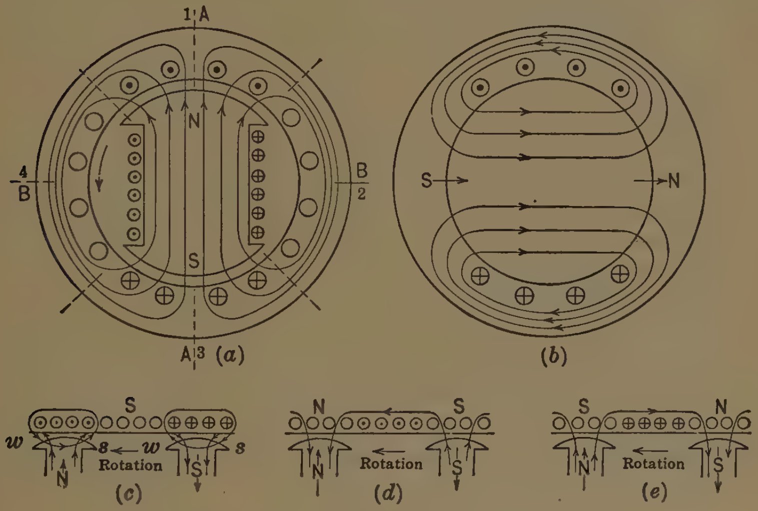

FIG. 296.—Armature Reaction for an Alternator.

But if the armature carries a balanced two-phase load, the armature currents due to this load will produce a resulting armature flux which appears as a two-pole field moving around the alternator air gap at synchronous speed, just as occurs in an induction motor stator. This two-pole field is of constant strength and is located 90 degrees from the exciting field produced by the direct current. The flux in the air gap of the machine is, therefore, the resultant of the flux due to the armature current shown in Fig. 296 *b* and the exciting current, shown in *a*. These are combined in Fig. 296 *c* and the result is that the main flux is distorted, two pole tips (*s*, *s*) having a higher and the two others (*w*, *w*) having a lower density than at no load. Since the armature flux is 90 degrees with respect to the main field, the effect of the armature current is to distort the main flux but not to change materially the total flux per pole. The effect is similar to that produced with direct-current machines when the brushes are on the neutral. With unity power factor, therefore, the armature reaction has a comparatively slight effect upon the voltage regulation. The wave form of the machine is, however, changed somewhat and is not so close to a sine wave as at no load.

310. Regulation at Lagging Power Factor.—Assume that the machine is carrying a lagging load at zero power factor. When the field magnet is in the position shown in Fig. 296 *a*, the armature current in phase *A* will be zero, and in phase *B* it will be a maximum. The armature current will produce a revolving flux of constant strength as before, but this flux will not be at right angles to the main flux and will be directly opposed to it as shown in Fig. 296 *d*. The armature current, therefore, produces a demagnetizing effect, but no distortion.

311. Regulation at Leading Power Factor.—With a leading load at zero power factor, the armature field coincides with the main field in such a way as to aid the main field (Fig. 296 *e*) and, hence, produces a magnetizing effect. We can, therefore, see the reason for the tendency for an alternator to increase its terminal voltage when carrying a load having a leading power factor.

With loads having a power factor less than unity but greater than zero, the effect of armature reaction is a combination of demagnetizing (or magnetizing) and distorting of the main field. The effect of armature reaction can be approximated by treating it like an additional reactance in the armature circuit as shown in Article 307 and by calculating the regulation by means of the synchronous impedance.

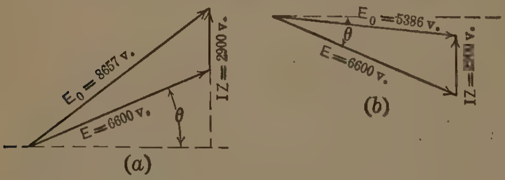

FIG. 297.—Regulation at Leading and Lagging Power Factors.

Example 1.—Referring to the alternator of Examples 1, 2, and 3, Article 307, it was found that the regulation at unity power factor is approximately 9.2 per cent. At 0.80 power factor lagging the regulation would be (Fig. 297 *a*).

$$E_0 = \sqrt{(6600 \times 0.8)^2 + (6600 \times 0.6 + 2900)^2}$$

$$= 8657 \text{ volts at no load.}$$

$$\frac{8657 - 6600}{6600} \times 100 = 31 \text{ per cent.}$$

At 0.80 power factor leading (Fig. 297 *b*), the regulation is

$$E_0=\sqrt{(6600\times 0.8)^2+(6600\times 0.6-2900)^2}$$

$$=5386 \text{ volts at no load.}$$

$$\frac{5386-6600}{6600}\times 100=-18 \text{ per cent.}$$

Hence, with a leading power factor the regulation tends to become negative; that is, the voltage falls as the load is thrown off.

It is apparent that if a constant voltage is to be maintained at the terminals of an alternator, more field current is required for lagging, and less for leading, than is required for unity power factor. As far as lagging loads are concerned, this may be the limiting condition in the alternator, as a machine designed for operation at loads of 95 or 100 per cent power factor may not be able to maintain normal voltage at 80 per cent power factor lagging, simply because the necessary field current would overheat the field windings or the exciter could not produce sufficient voltage. On the other hand, a large load at a leading power factor may cause the alternator voltage to be unstable and difficult of control. This sometimes occurs when an alternator is connected to a long, high-voltage transmission line which is lightly loaded. If the line charging current approximates full load on the alternator, its terminal voltage may rise to a dangerous value.

312. Losses and Efficiency.—The losses in an alternator are:

(*a*) Core loss, including hysteresis and eddy-current loss.
(*b*) Armature copper loss.
(*c*) Excitation loss, including loss in field rheostat.
(*d*) Friction and windage loss.

The core loss depends upon the induced e.m.f. and, therefore, is nearly constant for constant terminal voltage. The armature copper loss depends upon the load and is found as follows:

single-phase,	I^2R;
two-phase,	$2I^2R$;
three-phase (Y or delta)	$\frac{3}{2}I^2R$;

where I is the current per terminal and R is the resistance measured between terminals.[3] The excitation loss is affected by the load and also by the power factor, since it takes more current to maintain normal voltage with a lagging power factor than with unity power factor. The friction and windage loss is constant because the speed is constant. The efficiency is affected by the power factor because it is based on true power and not apparent power.

Example 1.—Referring to the alternator of Examples 1 and 2, Article 307, the resistance between terminals is $R = 0.103$ ohm and the full-load current is 184 amperes. The core loss is 39.5 kw. and the friction and windage loss 29.3 kw. The field current is 102 amperes at 125 volts, when carrying full load at 0.80 power factor.

The armature copper loss is

$$P_a = \tfrac{3}{2} \times 184^2 \times 0.103 = 5240 \text{ watts.}$$

The field loss including loss in field rheostat is

$$P_f = 125 \times 102 = 12{,}750 \text{ watts.}$$

Adding the losses we have

Core loss	39,500	watts
Armature copper loss	5,240	"
Excitation loss	12,750	"
Friction and windage loss	29,300	"
Total	86,790	watts

The efficiency is therefore

$$\frac{2{,}100{,}000 \times 0.8}{(2{,}100{,}000 \times 0.8) + 86{,}790} \times 100 = 95.09 \text{ per cent.}$$

313. Rating and Temperature Guarantees.—Alternators are ordinarily rated at the load which they are capable of carrying continuously without exceeding the temperature guarantees. The rating

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[3] The proof of this in the case of a three-phase machine is as follows. For a Y-connected machine, let r = the resistance between any terminal and the neutral. Then the resistance between terminals is $R = 2r$. The total copper loss is $3 \times I^2 r = \tfrac{3}{2} I^2 R$. For a delta-connected machine, let r be the resistance of each winding. Then the resistance between terminals is

$$R = \frac{1}{\dfrac{1}{r} + \dfrac{1}{2r}} = \frac{2}{3} r.$$

The current per winding is $\dfrac{I}{\sqrt{3}}$ and the total loss is

$$3\left(\frac{I}{\sqrt{3}}\right)^2 r = \frac{3}{2} I^2 R.$$

is expressed in kilovolt-amperes at 0.8 power factor. They are also capable of carrying a momentary overload of 150 per cent with rheostat set for rated load excitation. The temperature rise allowable is 50° C. except in the case of large machines having special insulation composed principally of mica, where considerably higher temperatures are allowed.

The standard voltages are 240, 480, 600, and 2400 volts. The usual voltages above these values are 6600, 11,000, and 13,200. The standard frequencies are 25 and 60 cycles.

314. Automatic Control of Alternator Voltage.—The regulation of modern alternators is rather poor, a regulation of 20 to 25 per cent at 0.80 power factor not being unusual. Therefore, it is generally necessary to provide automatic control of the voltage in order to maintain the terminal voltage sufficiently constant to meet the usual requirements. Compounding of the alternator field is not practicable and is no longer used while hand operation of the alternator field rheostat is suitable only for small machines or where the load does not change suddenly. For general service, therefore, an automatic voltage regulator, sometimes called a Tirrell regulator, is employed. There are several types of these voltage regulators, but the principle of operation is essentially the same. Variation of alternator field current to meet changes in load is accomplished, not by changing the setting of the alternator field rheostat, but by varying the exciter voltage. This is accomplished by means of relay switches which periodically short-circuit the field rheostat of the exciter. Changes in the exciter voltage, and, hence, in the alternator excitation, are secured by varying the rate at which these relay switches open and close. When the alternator requires a small exciting current, the relay switches open and close rapidly, thus leaving the field resistance connected in the exciter circuit a large part of the time, and, hence, giving a low voltage on the exciter. As a higher voltage is required, the speed with which the relay switches open and close is decreased, and the exciter field rheostat remains short-circuited for a longer period, with the result that the exciter voltage is higher. The maximum voltage is secured when the switches remain permanently closed, thus putting full field on the exciter. The relay switches shunting the field rheostat are caused to open and close by means of a magnet, controlled by a master relay which is in turn controlled by variations of the alternator terminal voltage to which it is connected. The action of a properly designed automatic regulator of the type described is very rapid, so that it is possible to maintain constant voltage with a rapidly fluctuating load, without causing more than a momentary change in voltage even when very heavy loads are thrown on or off. It is also possible, by the use of this type of regulator, to overcompound an alternator to compensate for the voltage drop in a feeder.

PROBLEMS ON CHAPTER XXXI

1–31. (*a*) What is the current per terminal of a 2500 kv-a., two-phase, 2400-volt alternator when carrying rated load?

(*b*) What size of rubber-covered wire would be used for the leads on the basis of 25 per cent overload allowed?

2–31. Find the size of varnished-cloth cable required for the leads of a 5000 kv-a., three-phase alternator rated at 11,000 volts, allowing 25 per cent overload.

3–31. What is the horsepower rating of a steam turbine required to drive a 2500 kv-a., three-phase alternator rated on the basis of 0.80 power factor and allowing 25 per cent overload? Efficiency of alternator is 96 per cent.

4–31. An 8-pole, three-phase alternator runs at 375 r.p.m. and has 144 slots with four conductors per slot. The flux per pole is 37,300,000 lines. The conductors for each phase of the winding are connected in series. The machine is Y-connected.

(*a*) What would be the voltage per phase if the windings were concentrated, with coils full pitch?

(*b*) The voltage between terminals is 6600. What is the phase constant?

5–31. The windings of a single-phase alternator consist of two groups of concentrated coils, connected in series. Each group generates a voltage of 200 volts, and the phase angle between groups is 30 degrees.

(*a*) What is the terminal voltage?

(*b*) What is the phase constant?

6–31. A 2500 kv-a. alternator has 40 poles and 240 slots, with 18 conductors per slot. The speed is 75 r.p.m. and the total flux per pole is 9,500,000 lines.

Calculate the effective voltage generated by one conductor.

7–31. The generator in Prob. 6–31 is a three-phase, Y-connected machine, with all the conductors for each phase connected in series. The pitch of the coils equals the pole pitch.

(*a*) Calculate the effective voltage for the group of coils which occupy at a given instant corresponding positions under the pole faces.

(*b*) The various groups in (*a*) are connected in series to form one phase of the three-phase winding. Calculate the phase constant for the winding, and the total voltage generated per phase.

(*c*) What is the voltage between terminals?

8–31. A 2500 kv-a., three-phase, 6600-volt, 25-cycle alternator has its windings Y-connected. The armature resistance for each of the three windings is 0.072 ohm. The synchronous impedance volts between terminals is 3070 for full-load current.

(*a*) What is the impedance of the generator?

(*b*) What is the reactance of the generator?

9–31. (*a*) Referring to the alternator of Prob. 8–31, calculate the approximate voltage regulation at unity power factor.

(*b*) Calculate the regulation for a load at 0.70 power factor lagging.
(*c*) Calculate the regulation for a load at 0.70 power factor leading.

10–31. A 2500 kv-a., three-phase, 60-cycle, 6600-volt alternator has a field resistance of 0.43 ohm and an armature resistance of 0.072 between each terminal and the neutral. The windings are Y-connected. The field current at full load, unity power factor is 200 amperes and at full load 0.80 power factor lagging it is 240 amperes. The friction loss is 35 kw. and the core loss 47.5 kw. Assume friction and core loss constant at either unity power factor or 0.80 lagging power factor.

(*a*) Calculate the full-load efficiency at unity power factor.
(*b*) Calculate the full-load efficiency at 0.80 power factor.

CHAPTER XXXII

SYNCHRONOUS MOTORS

315. Construction.—Synchronous motors are usually built with a stationary armature and a revolving field which is excited from a d.c. source. The arrangement of the parts is much the same as that used for a.c. generators;[1] in fact, an alternator will become a synchronous motor if electricity is supplied to it from an external source. Synchronous motors are generally of the polyphase type, single-phase synchronous motors being used only in very small sizes.

316. Starting a Synchronous Motor.—If a polyphase voltage were applied to the armature of a synchronous motor, a revolving flux would be produced, as in an induction motor. (Article 287.) This revolving flux, which has the same number of poles as the rotor or field magnet, would turn at synchronous speed, and hence would move across the pole faces of the rotor which is assumed to be stationary. When the d.c. excitation is applied to the field winding the field poles will be attracted or repelled by the revolving flux, according to the polarity of the poles which are under the influence of the flux. As an S pole of the revolving flux approaches an N pole of the field magnet, this force tends to turn the rotor in a direction opposite to the direction of motion of the revolving flux. (Fig. 298*a*.) As the S pole leaves the N pole of the field (Fig. 298*b*) the force is in the opposite direction, and tends to drag the rotor around with the flux. Therefore, each time a pole of the revolving flux passes a field pole, the turning force acts first in one direction and then in the other; hence, the resulting torque is zero and the field magnet remains stationary. For this reason, a synchronous motor cannot be started by the force action between the revolving flux and the d.c. field.

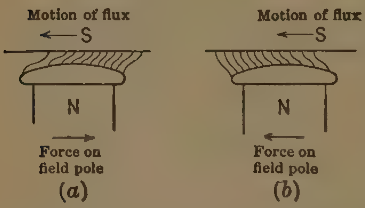

FIG. 298.—Forces on a Synchronous Motor Rotor with Field Excited.

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[1] See Chapter XXXI for details of construction of these machines.

The student should not, however, interpret this statement to mean that synchronous motors are not self-starting; on the contrary, by the use of auxiliary starting devices, it is possible to produce a starting torque comparable to that of a squirrel-cage induction motor. (See Article 321.)

317. Running Conditions.—If the rotor of a synchronous motor is driven at synchronous speed, it will lock in step with the revolving flux when the d.c. excitation is applied. The rotor would then be carried around at synchronous speed by the force due to the action of the revolving flux and the d.c. excitation. The machine would then become a synchronous motor. It was shown in Chapter XXX that the speed of an induction motor is always less than synchronism and that it varies with the load. The speed of a synchronous motor, however, is constant for all loads as long as the supply frequency is constant because there is no slip. The synchronous motor has a "pull-out" point similar to an induction motor. If this pull-out torque is exceeded, the motor will drop out of synchronism and stop.

In Fig. 299, is shown a two-phase two-pole synchronous motor with stationary armature winding. Four positions of the revolving flux are shown. Let the direction of the current or e.m.f. in a winding be positive when the current is flowing out at A or at B, Fig. 299 (*a*) or (*b*), that is; let a positive potential be induced when the N pole of the flux is opposite A or B. To produce a flux in the position shown in Fig. 299 *a* requires current in a positive direction in B and no current in A. This locates one point (*a*) on the curve of current I_B (Fig. 299 *e*). At this same instant (*a*) the e.m.f. induced in the winding A is a maximum in a positive direction and this therefore locates a point on the curve E_A' (Fig. 299 *e*) which shows the e.m.f. induced in phase A. The resistance of the windings would be small as compared with the reactance, and hence we may assume that the current I_B would lag 90 degrees behind the voltage E_B which is impressed on phase B. This fixes the phase position of E_B as shown in Fig. 299 *e*. After one-fourth of a cycle the revolving flux takes the position shown in Fig. 299 *b* and the quantities represented by the sine curves have values indicated at point (*b*) in Fig. 299 *e*. At this instant I_A is a maximum in a negative direction and I_B is zero. The voltage induced in B (E_B') is a maximum in a positive direction. To force the current I_A through phase A, requires an impressed e.m.f., E_A which is 90 degrees ahead of I_A. The phase positions of the currents and voltages are shown in Fig. 299 *e* and *f*. Suppose that the rotor were turning at exactly the same speed as that of the revolving flux and that the d.c. excitation was of such a polarity as to produce a direction of flux like that produced by the revolving flux which is shown in Fig. 299 *a*, *b*, *c*, and *d*. If the d.c. excitation is of such a strength that the induced e.m.f.'s E_A' and E_B'

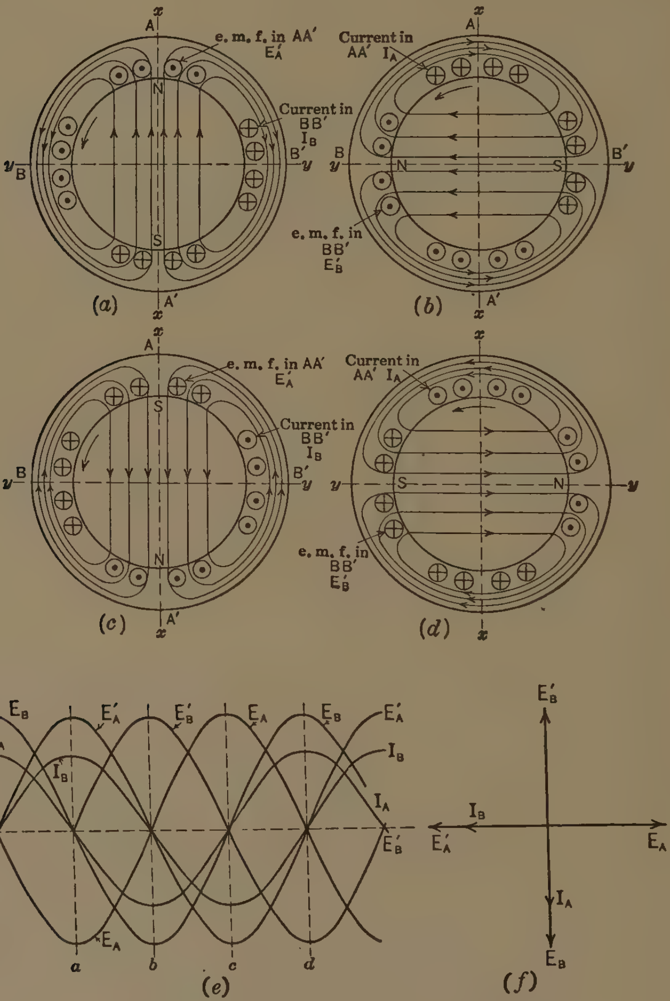


FIG. 299.—Principle of Operation of a Synchronous Motor.

were exactly equal to E_A and E_B, no current would flow in the stator windings, but there would still be the four voltages and the flux produced by the d.c. field. There would, however, be no torque tending to keep the rotor turning. If the d.c. excitation were reduced E_A' and E_B' would become less than E_A and E_B and there would be current in each phase. This is shown in Fig. 300 for phase A. The current would be 90 degrees behind the resultant voltage E_R if the resistance of the winding were neglected. The power input to phase A would be $P_A = E_A I_A \cos 90° = 0$; therefore, in spite of the fact that current flows in the windings, there can be no torque tending to turn the rotor. This can be seen in Fig. 300, where the condition corresponds to position (*a*) in Fig. 299. We have shown that at this instant I_A is zero and I_B is a maximum in a positive direction. (Fig. 299 *e*.) The current in B at this instant, therefore, tends to set up flux along axis *xx* (Fig. 300 *a*) in the same direction as the flux produced by the d.c. excitation. The resultant flux of the motor is, therefore, along the axis *xx*. But at this instant there is no current in the conductors of phase A, since I_A is zero. Hence, there is no torque tending to turn the rotor. Thus, in a

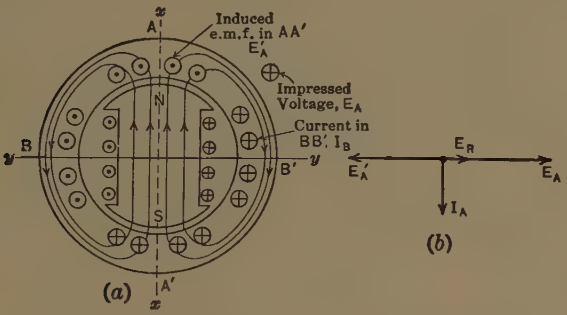


FIG. 300.—Effect of Reducing Field Excitation.

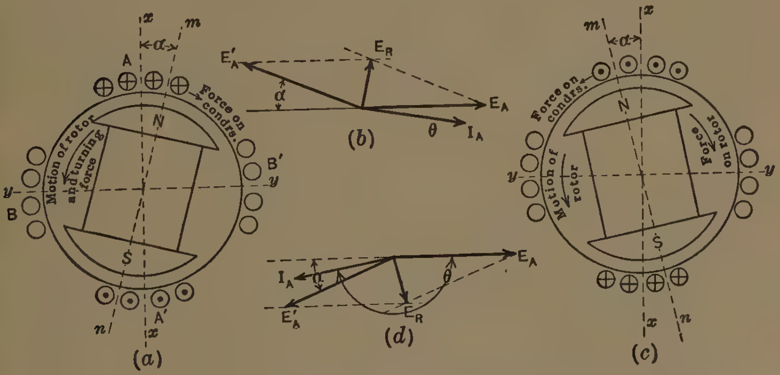


FIG. 301.—Diagrams for a Synchronous Motor.

synchronous motor, whenever the e.m.f. induced in the armature windings is exactly opposed to the impressed e.m.f. there can be no torque although current will flow in the motor armature if the voltages are not equal.

Assume that the induced e.m.f.'s E_A' and E_B' were made equal to the impressed voltages E_A and E_B by adjustment of the field rheostat. We have seen that when this is the case and the rotor has the position shown in Fig. 300 *a*, there would be no current. If, however, the rotor were displaced a small angle behind this position *xx* (Fig. 300), the impressed voltages and counter e.m.f.'s would no longer be exactly opposed and there would be a resultant voltage E_R, Fig. 301, which would cause a current to flow, even if $E_A = E_A'$. This current I_A would be 90 degrees behind E_R and would be nearly in phase with E_A. The power input would be

$$P_A = E_A I_A \cos \theta$$

and, hence, would no longer be zero because θ is less than 90 degrees. This power input would, therefore, serve to drive the machine as a motor. The current would now no longer be 90 space degrees from the position of the magnetic field (Fig. 301 *a*), so there would be a force on the armature conductors due to the field which would tend to move these conductors in a clockwise direction. As the conductors cannot move, however, the field is forced in the opposite direction, and this force keeps the rotor moving in a counter-clockwise direction. If the rotor were displaced ahead of the line *xx*, Fig. 301 *c*, the resultant voltage would be reversed, and the current would take the position shown in Fig. 301 *d*. The power is now

$$P_a = E_A I_A \cos \theta,$$

but θ is greater than 90 degrees; hence the power is negative. This means that the machine is no longer a motor but has become a generator and requires a source of power to drive it. This is also shown in Fig. 301 *c*. The direction of current is such as to force the field in a clockwise direction, opposing rotation. From this analysis, it may be seen that if a machine of this sort is to operate as a synchronous motor, the rotor must be behind the neutral position *xx*, Fig. 301, in order that it may carry a load. If the rotor coincides with this neutral position, no torque results even if current flows in the armature (Fig. 300). If it is ahead of the neutral position, the machine acts as a generator. Therefore, when a load is placed upon a synchronous motor it tends to slow down, but as soon as it moves a fraction of one revolution, or is displaced by the angle

α, current flows in such a direction as to keep it running with a fixed displacement angle α at exactly synchronous speed. If more load is thrown on the machine, angle α increases and thus the current is increased and a greater torque is produced. The torque of a synchronous motor increases as the displacement angle α increases until a maximum value is reached, beyond which with larger values of α and I_A the torque is less. This maximum torque is also called the "pull-out" torque. A synchronous motor will drive its load at a constant speed as long as the load does not exceed this pull-out value; if it does, the motor will drop out of synchronism and will stop. The synchronous motor, therefore, like the induction motor, has a maximum torque value, but the difference is that the synchronous motor maintains constant speed up to the pull-out point, whereas the slip of an induction motor at pull out is large. The maximum torque of a synchronous motor is proportional to the impressed voltage. It depends also upon the strength of the d.c. field. Increasing the field strength gives a greater pull-out torque.

318. Mechanical Analogy.—The action in a synchronous motor can be compared to the transmission of mechanical power through two shafts connected by a spring coupling (Fig. 302). One shaft is provided with a flange F and the other with a hub A with two arms. The two are connected by the springs SS. If both shafts were revolved at exactly the same speed by independent mechanical means, the member A would take the position xx as shown in Fig. 302, and there would be no tangential force exerted by the springs. This corresponds to the condition in the synchronous motor when the field pole is exactly in line with the revolving flux, position xx, Fig. 300. Referring again to the mechanical coupling Fig. 302, if the driving force were removed from the shaft which is connected to the member A, and a retarding force were applied instead, A would tend to slow down and, in doing so, would move to a new position mn. As soon as this occurs, the springs SS exert a tangential force acting in the direction of rotation, so that A would be driven by F through the medium of the springs. The speed of the two shafts would, however, be exactly the same, but there would be an angular displacement of the two. In the case of a synchronous motor (Fig. 301), if there is a retarding force due to a mechanical load on the rotor, it will tend to slow down, but, in doing so, will move backward

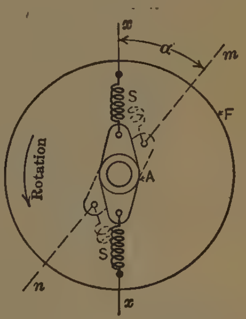


FIG. 302. — Mechanical Analogy for a Synchronous Motor.

through a small angle α. There will then be a tangential force which will keep the rotor turning at exactly the same speed as the revolving flux. The angle α will increase with increased load. In a synchronous motor, this angle is about 20 to 25 electrical degrees when the motor is carrying full load.

319. Effect of Varying the Field Strength.—The power factor of an induction motor is fixed by the load and varies only as the load varies. The power factor of a synchronous motor carrying a definite load may be unity or less than unity, either lagging or leading, depending upon the strength of the d.c. field. In Fig. 303 *a* is a vector diagram for a synchronous motor carrying full load with the field adjusted to give unity power factor. E_A represents the impressed or terminal voltage of the motor and E_A' its counter e.m.f. Assume that the mechanical load is kept constant and the d.c. field strength increased. This would increase the voltage E_A' (Fig. 303 *b*) and cause the current to lead the impressed voltage E_A. The current would also be greater because the product $E_A I_A \cos \theta$ must be constant since the mechanical load on the motor is constant. If the field strength were decreased (Fig. 303 *c*) the current would lag behind the impressed voltage E_A. Therefore, a synchronous motor operated with a weak field produces a lagging load on the line and with a strong field produces a leading load. The variation of power factor and current at different loads and varying field excitation is shown by *V* curves like those in Fig. 304. It will be seen that the current is a minimum at unity power factor and is greater for either lagging power factor (weak field) or leading power factor (strong field).

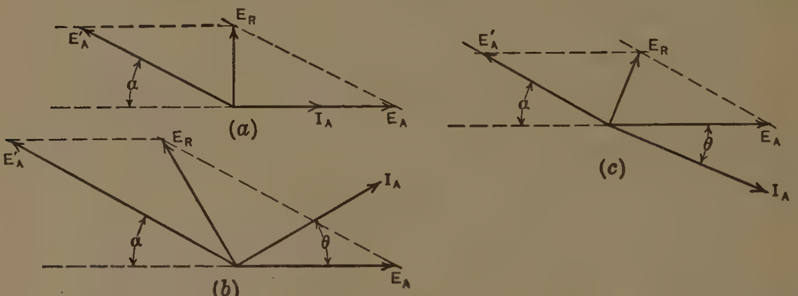


FIG. 303.—Effect of Change of Field Strength of a Synchronous Motor. (*a*) Unity power factor, normal field; (*b*) Leading power factor, strong field; (*c*) Lagging power factor, weak field.

320. Starting Synchronous Motors.—Synchronous motors are usually made self-starting by providing the rotor with a squirrel-cage winding similar to that used in an induction motor. This consists of brass or copper bars placed in slots in the pole faces (Fig. 305). These bars are connected together by end rings. This arrangement is called a damper or amortisseur winding, because it not only provides the necessary starting torque but also serves to damp out oscillations or "hunting" of the rotor when the machine is operating at synchronous speed. (See Article 327.) During the starting period, the d.c. excitation is removed and the revolving flux produced by the alternating current in the armature windings acts on this squirrel-cage winding as it does in the induction motor, thereby producing a starting torque. If the field circuit is left open during the starting period, a high voltage is induced therein, due to the armature flux. This voltage may be as high as 2000 volts, and, therefore, it would be dangerous to come in contact with the field circuit of a synchronous motor while it is being started. Because of this high voltage, it is customary to provide extra insulation on the field winding to protect it against breakdown during the starting period. If the field winding is short-circuited, the induced voltage is reduced, due to the effect of the current which flows in the field circuit, but under these conditions the starting torque is reduced. If the field circuit is closed through a resistance, the strain on the insulation is reduced while the effect on the torque is less than when

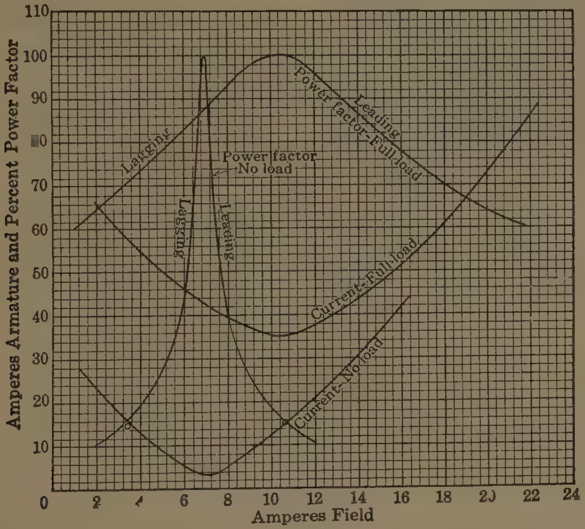


FIG. 304.—V-curves for a Synchronous Motor. 15 kv-a., three-phase, 60-cycle, 220 volts.

the field is short-circuited. It is customary, therefore, to close the field circuit through the regular field rheostat or, if the motor has a direct-connected exciter, to leave the exciter armature connected to the field during the starting period. As the machine approaches synchronous speed, there is an increased torque produced if the field winding is short-circuited. When starting in the manner described, a reduced a.c. voltage produced by auto-transformers, is applied to the armature until it reaches a speed slightly less than synchronous speed due to the induction motor action described. The d.c. excitation is then applied, which thereby causes the motor to "pull in" and run at synchronous speed. The full a.c. voltage is then applied, and the motor is ready to carry its load. If the field excitation were applied after the motor had been shifted to full voltage, a very much greater rush of current would take place when the motor pulled into synchronism.

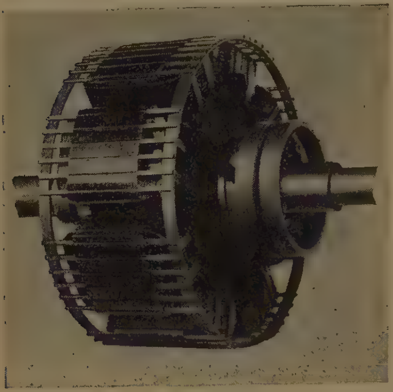


FIG. 305.—Rotor of Synchronous Motor. Showing cage damper winding on pole faces. *The Westinghouse Electric & Mfg. Co.*

When a synchronous motor is started as an induction motor in the manner just described, it draws a very heavy current from the supply, in fact the kv-a. demand is somewhat greater than that of a squirrel-cage motor when starting. When the motor is large as compared with the generator capacity or when the starting current must be limited, it is necessary to use a starting motor which may be an induction motor specially designed to produce a high torque, or the exciter itself may be used as a motor if there is a source of direct current available. When starting in this manner, the a.c. supply is not connected to the machine until it has reached exactly synchronous speed as determined by a synchronism indicator. (See Article 326.) When the motor is started in this way, it is disconnected from its load until it has been put in synchronism, whereas, self-starting synchronous motors are designed to start a considerable load. The starting performance of synchronous motors may be varied by changes in design and usually it is necessary to have a knowledge of the torque required to start the load before the motor can be designed properly. A self-starting synchronous motor designed to start a motor-generator set, a frequency changer, or similar loads requiring a small torque below synchronism, would require a starting torque of about 15 per cent of full-load torque in order to overcome the static friction. As soon as the machine starts, the torque decreases and at the pull-in point, which occurs at about 95 per cent of synchronism, about 3 per cent only of full-load torque is required. A self-starting synchronous motor designed to start a centrifugal pump, or a blower, would be required to produce a larger starting torque. For example, about 30 per cent of full-load torque would be required to overcome the static friction. At the pull-in point, at 95 per cent of synchronism, the torque is about 50 per cent of full-load torque. The torque due to the squirrel-cage winding on the field poles is zero at synchronism, so that the field magnets must be excited at the pull-in point in order to synchronize the motor. A synchronous motor must have sufficient torque to start its load and bring it to a speed within approximately 2 to 5 per cent of synchronism; otherwise, the motor will not "pull-in" when the d.c. excitation is applied. The required torque at the pull-in point depends upon the kind of load on the motor; hence the designer must know what sort of a load the synchronous motor must start before he can properly design a machine to give the necessary starting and pull-in torques.

321. Performance of Commercial Motors.—The starting characteristics of the synchronous motor are varied to suit the requirements of the load. Some loads, such as motor-generator sets, require a torque at starting higher than at pull-in, while others, like centrifugal pumps, require a higher pull-in torque due to the churning effect of the water in the pump as it approaches full speed. The motor, therefore, must be adapted to meet the load requirements. One manufacturer builds self-starting synchronous motors which have a starting torque equal to 50 per cent of full-load running torque, with 70 per cent voltage applied. The pull-in torque is about 50 per cent of full-load torque. As these machines have direct-connected exciters with the field adjusted for unity power factor operation at full load, no field rheostat is required. The armature of the exciter is left connected to the motor field at all times, thus requiring no manipulation on the part of the operator, other than that of applying reduced voltage to the motor and of throwing over to full voltage after the machine has pulled into synchronism. The starting process is, therefore, as simple as that for starting a squirrel-cage induction motor.

The pull-out or maximum torque of a synchronous motor may be varied somewhat by proper design, but is usually 1.5 to 3.0 times full load torque, depending upon the design. The pull-out torque is proportional to the strength of the d.c. field and the impressed voltage. Anything which causes the supply voltage to drop results in a lower pull-out torque. Therefore an excessive drop in this voltage might result in the motor becoming stalled if it were carrying a heavy load. Synchronous motors driving reciprocating machines, such as air compressors or pumps, are subject to variation of load torque at different times during each revolution. This would cause a periodic variation of the displacement angle of the rotor and a corresponding fluctuation in the current drawn from the supply. Heavy flywheels are, therefore, used to reduce this fluctuation.

322. Use of a Synchronous Motor to Change the Power Factor of a System.—A synchronous motor can be used to change the power factor of the system to which it is connected by adjusting the field excitation of the motor. Where the load consists principally of induction motors, the system power factor would seldom be higher than 0.80 lagging, and it is more likely to be considerably lower. Therefore, if the power factor can be raised to approximately unity, better voltage regulation and lower system losses will result. To improve the power factor, synchronous motors are connected to the system, preferably at one or more substations, and these motors are operated at a leading power factor in order to produce a leading component of current to counteract the lagging component of the induction motors. If the synchronous motors are operated without mechanical load, at approximately zero power factor leading, they are called synchronous condensers; because, like static condensers, they produce a leading current. Where a large leading current is required, it may be secured more cheaply by the use of synchronous condensers than by static condensers, such as were described in Article 68.

Example 1.—It is desired to install a synchronous motor to improve the power factor of an induction motor load aggregating 1000 kw. at 0.75 power factor. Assume that the synchronous motor is to operate at a power factor of 0.70 leading.

(*a*) Calculate the size of synchronous motor required to give unity power factor for the combined induction and synchronous motor loads.

(*b*) How much mechanical load could the synchronous motor carry?

(*a*) The total kv-a. for the induction motors is $1000 \div 0.75 = 1333$ kv-a. (Fig. 306.)

The reactive component is

$$1333 \times \sin\theta = 1333 \times 0.661 = 882 \text{ kv-a.}$$

If the total power factor is to be unity, the reactive component of the synchronous motor must also be 882 kv-a. Hence the input of the synchronous motor must be

$$\frac{882}{\sin\phi}=\frac{882}{0.713}=1235 \text{ kv-a.}$$

The power input to the synchronous motor is

$$1235\times\cos\phi=1235\times0.70=865 \text{ kw.}$$

Neglecting the losses, the mechanical power available from the synchronous motor would be

$$865\div0.746=1160 \text{ hp.}$$

As a rule, it is not economical to install sufficient capacity in synchronous machinery to make the power factor exactly unity. If the resulting power factor is 0.90 or 0.95 lagging, this is sufficiently close to unity to give satisfactory voltage regulation, and the additional cost of equipment to raise the power factor to unity is usually not justified.

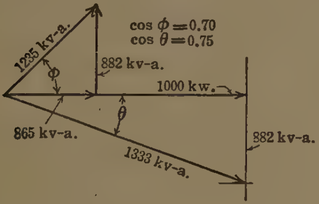


FIG. 306.

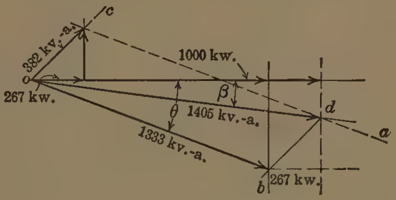


FIG. 307.

FIG. 306.—Diagram for a Synchronous Motor Used to Improve Power Factor. Unity power factor.

FIG. 307.—Diagram for a Synchronous Motor Used to Improve Power Factor. Lagging power factor of 0.90.

Example 2. Referring to Example 1, find the size of synchronous motor required to raise the total power factor to 0.90 lagging, assuming that the synchronous motor operates at 0.70 leading power factor. Referring to Fig. 307, draw *o-a* at an angle β such that $\cos\beta=0.90$. The line *o-b* represents the induction motor kv-a. From point *o*, the line *o-c* is drawn at an angle ϕ to represent the synchronous motor kv-a.; where $\cos\phi=0.70$. Line *b-d* is drawn parallel to *o-c*. The point *d*, where this line cuts *o-a*, gives *o-d* which represents the total kv-a. The angle *o-b-d*$=93.1°$. The angle *o-d-b*$=71.3°$.

Let x equal the total kv-a. load. The induction motor load is

$$o\text{-}b=\frac{1000}{0.75}=1333 \text{ kv-a.}$$

$$\text{Then }\frac{\sin 93.1°}{x}=\frac{\sin 71.3°}{1333}$$

and $x = 1405$ kv-a.

The power supplied to the synchronous motor is

$$P_S = (1405 \times 0.90) - 1000 = 267 \text{ kw.}$$

Hence, the input to the synchronous motor is

$$\frac{267}{0.70} = 382 \text{ kv-a.}$$

If the efficiency of the synchronous motor were 90 per cent, the output would be $267 \times 0.90 = 240$ kw. or 322 hp. Therefore, a synchronous motor rated at 382 kv-a. input, and operated at 0.70 power factor leading, would raise the power factor of the system to 0.90 and would also drive a mechanical load of 322 hp.

323. Use of a Synchronous Condenser to Control the Voltage of a Transmission Line.—The problem of securing satisfactory voltage regulation at the end of a long, high-voltage transmission line is a difficult one, owing to the large charging current of the line and its high reactance. At light loads, the capacity current of the line causes the generator voltage to rise considerably above normal, and it is difficult to maintain the correct voltage at the substation end of the line. With heavy loads at a lagging power factor, the large reactive drop in the line results in low voltage at the substation. It is common practice to install a synchronous condenser in the substation and to control the field of this machine by means of a voltage relay. When the load on the system is light, the voltage at the substation would tend to rise above normal. This relay would weaken the synchronous condenser field, causing it to take a lagging load from the line and, thereby, prevent the rise in voltage. When the load on the substation is heavy and has a lagging power factor the substation voltage would tend to fall below normal; the relay would then increase the field of the synchronous condenser so that it would take a leading load which would counteract the effect of the lagging load and keep the substation voltage normal.

324. Parallel Operation of Alternators.—In order that two alternators shall operate in parallel, it is necessary that the voltages of the two machines shall be equal and opposed to each other at each instant. To fulfill these requirements it is necessary that:

(*a*) The effective value of the induced voltages of the two machines shall be the same.

(*b*) The frequency of both machines shall be exactly the same.

(*c*) The wave form of the two shall be alike.

(*d*) The phase position shall be such that the two voltages oppose each other, considering the circuit between machines.

(*e*) For polyphase machines, the phase rotation shall be the same.

All of these requirements must be fulfilled if there is to be no circulating current between the two machines. (Fig. 308.) In practice, however, a small difference in effective voltage or in wave form is not objectionable as the circulating current which flows is not excessive. The frequency of both machines must, however, be exactly alike. This means that, if they have the same number of poles, they must operate at exactly the same speed; that is they must operate synchronously. Furthermore, the voltages of the two machines must be 180 degrees apart considering the circuit between machines. (Fig. 308 *b*.) Phase rotation is defined as the order in which successive terminals reach a positive maximum value. Thus, suppose that the corresponding terminals of the two machines are numbered 1, 2, and 3, Fig. 309. At a certain instant during one revolution terminals 1_A and 1_B will be positive with respect to 2_A and 2_B. If the phase rotation of both machines is the same, the next terminal to be positive would be 2_A and 2_B, then 3_A and 3_B. If the phase rotation of machine *B* were reversed; terminal 3 would become positive before 2. The connections between machines must be so made that the terminals connected to the same bus-bar shall have the same polarity at the same instant. It is not necessary to reverse the direction of rotation of either generator to produce correct phase rotation; as this can be done by a change of connec-

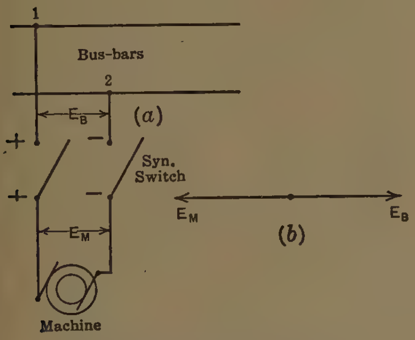
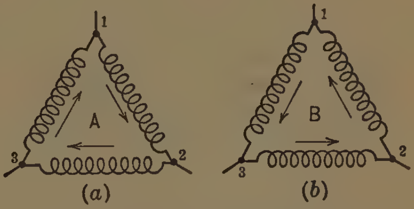


FIG. 308.—Parallel Operation of Alternators.



(*a*) (*b*)

FIG. 309.—Phase Rotation. (*a*) Rotation 1–2–3; (*b*) Rotation, 1–3–2.

tions. The process of securing the proper phase rotation is called "phasing out." This test is required only when the machines are paralleled for the first time. Subsequently to this, it is necessary only to synchronize the two, that is connect them together when their voltages oppose each other as illustrated in Fig. 308 *a*.

It would not be possible, by any form of mechanical governor on the prime mover, to keep two alternators at exactly the same frequency and with the proper phase angle between the voltages; instead, this is accomplished by a circulating or synchronizing current which flows between machines. Assume that we have two alternators producing equal terminal voltages and that they are operating at exactly the same frequency. If they are connected together when these two voltages are 180 degrees apart (Fig. 310), no current can circulate between the machines because the resultant voltage is zero. (See Article 317.) If the prime mover of B tends to slow down, the rotor of B is displaced backward and the e.m.f. of B is represented by E_B'. (Fig. 310 *b*.) We have seen that this would cause a current to flow in such a direction as to produce motor action (Article 317) which would tend to pull the machine back to the normal position E_B. Similarly, if the prime mover tended to increase in speed, the voltage of B would take a position of E_B'', when the current which flows would produce a generator action tending to slow down the machine and force it back to position E_B. Any tendency of the rotor of one machine to move away from its normal position, therefore, results in a current circulating between the two machines in such a direction as to pull the rotor back to the normal position. This current maintains exact equality of frequency and phase position and is, therefore, called the synchronizing current. In normal operation, there is more or less synchronizing current to compensate for inequalities in the driving torque, and the rotors of the alternators oscillate slightly on either side of their normal position. In the case of steam-turbine driven units, variations of driving torque are usually small and the synchronizing current is small. In the case of reciprocating units, there is a variation of

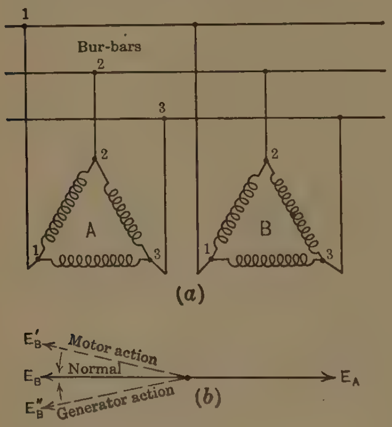


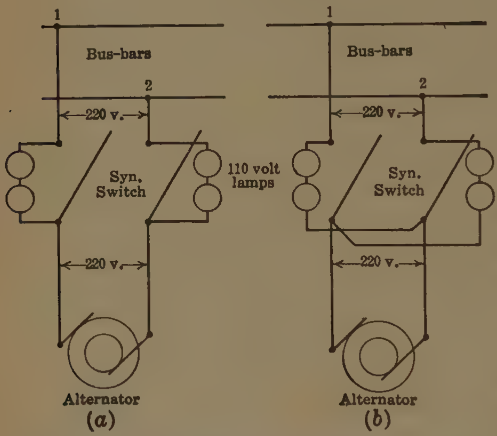
FIG. 310.—Synchronizing Effect in an Alternator.

torque during each revolution. Consequently there is a larger synchronizing current. When two alternators are carrying an external load, the normal position of the rotors of the two machines would be such as to maintain the normal phase position E_B as shown in Fig. 310 *b* and reduce the synchronizing current to zero. If one prime mover tends to slow down, the rotor of this machine will be displaced behind its normal position with respect to the other machine. This causes a *motor* current to flow which tends to pull the rotor ahead again. This current also flows in the other machine and its direction in this machine is such as to produce a *generator* action. The opposite effect occurs if a machine tends to move ahead of its normal position. In this case the synchronizing current produces generator action in the machine which is ahead, which pulls the rotor back to its normal position.

If the field strength of machine *B* is increased, its generated e.m.f. tends to be higher than *A* but of course the terminal voltage of both machines must remain the same since they are connected together. We have seen (Article 319) that a circulating current will flow between machines in such a direction as to tend to weaken the field of machine *B* and strengthen the field of *A*, but this current is practically 90 degrees from the terminal voltage and, therefore, represents very little power. The division of load between alternators is, therefore, affected only slightly by changes in the field strength of the machines. This is different from the action in d.c. generators where the division of load can be controlled entirely by adjustment of the field strength. The adjustment of an alternator field rheostat, therefore, has practically no effect upon the power delivered by the machine, but it does change the armature current. This would change the power factor of the load carried by the individual machines but could not, of course, affect the power factor of the total load, as this is fixed entirely by the loads on the various feeders.

A steam engine can be made to carry additional load by allowing it to slow down slightly so that the governor can act and admit more steam, or it may be made to carry more load without change in speed by changing the adjustment of the governor. In the case of an alternator in parallel with others, the amount of load which it carries cannot be adjusted by changing the speed, since it is held at synchronous speed by the synchronizing current. Therefore, the alternator can be made to take a greater proportion of the total load only by adjusting the governor of the prime mover so that more steam is admitted to the engine, thus allowing it to carry more load at the same speed. Adjustment of load between alternators is, therefore, made by adjustment of the engine governor and not by change in the excitation.

325. Synchronizing and Phasing Out Alternators.—In the preceding article, it was shown that before two alternators are connected in parallel, they must both produce exactly the same frequency and the voltages of the machines must be opposed to each other. The process of paralleling two alternators, after making suitable tests to insure that these conditions are fulfilled, is called synchronizing. These tests may be made by means of incandescent lamps connected across the terminals of the switch used for paralleling. (Fig. 311.) The switch is left open and the machine brought up to normal speed. The terminal voltage of the incoming machine is then adjusted to be approximately the same as the bus-bar voltage, 220 volts in the case shown in Fig. 311. If the voltages of machine and bus were exactly opposed to each other, there would be no difference of potential across the switch, and the lamps would be dark. If the voltages were not 180 degrees apart, however, there would be a resultant voltage which would cause the lamps to burn more or less brightly depending upon the phase angle between the voltages. Obviously the switch should be closed when the voltage across the switch is zero, that is when the lamps are dark. If the frequency of the incoming machine were not exactly the same as that of the bus, the lamps would be alternately bright and dark at frequent intervals, indicating that the phase angle between the two voltages is changing. The speed of the incoming machine is adjusted until the pulsations of the lamps are rather slow, and the switch is closed when the lamps are dark. By cross-connecting the lamps (Fig. 311 *b*), the proper time for closing the switch would be indicated when the lamps are bright. This gives a more accurate indication of the point when the voltages are exactly opposed because, with the dark method (Fig. 311 *a*) there

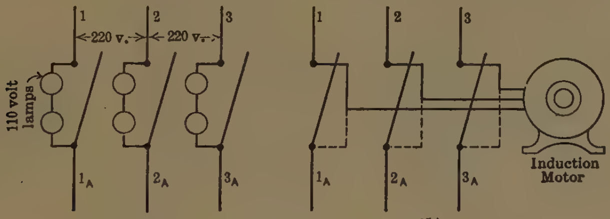


(*a*) (*b*)

FIG. 311.—Synchronizing Connections. (*a*) Dark-lamp synchronizing; (*b*) Bright-lamp synchronizing.

may be a considerable resultant voltage but still not enough to show in the lamps. For high-voltage machines, transformers are used to step down the machine voltage to a value suitable for the lamps, but the principle of operation is the same.

The process of synchronizing polyphase machines is the same as for single phase, but, when the former are first put in service, a preliminary test must be made to check the connections. This test is called "phasing out" a machine and must always be applied to a polyphase synchronous machine, whether alternator, synchronous motor, or synchronous converter, before the machine can be connected to the system for the first time. The phasing-out test is made to check the phase rotation of the new machine and to ensure that it corresponds to that of the supply. The term phase rotation does not indicate the direction of rotation of the machine, but refers to the order in which successive terminals become positive. (See Article 324.) A polyphase machine can be phased out by means of lamps connected as shown in Fig. 312. If the phase rotation of the new machine and the bus-bars are the same, all lamps will be dark or bright at the same instant, but if the new machine has the wrong phase rotation, there will never be a time when all the lamps will be dark simultaneously; instead, the lamps in the different legs of the circuit will be successively dark. Hence, it is not possible to find an instant when the switch should be closed. To change the phase rotation of a machine it is not necessary to change its direction of rotation, although, if that were done, the correct phase rotation would be secured; instead, any two leads may be interchanged as for example, 1_A and 2_A (Fig. 312). In the case of a two-phase machine, the leads of one phase would be interchanged. Another convenient method for checking phase rotation is to use an induction motor (Fig. 312 *b*) running it first from the bus-bars and then shifting the connections to the new machine, taking care that the same lead

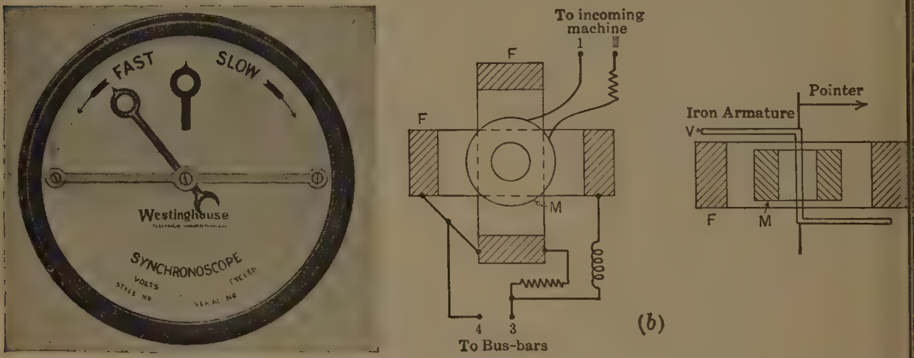


(*a*) (*b*)

FIG. 312.—Methods of Phasing-out Polyphase Machines.

from the motor is connected to corresponding terminals of bus-bars or machine in each case. If the motor revolves in the same direction when run from bus-bars or from the machine, the phase rotation is correct; if it reverses the rotation is reversed. After a polyphase machine has once been phased-out, it is necessary to synchronize only on one phase, because the other phases will be correct as long as the machine connections are not changed. A machine, therefore, need be phased-out only when the connections are changed, but it must be synchronized every time it is paralleled with another machine. Synchronizing connections are therefore the permanent connections and are made single phase.

326. The synchronoscope is an instrument used in paralleling alternators. One type of this device is shown in Fig. 313. The pointer is free to revolve in either direction and has no control springs. When connected to an alternator which is to be paralleled with the bus-bars, the pointer revolves in either the fast or slow direction, thereby indicating how the speed of the alternator should be changed. When the frequency of the bus-bars and the alternator is the same, the pointer stands still. When the pointer is over the index at the top of the scale, the correct position is indicated and the machine switch can be closed, provided the pointer remains stationary at that point or is moving slowly. The principle of operation is shown in Fig. 313 *b*. Two stationary coils *F*, *F*, set at an angle, are supplied with potential from the bus-bars. By means of a split-phase arrangement,[2] a rotating field is produced by these coils. The shaft carrying



(*a*) (*b*)

FIG. 313.—Synchronoscope. *The Westinghouse Electric & Mfg. Co.*

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[2] The method of producing a rotating field by phase-splitting is explained in Chapter XXXV, Article 346.

the pointer also has a small soft iron armature, *V*, which is surrounded by the coils *FF* and is under the influence of the revolving field which they produce. A third coil, *M*, which is also stationary, is connected to the incoming machine and magnetizes the iron armature *V*, with an alternating polarity. This magnetic effect is not constant but drops to zero every half cycle. The armature will, therefore, set itself in such a position that it will coincide with the rotating field when the coil *M* is producing a maximum strength of field. The pointer, therefore, will change its position as the phase of the incoming machine changes. If the frequency of bus-bars and machine is different, this phase angle changes and the pointer revolves; if the frequencies are alike, the pointer stands still. When the pointer is over the index at the top, the bus-bar and machine voltages are opposed and the switch can be closed. The synchronoscope gives a more accurate indication than the synchronizing lamps and also indicates whether the incoming machine is fast or slow. It is generally used for synchronizing large machines.

327. Hunting of Synchronous Machinery.—When two alternators are operating in parallel, it has been shown that their rotors maintain the same relative position, and the voltages of the two machines are opposed. (Article 324.) Any variation of the driving torque will tend to change this angle and cause a synchronizing current to flow, which will pull the two machines back to the proper phase position. In the case of reciprocating steam or gas engines, the turning torque varies during each stroke, and this variation is partly corrected by the use of flywheels. This results in a regular motion of the rotor of the alternator on either side of the normal position with a resulting synchronizing current. This is called "hunting." There is always more or less hunting, particularly in the case of reciprocating-engine drives, but sometimes these oscillations become so great as to produce a very large synchronizing current and in some cases to cause the machines to drop out of step. Hunting may also occur between a synchronous motor or converter and the driving alternator, if the rate of variation of the angular velocity of the alternator happens to coincide with the natural period of vibration of the rotor of the motor. Under these conditions, a heavy circulating current would flow and the motor would probably fall out of synchronism. When excessive hunting occurs, it may sometimes be corrected by adjustment of the dash-pot on the governor to make it somewhat more sluggish. Another method is to increase the weight of the flywheel on the prime mover. With alternators driven by reciprocating engines, these remedies are not sufficient, so damper windings

(See Article 320) are placed on the field poles. Any variation of angular position of the rotor makes the armature flux cut this damper winding and produce a current therein, which tends to oppose the motion of the rotor, thus damping out the oscillation. Particularly in gas-engine driven alternators, it is necessary to have very efficient damper windings. For turbo-generators or machines driven from water wheels, the turning effort is more nearly uniform; hence damper windings are ordinarily not used. In the case of synchronous motors, the starting winding in the pole faces serves as a damper winding to reduce the tendency to hunt. Synchronous converters are generally provided with damper windings for the same purpose. (See Article 335.)

PROBLEMS ON CHAPTER XXXII

1–32. Two 2500 kv-a., 2300-volt, three-phase generators are arranged to be coupled together mechanically and driven from the same engine. The full-load impedance voltage for each machine is 1000 volts between terminals. The resistance of the windings may be neglected.

(*a*) Generator *A* is so coupled to generator *B* that the voltages of the two machines are exactly opposed when the switches are closed. (180 degrees out of phase considering the circuit through the armatures of the two machines.) If the two voltages are adjusted to normal value, and the switches closed, what current would flow?

(*b*) If the machines were coupled as in (*a*) and the voltage of *B* is made 3000 and *A*, 2300 before the switches are closed, calculate the current which would flow.

(*c*) What is the power in each machine?

2–32. The coupling between *A* and *B* is changed, so that the voltage of *B* is 190 degrees behind *A*. The voltages of *A* and *B* are each made 2300 volts and then the switches are closed.

(*a*) What current would flow?

(*b*) What is the power for *A* and *B*?

(*c*) If the angle were made 220 degrees, what would be the current and power for each machine?

3–32. If the coupling were changed so that the angle were 170 degrees between *A* and *B*, what would be the current and power in each machine?

4–32. The machines of Prob. 1–32 are so coupled that their voltages are 180 degrees apart. The voltages are made normal, and then the machines are connected together. The coupling is then opened, leaving *A* alone driven by the engine.

(*a*) What effect will this have on the speed of *B*?

(*b*) If a mechanical load were placed upon *B*, what effect will this have on the speed?

(*c*) What effect would this load have upon the phase angle between the voltages of the two machines?

5–32. Assume a three-phase, 2500 kv-a., 2300-volt, synchronous motor, operated at a 45-degree lead of current with respect to voltage. Assume motor is operating at full kv-a. rating.

(*a*) What is the mechanical horsepower output assuming an efficiency of 95 per cent?

(*b*) With three-phase induction motors giving a power factor of 0.80, calculate what horsepower in these motors could be used if the lagging component is to be entirely neutralized by the synchronous motor.

(*c*) Calculate the same as in (*b*) except that the resulting power factor is to be 0.90 lagging.

6–32. Assume a load of induction motors aggregating 1000 kw. at 0.80 power factor.

Assume a synchronous motor is to be operated at 0.70 power factor leading.

(*a*) Find the kv-a. rating of the synchronous motor required to neutralize entirely the wattless component of the induction motor load, and to give unity power factor for the entire load.

(*b*) Calculate the same if the entire load is to have a power factor of 0.95 lagging.

(*c*) Calculate the same if the entire load is to have a power factor of 0.90 lagging.

7–32. A certain plant has a synchronous motor load of 500 hp. at 0.80 power factor leading, an induction motor load of 1500 hp. at an average power factor of 0.75 lagging and a lighting load of 100 kw. Assume efficiency of synchronous motors is 92 per cent, of induction motors 90 per cent, and line losses 8 per cent.

(*a*) Calculate the total load, in kilowatts, at the power house.

(*b*) What is the total kv-a. at the power house?

(*c*) What is the total power factor?

8–32. Two 2500 kv-a., 2300-volt, 60-cycle, three-phase alternators are to be connected in parallel. The impedance volts at full load current is 1000 for each machine.

(*a*) Generator No. 1 is adjusted to 2300 volts and No. 2 to 1800 volts, and then the two are synchronized. What would be the resulting current if they are paralleled exactly in step as indicated by the synchronoscope?

(*b*) If each generator were adjusted to 2300 volts and they were then thrown together exactly 180 degrees out of step as shown by the synchronoscope, what would be the current which would flow? It should be noted in this, and the two problems following, that the momentary rush of current when the machines are thrown together is greater than the calculated values. (See Article 226.)

9–32. If the voltages for Prob. 8–32 are normal and the machines are thrown in 10 degrees out of step as shown by the synchronoscope, what would be the current?

10–32. In Prob. 9–32, assume that the circulating current lags 80 degrees behind the resultant voltage.

Calculate the power for each machine.

11–32. Draw a diagram of connections for phasing-out a three-phase, 2400-volt alternator, using voltage transformers and incandescent lamps. Specify voltage ratings of transformers and lamps.

CHAPTER XXXIII

SYNCHRONOUS CONVERTERS

328. Principle of Operation.—We have seen (Article 56) that the e.m.f. generated in the coils of a d.c. generator is alternating, therefore, if connection were made directly to the coils of such a machine, an alternating e.m.f. would be produced at the taps. In Fig. 314, is shown a bipolar d.c. generator with an armature winding having two taps *a* and *b* at diametrically opposite points. When this armature is driven mechanically in the usual way, an alternating e.m.f. is produced at the two slip-rings which are connected to taps *a* and *b*, while, at the same time, a steady d.c. voltage will be produced at the brushes which bear on the commutator. At the instant when the armature is in the position shown in Fig. 314, the a.c. voltage would equal the d.c. voltage and would be 100 volts in this case, because the a.c. taps are in direct contact with the two d.c. brushes. When the armature has turned one-quarter of a revolution, tap *a* will be in position 2, opposite the N pole, and tap *b* will be at 4. The e.m.f. between 2 and 4 is the resultant of the e.m.f.'s of the conductors in either path, but this is zero, since the voltage induced in the conductors between 1 and 2 is exactly equal and opposite to the voltage between 1 and 4. Therefore, with the armature in this second position with taps, *a* and *b* at 2 and 4 respectively, the a.c. voltage is zero. Similar reasoning will show that when tap *a*

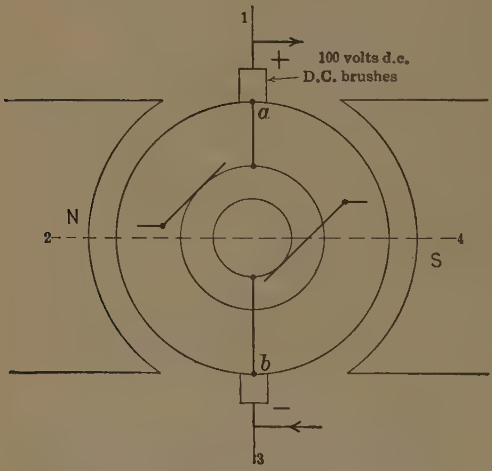


Fig. 314.—Diagram for a Single-phase Converter.

is at 3, the a.c. voltage is opposite to that corresponding to position 1 and in position 4 it is again zero. Therefore, one revolution of the armature causes the a.c. voltage to pass through one cycle. When the taps are in line with the brushes, the a.c. voltage is a maximum; hence, the maximum value of the a.c. wave is equal to the d.c. voltage or 100 volts in the case illustrated. If the a.c. wave is sinusoidal, the effective voltage between the a.c. terminals is

$$E_{ac}=E_{dc}\frac{1}{\sqrt{2}}=0.707E_{dc},$$

or 70.7 volts in this case. This ratio is fixed, and therefore, if the d.c. voltage is changed by varying the field strength, the a.c. voltage will also be changed because both voltages are produced in the same armature winding by the same field flux. A machine of this type, when driven mechanically, is called a double-current generator because it can supply both alternating and direct current at the same time. If the slip-rings of this machine were connected to an a.c. supply, it would operate as a synchronous motor and at the same time, there would be a d.c. voltage at the brushes, and the machine would convert from a.c. to d.c. When operated in this manner, the machine is called a synchronous converter. If the d.c. side of the

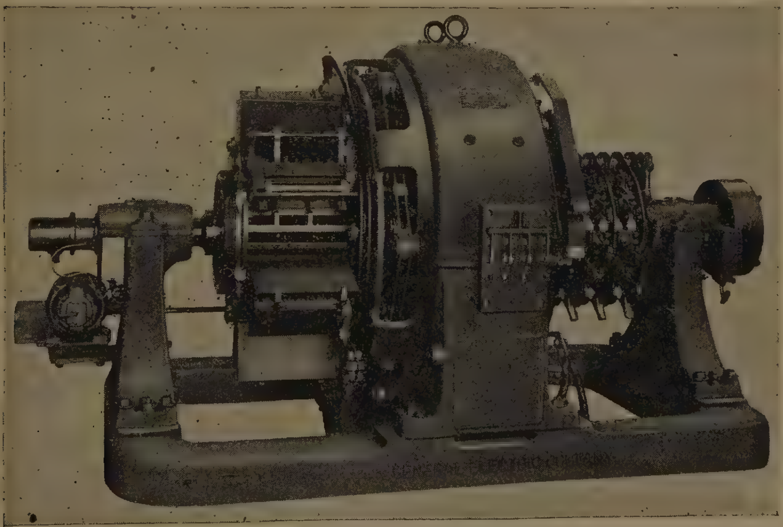
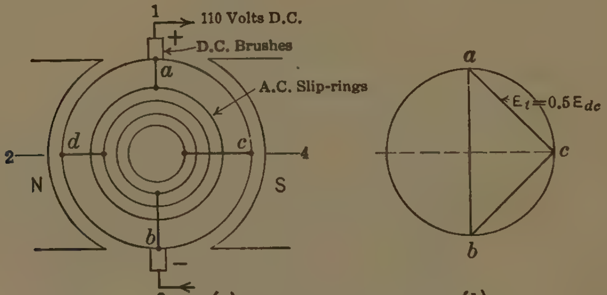


FIG. 315.—Synchronous Converter. 500 kw., 600 volts, 60 cycles. Equipped for automatic operation. *The General Electric Co.*

machine were connected to a source of power, the machine would operate as a d.c. motor, and an alternating current could be taken from the slip-rings. When operated in this manner, the machine is called an inverted synchronous converter. Synchronous converters are generally used to convert from alternating to direct current.

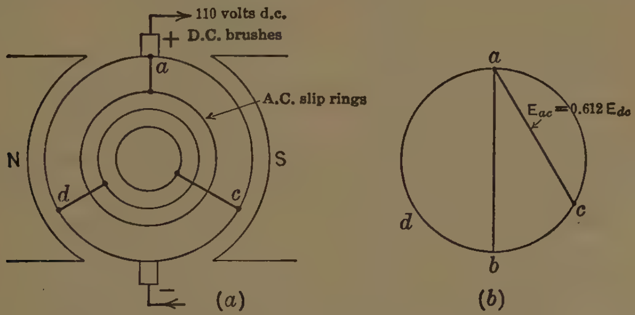
Synchronous converters have their stationary field magnets excited by a shunt winding, and sometimes a series winding is also provided for voltage regulation. Commutating poles are also very commonly used. As constructed, the synchronous converter has very much the same appearance as a d.c. generator except for the slip-rings, which are at the opposite end of the armature core from the commutator. (Fig. 315.) It will also be noted that the commutator is proportionally larger for a converter of a given size of frame because a considerably greater kilowatt output is obtained from a machine operated as a converter compared with its output as a d.c. generator. (See Article 333.) The machine illustrated in Fig. 314 is a single-phase converter. These are used only in very small sizes, for special purposes, such as the operation of arc lamps for moving picture projectors. For general power service, polyphase converters are employed. The principle of operation is the same, but more taps and slip-rings are required.



(a) (b)

FIG. 316.—Diagram for a Two-phase Converter.

329. Polyphase Converters.—Assume that four taps were made at equidistant points on a bipolar armature (Fig. 316) and four slip-rings with brushes were used. Between taps *a* and *b*, a single-phase a.c. voltage of 70.7 volts effective would be produced as was explained in the previous article. Similarly an a.c. voltage of 70.7 volts would be produced between taps *d* and *c*, but this voltage would be a maximum when the first voltage was zero; that is, these two single-phase voltages are equal and 90 electrical degrees apart. Hence the two, considered together, result in a two-phase or quarter-phase system. The machine would be known as a two-phase converter and could be operated from a two-phase, four-wire supply. It can be shown that, if the diameter of a circle drawn to a suitable scale represent the single-phase voltage between two taps diametrically opposite on the armature, then the voltage between taps at any other points on the armature is represented by the length of the chord connecting the proper points on the circle. Thus the diameter *ab*, Fig. 316 *b*, represents the voltage of one phase of the two-phase machine. Then the voltage between taps in this machine, *a* to *c*, or *c* to *b*, etc., is equal to the length of the chord connecting these points. If E_{ac} is the phase voltage represented by the diameter *ab*, then the voltage between adjacent taps is:



(*a*) (*b*)

FIG. 317.—Diagram for a Three-phase Converter.

$$E_t=\sqrt{2\times\left(\frac{E_{ac}}{2}\right)^2}=\frac{E_{ac}}{\sqrt{2}}$$

But

$$E_{ac}=\frac{E_{dc}}{\sqrt{2}}$$

Hence, the voltage between taps *b* and *c* is

$$E_t=\tfrac{1}{2}E_{dc},$$

or 50 volts for the particular machine being considered.

For a three-phase converter, three taps equally spaced and three slip-rings would be used. (Fig. 317.) The voltages between these taps would be alike and would be $E_{ac}=0.612E_{dc}$. In many applications six-phase converters are used. These would have six taps with six rings for a two-pole machine. The voltage between adjacent taps would be 0.354 E_{dc}. Six-phase converters are operated from a three-phase supply with a special arrangement of the transformer connections. (See Article 330.) Synchronous converters as ordinarily built have more than two poles (see Fig. 318), but the number of a.c. rings is the same regardless of the number of poles. There would, however, be a tap connected to each ring for each pair of poles so that the entire armature winding may be effective in carrying the

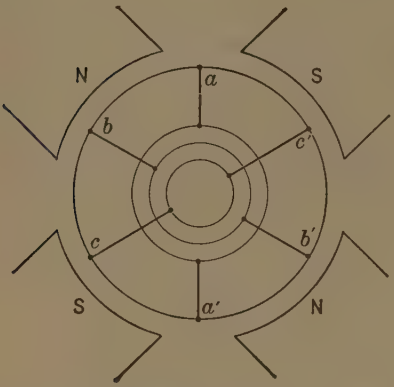


FIG. 318.—Diagram for a Four-pole, Three-phase Converter.

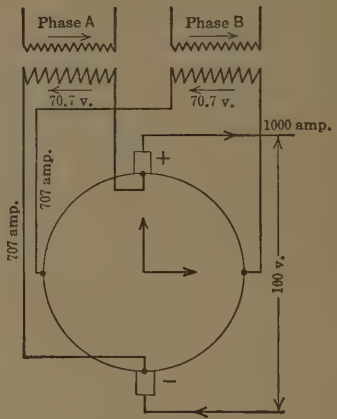


FIG. 319.—Connections for a Two-phase Converter.

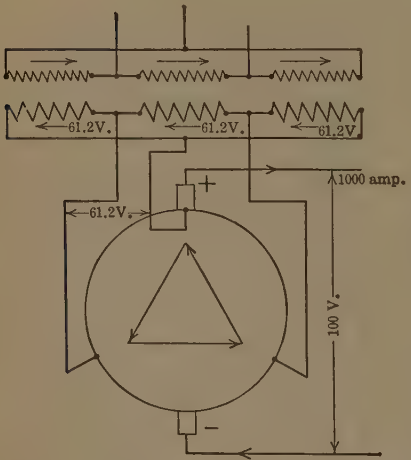


FIG. 320.—Connections for a Three-phase Converter. Delta-delta connection of transformers.

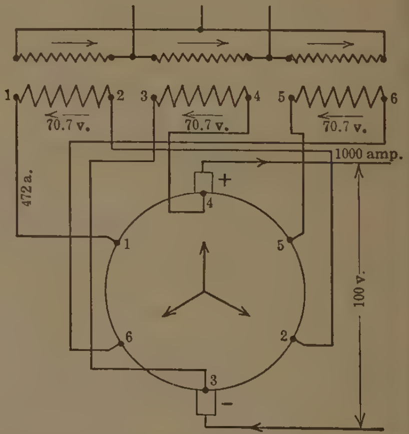


FIG. 321.—Connections for a Six-phase Converter. Diametrical connection of transformers.

current. Figure 318 shows connections for a three-phase, four-pole converter. It should be noted that points connected to the same ring must be located at corresponding points on the armature so that the voltages induced at these points shall be equal and have the same polarity. Otherwise there would be a short-circuit.

330. Converter Connections.—The connections for a two-phase converter are shown in Fig. 319. The slip-rings are omitted for the sake of simplicity. It should be noted that the two transformers are connected across diameters 90 degrees apart, as would be necessary in order to produce the proper phase relation of the voltages. Connections for a three-phase converter are shown in Fig. 320. The secondaries of the transformers may be connected in Y if desirable, in which case the neutral could be used as the return for a three-wire system.

Two common methods for connection of the six-phase converter are used. The diametrical (Fig. 321) treats the three transformer secondaries as three single-phase circuits and there are no interconnections between transformers except in the armature. The double-delta system (Fig. 322) requires that the secondary winding of each transformer be divided into two parts and be used to make two separate deltas. One of these is a reversed delta in order to obtain the proper polarity on the converter. It is obvious that the diametrical connection gives greater simplicity in the wiring, and this is the type generally employed. Either method requires the same current per lead and, hence, the same amount of copper for connections. Synchronous converters are extensively used for supplying Edison three-wire systems. The neutral return is made through the converter transformers. If the unbalanced current is

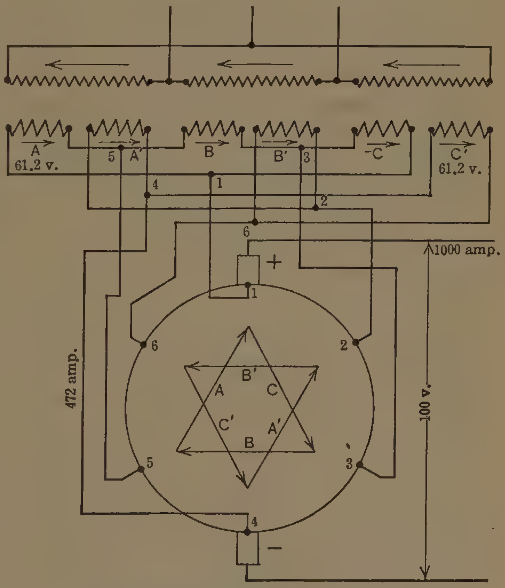


Fig. 322.—Connections for a Six-phase Converter. Double-delta connection of transformers.

small, an ordinary Y-connection can be used; for large neutral currents, however, special arrangements of the transformer secondaries are necessary in order to avoid unbalanced magnetic effects in the transformers, due to the neutral currents.

331. Voltage Ratios.—It is evident that the voltage required for the secondary side of the transformers used with a converter depends upon the method of connection. These voltages are as follows:

	Voltage between Adjacent Rings	Phase Voltages
Single-phase, 2 rings	$0.707E_{dc}$	$0.707E_{dc}$
Three-phase, 3 rings	$0.612E_{dc}$	$0.612E_{dc}$
Two-phase, 4 rings	$0.500E_{dc}$	$0.707E_{dc}$
Six-phase, diametrical, 6 rings	$0.354E_{dc}$	$0.707E_{dc}$
Six-phase, double delta, 6 rings	$0.354E_{dc}$	$0.612E_{dc}$

These ratios are the theoretical values; in an actual machine they may be slightly different. They will also be somewhat higher at full load than at no load, due to voltage loss in the converter armature. It should be noted that as long as the voltage at the slip-rings is held constant, the d.c. voltage will be practically constant regardless of any variation of the field strength.[1] Special means are provided, therefore, for varying the d.c. voltage of converters. (See Article 334.)

332. Current Ratios.—If it is assumed that the converter is operated at unity power factor and that the losses are neglected, the kv-a. input equals the kw. output and the current ratios can be easily calculated.

Example 1. Assume that a converter is supplying 1000 amperes at 100 volts d.c. The d.c. output is then 100 kw. and the a.c. input is also 100 kw. For a three-phase converter with 100 volts d.c., the theoretical voltage between a.c. terminals is 100×0.612 or 61.2 volts. Hence, the current in each a.c. lead is

$$I_{ac}=\frac{100{,}000}{\sqrt{3}\times 61.2}=943 \text{ amperes}$$

The ratios for other converters can be calculated in a similar manner giving results as follows:

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[1] Except for the split-pole converter, see Article 334.

CURRENT RATIOS FOR CONVERTERS[2]

Single-phase, 2 rings........................ $I_1 = 1.414 I_{dc}$.
Three-phase, 3 rings........................ $I_3 = 0.943 I_{dc}$.
Two-phase, 4 rings.......................... $I_2 = 0.707 I_{dc}$.
Six-phase, 6 rings. Either double-delta or diametrical........................... $I_6 = 0.472 I_{dc}$.

If the machine operates at a lower power factor and efficiency, the a.c. current for a given d.c. current will, of course, be increased.

Example 2.—Referring to the machine in Example 1, calculate the a.c. current for a power factor of 0.90 and an efficiency of 0.91.

$$I_{ac} = \frac{100{,}000}{\sqrt{3} \times 61.2 \times 0.90 \times 0.91} = 1150 \text{ amperes.}$$

333. Ratings of Converters.—The capacity of a synchronous converter may be limited by commutation, or heating, or both. As far as the d.c. side of the converter is concerned, the current which flows to the d.c. load is a *generator* current and is in the same direction as the e.m.f. induced in the armature. The alternating current which enters the armature through the slip-rings is a *motor* current and flows in opposition to the e.m.f. induced in the armature. The motor current produces the driving force necessary to supply the losses and furnish the force to balance the retarding effect of the generator current. In the usual motor-generator set, the generator and motor currents are in separate circuits and there is a double transformation. The electrical energy in the alternating-current form is first converted to mechanical energy in the motor and is then transmitted through the shaft to the generator, where it is transformed to electrical energy again. In the converter this double transformation takes place in the same conductors, and, obviously, the motor current must have a direction opposite to the generator current. Hence, the resultant current is the difference of these two currents, and it has an effective value much less than either; that is, the actual current which flows in the armature winding is less than either the direct or the alternating current in the external circuits. The current which actually flows in an individual coil is not a steady value of current such as would exist in a d.c. generator, but has a very irregular wave form which varies for different parts of the armature. The coils near the taps carry a current having a larger effective value, for a fixed d.c. load, than coils midway between taps. As a result, the heating of the armature is not uniform as is the case in a d.c. generator, but there are hot spots near each tap. The more taps there are, the more this heating is distributed because the incoming alternating current enters at more points, and the amount at any one point is less. The carrying capacity of a converter, therefore, is determined not by the average heating, but by the temperature of the hottest points which are the tap coils. As a result, a machine of a certain size will carry a greater d.c. load according as the number of taps (or slip-rings) is increased. In general, a certain d.c. generator if changed to a converter would carry considerably more d.c. load without overheating. This is shown by the following table:

---

[2] Unity power factor and 100 per cent efficiency assumed.

RELATIVE CAPACITY OF A MACHINE OF GIVEN SIZE WHEN OPERATED AS A CONVERTER

How Operated	Capacity, Unity Power Factor, Per Cent	Capacity, 0.9 Power Factor, Per Cent
D. C. Generator	100	100
Single-phase converter (2 rings)	85	74
Three-phase converter (3 rings)	134	110
Two-phase converter (4 rings)	164	127
Six-phase converter (6 rings)	196	147

This means that the output of a field frame and armature of a given size, when used as a six-phase converter, is nearly double the output of the same machine when used as a d.c. generator. The machine, if used as a converter, would of course require a larger commutator and correspondingly greater brush area in order to carry the increased d.c. current. A six-phase converter may be operated as a three-phase machine, but the safe output would be reduced in the proportion shown in the table. If the converter is operated at a power factor less than unity, the armature heating is greatly increased or in other words the safe output decreased, as is shown by the table above. This is due, not alone to the fact that a larger alternating current is required for a given d.c. load, but also because the alternating current is displaced in phase with respect to the direct current in the conductors, so that the resulting current is larger. For this reason, it is important to operate converters close to unity power factor when they are carrying their rated load, although at light loads it is sometimes desirable to run at lower power factors.

As far as armature reaction in the converter is concerned, the effect of the generator current is largely neutralized by the effect of the motor current so that the resulting armature reaction is much less than for a corresponding d.c. machine. The commutation is, therefore, better in the converter than for a similar machine operated as a d.c. generator. Commutating poles are used on converters and perform the same function as in the case of d.c. generators.

334. Methods of Voltage Adjustment.—It was shown in Article 328, that for a converter the ratio of a.c. to d.c. voltage is fixed and cannot be changed by changing the field strength. The converter acts as a synchronous motor on the a.c. side and, therefore, if the field strength is increased the machine will take a leading current from the a.c. supply, and the armature reaction of the converter will be changed in such a way as to neutralize the effect of the change in field strength; hence the d.c. voltage will not change. Therefore, if the d.c. voltage of a converter is to be changed, there must be a corresponding change in the terminal voltage on the a.c. side of the converter. The following methods are used to change the a.c. voltage:

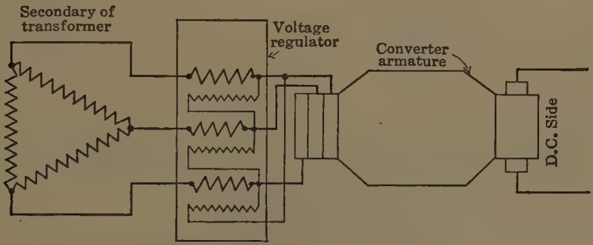


FIG. 323.—Voltage Adjustment of a Converter by Means of an Induction Regulator.

(1) Induction regulator.
(2) Synchronous booster.
(3) Reactance in the a.c. circuit.

The induction regulator (1) (Article 284) is used as a boosting transformer of variable voltage to raise or lower the voltage at the slip-rings (Fig. 323). It is customary to arrange the regulator so that its voltage may add to or subtract from the supply voltage to reduce the size of regulator required. The booster converter (2) has an alternator with a revolving armature which is mounted on the converter shaft with its armature winding in series with the connections between the slip-rings and the converter winding (Fig. 324). The voltage generated by this booster alternator has the same frequency as the main supply and can be varied in magnitude in order to adjust the voltage supplied to the armature by the required amount. The field of this machine is excited by direct current and is arranged so that it can be reversed in polarity in order that the booster may buck or boost the main voltage. A booster converter is shown in Fig. 325.

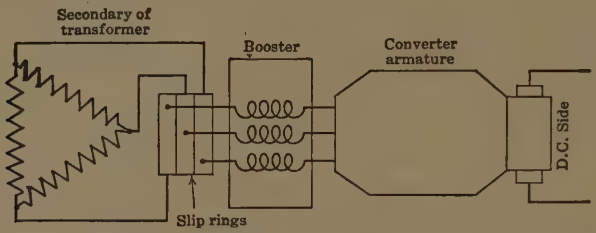


FIG. 324.—Voltage Adjustment of a Converter by Means of a Synchronous Booster.

The d.c. voltage of a converter may be varied by the use of reactance in the a.c. circuit of the converter, Fig. 326. It was shown (Article 232), that if a load having a lagging power factor flows over a circuit having inductive reactance, such as a transmission line, the terminal voltage decreases; whereas, if the load has a leading power factor, the terminal voltage rises. By connecting an inductive

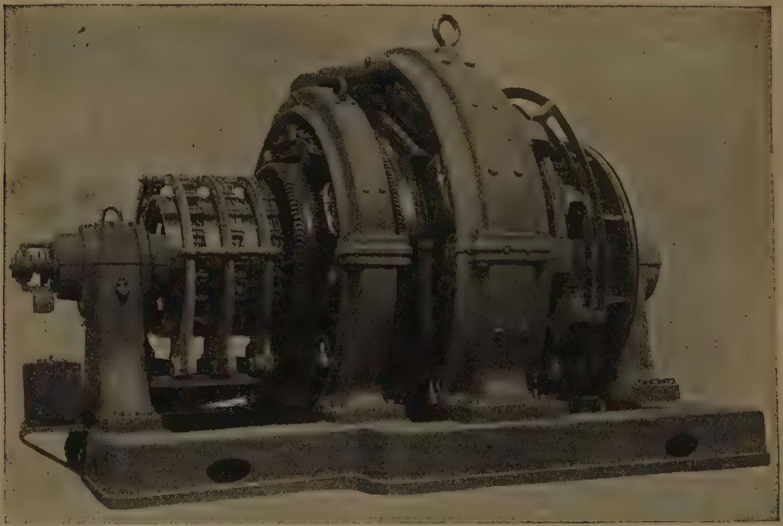


FIG. 325.—Booster Converter. 8000 amperes, 60 cycles. *The Westinghouse Electric & Mfg. Co.*

reactance as shown in Fig. 326, the a.c. voltage of the converter may be changed, thereby changing the d.c. voltage. If the field strength of a converter is increased above normal the machine will operate with a leading power factor. This would cause a rise in voltage on the converter, whereas a decrease of field strength would produce a lagging power factor and cause the d.c. voltage to drop. Hence, the d.c. voltage can be regulated by means of a series winding on the main poles of the converter, connected exactly the same as for a compound generator.

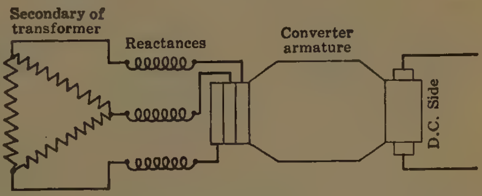


FIG. 326.—Voltage Adjustment of a Converter by Means of Series Reactance.

We have already stated that the voltage ratio of a converter is fixed, and that is true for the usual type of converter. A special type called the split-pole converter, can, however, vary the d.c. voltage while the a.c. voltage at the terminals remains constant. In this machine, the main field is divided into two parts having separate exciting windings. The d.c. voltage can be changed by adjusting the total flux per pole, whereas the effective value of the a.c. voltage depends upon the wave form. Therefore, by changing the relative amount of excitation of the two parts of the poles, it is possible to increase the total flux, thereby increasing the d.c. voltage and also to change the distribution of the flux in the air gap in such a way as to distort the a.c. wave form and still maintain a constant effective value of a.c. voltage.

All of the methods described have been used in practice, when an adjustment of the d.c. voltage is necessary while the machine is running. Changes in voltage can also be made by changing the taps on the transformers, and it is customary to provide these transformers with several taps, usually on the high voltage side, so that the converter can be adapted to changed line conditions. When once adjusted, however, it is not possible to shift to another voltage tap without shutting down the machine and disconnecting the transformer leads.

Where a considerable variation of voltage is required, either the induction regulator or the booster converter is used. The tendency in modern practice is to use the booster converter because it is lower in first cost, occupies less floor space, and its operation is simpler. Theoretically there is no limit to the range of voltage available by using either a synchronous booster or an induction regulator, as it is simply a matter of getting enough boosting voltage. Ordinary requirements, however, are met by machines giving 10 per cent buck or boost, thus giving a total voltage range of 20 per cent. The booster converter is extensively used for electrolytic work and for supplying Edison three-wire systems.

The use of series reactance is restricted to applications where only a moderate change in voltage is necessary to give flat compounding on the d.c. side or possibly a slight over-compounding. A large variation of voltage is not feasible because of the low power factor which would be required, which would increase the heating of the converters (Article 333). The necessary series reactance may consist of a separate device, or it may be secured by increasing the leakage reactance of the main transformers. The latter method is the one generally employed. A reactance voltage of 15 per cent is usually sufficient. Converters regulating by series reactance are extensively used for railway and general power service. The split-pole converter was at one time used extensively for Edison systems, but it is now being displaced by the booster converter. The split-pole converter gives trouble in commutation on rapidly fluctuating loads and is more likely to overspeed if the a.c. supply fails when it is connected to a d.c. source such as a storage battery.

335. The methods of starting converters which are commonly used are:

(1) A starting motor
(2) Self-starting from the a.c. side.
(3) Self-starting from the d.c. side

The starting motor, when used, is generally a squirrel-cage induction motor designed to produce a large starting torque. The high rotor resistance required is no disadvantage in this case since the motor is disconnected as soon as the converter is synchronized. The motor is designed to have a synchronous speed somewhat higher than the converter speed, usually by building the starting motor with one pair of poles less than the number of converter poles. When a converter is started in this manner the transformers are usually connected to the converter slip-rings, and the set is synchronized on the high tension side of the transformers. As the machine comes up to speed, the voltage of the converter is adjusted to approximately normal value, by means of the field rheostat. The exact adjustment of speed which is necessary in order to synchronize is accomplished by changing the converter field current. If the speed is too high it can be decreased by raising the converter voltage slightly by means of the field rheostat. This increases the losses of the outfit and causes an increased slip of the driving motor. Weakening the field would raise the speed. The machine is synchronized by lamps or a synchronoscope in the same manner as an alternator.

A suitably designed converter may be started by applying reduced voltage to the a.c. slip-rings. This reduced voltage is usually secured by taps on the secondary side of the main transformers. (Fig. 327.) The converter field poles have a squirrel-cage winding imbedded in the pole faces, and the machine, therefore, starts by induction motor action in the same way as a self-starting synchronous motor. In the case of a converter, the revolving flux produced by the alternating current in the armature moves at synchronous speed when the armature is standing still. This revolving flux cuts the stationary squirrel-cage winding on the field poles and produces a force tending to move the squirrel-cage winding in the same direction as the revolving flux. But this squirrel-cage winding cannot move; hence the reaction on the armature causes it to turn in a direction opposite to the revolving flux. As the armature speeds up, the flux produced by the armature current decreases in speed relative to the stationary field magnet until the point of synchronism is reached, when the armature flux stands still and links with the stationary field poles of the converter. During the starting process, an a.c. voltage appears at the d.c. brushes, and a high voltage alternating e.m.f. is induced in the field winding, the same as occurs with a synchronous motor (Article 320). Sometimes the field circuit is broken up into several parts during the starting period in order to reduce the strain on the insulation, but the more common method is to leave the shunt field circuit connected to the d.c. brushes through the field rheostat. When using the latter method, the kv-a. required for starting is only slightly greater than when the field is open and, therefore, the

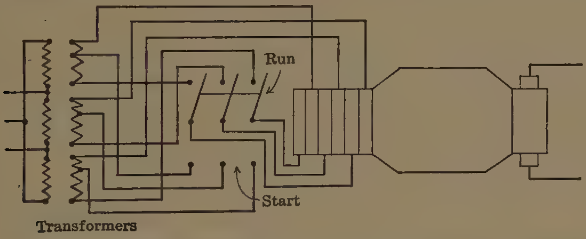


FIG. 327.—Diagram of Connections for Starting a Converter from the A.C. Side. Six-phase, diametrical connection.

former method is preferable because it eliminates the high voltage in the field winding. After the machine has pulled into synchronism on the reduced voltage, full voltage is applied. The starting voltage usually is about one-third normal voltage and only one starting point is required. The starting kv-a. seldom exceeds the rated kv-a. and in some designs is only about 50 per cent of rated kv-a. A converter started from the a.c. side may lock into synchronism in such a way as to give the wrong polarity on the d.c. side. The field winding is provided with a two-throw switch so that the field can be thrown directly across the d.c. brushes. In starting, this switch is set in such a position as to place the field rheostat in series with the field. If the machine synchronizes with reversed polarity, the field switch is thrown to the other position which puts full voltage on the field with reversed polarity. This causes the armature to fall out of synchronism momentarily and to slip one pole. The field switch is then thrown back to the running position with the rheostat in circuit, when the polarity will be correct. Adjustment of polarity is made while the machine is still operating from the starting voltage. When compound converters are operated in parallel, the series field of the incoming machine may be placed in parallel with the others by closing the equalizer switch and the line switch on the same side of a converter, leaving the other line switch open. This causes current to flow through the series winding of the incoming machine in such a direction as to give a weak field of the correct polarity which will assist the converter to pull into step with the correct polarity at the d.c. brushes. When commutating-pole converters are started from the a.c. side, a large current flows in the coils short-circuited by the brushes, because of the strong field surrounding these coils due to the commutating poles. This causes excessive sparking and burning of the brushes. For this reason, all the d.c. brushes except one of each polarity are raised from the commutator during the starting period. The two brushes remaining on the commutator are narrower than the others and provide the necessary current for exciting the field. With non-commutating pole converters, it is not necessary to raise the brushes as the field surrounding the coils short-circuited by the brushes is not enough to cause harmful sparking.

A converter may be started from the d.c. side as a shunt motor where a source of direct current of the proper voltage is available. This may come from a storage battery, another converter, or another sub-station. When started in this manner, the machine is synchronized the same as when a separate starting motor is used. If the transformers are connected to the converter armature during the starting period, the starting current is increased because the transformer windings form a low resistance path for the starting current. Where it is necessary to leave the transformers connected, as would be the case in large converters, special provision is made to reduce the static friction by introducing oil under pressure into the bearings. When this is done, the converter starts easily.

The a.c. method of starting is much better than the others, due to its simplicity, because the machine is self-synchronizing. This method is now standard practice and is used generally, except where it is necessary to reduce the starting current to a minimum. The power factor when starting is low, and hence, when the kv-a. rating of the converter is approximately the same as that of the generator supplying it, the a.c. starting method should not be used if other apparatus is operated from the generator at the same time.

It should be noted that converters, when first put into service, must be phased-out so that they will have the correct direction of rotation to suit the arrangement of the d.c. brushes. Phasing-out may be done by starting with reduced voltage or by the use of lamps in the different phases, exactly the same as in phasing-out an alternator.

336. Parallel Operation of Converters.—Converters may be operated in parallel on the d.c. side in the same manner as for d.c. generators. If the machines have a compound winding, equalizer connections are required to stabilize the operation. Adjustment of load between machines is accomplished by changing the d.c. voltage of the machine. In general, converters should not be paralleled on both the d.c. and the a.c. side because this gives a metallic connection of low resistance between machines on both a.c and d.c. sides. Local currents are very likely to flow through this circuit so that parallel operation in this manner is not always satisfactory. The machines can, however, be paralleled on the d.c. side and operated from the same high-tension bus-bars. This method, which is the one most commonly used, requires a separate bank of transformers for each converter. Hunting may occur with synchronous converters, since they act like synchronous motors as far as the a.c. side is concerned. With modern machines, however, serious hunting is not very common. The squirrel-cage winding which is imbedded in the field poles tends to prevent hunting in the same way as in a synchronous motor. (Article 327.) An excessive resistance drop in the supply system or fluctuations in the supply voltage may result in hunting. Because they are synchronous machines, converters may drop out of synchronism if the supply voltage drops momentarily, due to a short-circuit or other disturbance on the line.

If a converter is operated in parallel with other machines or any other source of direct-current, the machine would turn into an inverted converter if the a.c. supply failed. This might result in overspeeding, due to the differential action of the series field winding. For this reason, the d.c. circuit breaker is interconnected with the a.c. breaker so that, if the latter opens, the d.c. supply will also be cut off.

337. The Inverted Converter.—When a synchronous converter receives power from a direct-current source and delivers alternating current, it is called an inverted converter. As far as the d.c. side is concerned, it has somewhat the same characteristics as a d.c. shunt motor, while, on the a.c. side it is like an alternator. The kv-a. rating would be the same as when operating as a direct converter, and the voltage ratio is nearly the same. If an inverted converter is paralleled with an alternator on its a.c. side the converter speed will be fixed by the alternator speed since they must operate in synchronism. If, however, the converter is the only source of alternating current, the speed will be affected by a change in field strength of the converter, and the machine will act like a shunt-wound motor. An inductive load on the a.c. side of an inverted converter when not connected to an alternator would, therefore, tend to weaken the converter field and would cause the machine to speed up. In fact, the speed of an inverted converter is likely to be unstable unless it is connected on the a.c. side to a system of constant frequency or unless the machine has its fields excited by a special exciter direct connected to the converter, so that any tendency for the machine to increase its speed would be corrected by an increase in exciter voltage and a stronger field. Inverted converters are provided with a centrifugal type of over-speed device which opens the line connections in case the converter overspeeds.

338. Commercial Standards.—The standard d.c. voltages for converters are 250, 275, 600, and 1500 volts. Single-phase converters are built only for small capacities. The polyphase converters are usually six-phase for sizes above 100 kw. because of the greater output possible from the same frame. It is customary to provide each converter with its own bank of transformers because operation in parallel on both d.c. and a.c. sides is, in general, not desirable. These transformers are usually provided with several taps on the high-tension winding so that the secondary voltage can be adjusted to give the required d.c. voltage even when the line voltage differs slightly from the rated value. Modern converters are ordinarily equipped with commutating poles. Converters are provided with speed-limit devices which open the circuit breakers and cut off the d.c. supply if the speed of the machine rises a few per cent above normal.

339. Synchronous Converters vs. Motor-generator Sets.—Both of these types of conversion apparatus are extensively used in practice. Converters cost less and occupy less floor space than motor generator sets. The full-load efficiency of the converter with transformers is from 3 to 6 per cent higher than that of the motor-generator set without transformers. Transformers must be used with converters, whereas motor-generator sets may be designed to operate directly from the supply voltage if it is not above 15,000 volts. Converters are at a disadvantage where there are such violent fluctuations of the supply voltage or frequency that synchronous apparatus could not be used successfully. Under these conditions, an induction motor-driven generator set would be preferable. If correction of the system power factor is desirable, a synchronous motor-generator set may be used to advantage.

PROBLEMS ON CHAPTER XXXIII

1–33. Indicate by a circle the d.c. winding for a two-pole synchronous converter, and show taps and slip rings for:

(*a*) A single phase converter.
(*b*) A quarter-phase converter.
(*c*) A three-phase converter.
(*d*) A six-phase converter.

2–33. Indicate by a circle the d.c. winding of a six-pole synchronous converter, and show taps and slip rings for:

(*a*) A three-phase converter.
(*b*) A six-phase converter.

3–33. A 250-volt d.c. generator is to be used for a synchronous converter to change from a.c. to d.c. What would be the theoretical a.c. voltage for:

(*a*) A single-phase converter?
(*b*) A three-phase converter?
(*c*) A six-phase converter?

4–33. Given a 250-volt d.c. converter delivering 100 kilowatts. Using the theoretical ratios (and neglecting losses), calculate:

(*a*) The a.c. current in each line wire if the machine is a single-phase converter.
(*b*) Calculate the same for a three-phase converter.
(*c*) Calculate the same for a six-phase converter.

5–33. A 1000-kw., 600-volt d.c. generator is to be used as a synchronous converter. On the basis of unity power factor, calculate:

(*a*) The d.c. current when run as a generator at rated load.

(*b*) The d.c. current when the machine is operated as a three-phase converter and has the same armature heating as for the d.c. generator.

(*c*) The d.c. current when run as a six-phase converter with heating same as when run as a d.c. generator.

(*d*) Calculate the kilowatt output for each case above.

It is assumed in this problem that the commutator is large enough to carry the additional load when operated as a converter.

6–33. A 2000-kw., 600-volt converter is to be operated from an 11,000-volt supply.

Using the theoretical voltage ratios, determine proper voltages for the transformers and show a diagram of connections for:

(*a*) A two-phase converter.
(*b*) A three-phase converter, transformer secondaries in delta.
(*c*) A three-phase converter, transformer secondaries in Y.
(*d*) A six-phase converter connected in double-delta.
(*e*) A six-phase converter with diametrical connection.

7–33. Referring to Prob. 6–33: assume a converter efficiency of 96 per cent, a transformer efficiency of 98 per cent and a power factor of 0.90. Calculate the current in the primary and secondary circuits of the transformers for each type of converter given in Prob. 6–33.

8–33. (*a*) Using data of Probs. 6–33 and 7–33, determine the kv-a. rating of the transformers required in each case of Prob. 6–33.

(*b*) What would be the voltage rating of the transformers in each case?

CHAPTER XXXIV

MOTOR-GENERATOR SETS AND CURRENT RECTIFIERS

340. A.C.–D.C. Conversion.—Motor-generator sets for this purpose may be driven either by a polyphase induction motor or by a synchronous motor. The former has the advantage of simplicity since the motor requires no d.c. excitation and is easily started. It has a lagging power factor, however, which is a disadvantage in many applications. Modern types of synchronous motors are as easy to start as are induction motors and they require very little attention while running. The synchronous motor has an advantage over the induction motor due to the fact that it is possible to operate at unity power factor, or if desired, the synchronous motor may be adjusted to produce a leading power factor and thus improve the total power factor of the system. (Article 322.) For this reason, synchronous motor-generator sets are frequently used instead of synchronous converters because many power contracts now put a penalty on low, lagging power factors and, therefore, it is more economical to use motor-generator sets. Induction motor driven sets are more commonly used for small units or where the power factor is not important. In some cases voltage fluctuations of the system are so violent that there would be danger of synchronous machinery falling out of step; in such cases, induction motor-driven sets are preferable. This type of set is also used for furnishing excitation for alternators, because it will not drop out of step with a drop of voltage due to a short-circuit, but will slow down and come up to speed again when the voltage and frequency become normal.

341. A.C.–A.C. Conversion.—Where it is necessary to interchange power between two systems having different frequencies, a motor-generator set consisting of an alternator driven by a synchronous motor is usually employed. With such a set, either machine may act as a motor, while the other acts as a generator, thus allowing a transfer of power in either direction between the two systems. Since both machines operate synchronously, the speed and number of poles are fixed by the frequencies of the two systems. For conversion between 25 and 60 cycles it will be seen that there are only a few combinations which will give the correct frequencies for both systems. One of these is 300 r.p.m. with 10 poles on the 25-cycle; and 24 poles on the 60-cycle machine. According to Article 317, the displacement angle of a synchronous motor field must increase with load. In a set of this type, however, the fields of the two machines are rigidly keyed to the same shaft; therefore, in order to permit adjustment of the load, it is necessary to arrange the stator or armature of one of the machines so that it can be rotated a few degrees, thereby changing the angle between the revolving magnetic field produced by the stator and the rotating field of the set. The synchronizing of a motor-generator set of this type is difficult because both machines must be synchronized, and this sometimes necessitates "slipping poles" a number of times until the correct phase position is obtained.

CURRENT RECTIFIERS

342. Synchronous Commutator.—Alternating current can be changed into pulsating direct current by means of a commutator so arranged that it reverses the connections at the same instant that the current reverses. If this is done, the commutator must be driven by a synchronous motor from the source of alternating current. This device is limited to use on small currents because of the difficulty of reversing the current at exactly zero value. The most common application of this device is that used for the Cottrell process, for precipitating dust in gases (Article 71), where a high voltage direct current is required. The rectifying contacts do not touch the segments but are separated by a short gap which the high voltage readily jumps. This saves wear of the contacts. The arrangement is shown in Fig. 328.

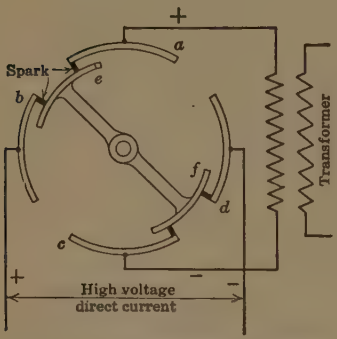
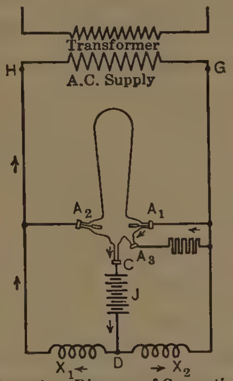


FIG. 328.—Synchronous Commutator. Used for rectifying a high-voltage alternating current. Segments *a*, *b*, *c*, and *d*, are stationary. Segments *e* and *f* are insulated from the arm and rotated at synchronous speed.

343. The mercury-arc rectifier, and the hot cathode rectifier, which is described in the next article, depend for their action upon the conduction properties of "ionized" gases. Heated metals give off extremely minute particles of negative electricity called electrons.

If these electrons are emitted in an electrostatic field, they are given a high velocity and are then capable of breaking up atoms of gas which may be in the vicinity of the electrode and thereby ionizing the gas. When the gas is ionized in this way, it becomes a conductor of electricity. The ionized gas is, however, conducting in one direction only so that current can flow to the hot metal but cannot flow away from it. This arrangement produces a sort of valve action which is used to rectify alternating current. The mercury-arc rectifier which works on this principle has a glass bulb exhausted to a low pressure, in which there is a cathode or negative terminal consisting of a small pool of mercury. There are usually two positive terminals or anodes (Fig. 329) composed of iron or graphite. If a potential were applied between A_1 and C in such a manner as to tend to send current through the tube from A_1 to C, no current could flow as there would be no conducting path. A difference of potential would, however, exist between A_1 and C with A_1 positive with respect to C. A mercury arc must be started in order to obtain mercury vapor in the tube. This is accomplished by a starting anode A_3 which is located close to C and is so arranged that by tilting the tube slightly a mercury bridge is formed by way of A_3 and C. When this is broken, the current which flows by this path produces a mercury arc. This at once produces electrons which are given a high velocity by the difference of potential between A_1 and C, thus ionizing the mercury vapor so that the entire tube is filled with ionized gas and current will flow through this gas from



Elementary Diagram of Connections
(a)

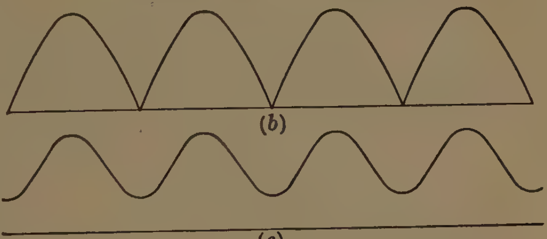


FIG. 329.—Mercury-arc Rectifier. (*b*) Pulsating wave when no reactances are used; (*c*) Wave smoothed out by use of reactances.

A_1 to C and thence through the external circuit. . (Fig. 329 *a*.) As soon as A_1 becomes negative, the action stops so that, under such circumstances, only one-half the a.c. wave flows and the arc is extinguished each half cycle. Another anode (A_2) is, therefore, provided, and this is so connected to the a.c. supply that A_1 and A_2 are alternately positive. The current then flows for one-half cycle from A_1 to C, thence through the external circuit, and for the next half-cycle from A_2 to C. By this means, the alternating current is changed into a pulsating direct current (Fig. 329 *b*). To stabilize the action and prevent the arc from being extinguished each time the current becomes zero, inductances X_1 and X_2 (Fig. 329 *a*) are used, thus smoothing out the wave and giving a form like that shown in Fig. 329 *c*. The mercury-arc rectifier is made for both high and low voltages to suit requirements. The current capacity of the ordinary glass bulb is, however, limited to rather small values. This type of rectifier is used extensively for supplying series d.c. lighting systems from a.c. circuits, the current being usually from 4 to 6.6 amperes and the voltage 5000 to 7000 volts. The device is also used for charging storage batteries, in this case the usual current capacities being from 10 to 50 amperes and 20 to 120 volts. This type of rectifier is also used for supplying the arc lamps used in motion-picture projectors. The mercury rectifier has not as yet been employed in this country for large currents, but in Europe rectifiers using steel tanks instead of glass bulbs and having a capacity of 900 amperes at 800 volts are in use. The difficulty of manufacturing this type of apparatus and maintaining the proper vacuum seem to be the principal reasons why they are not used in the United States.

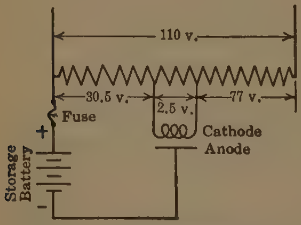


Fig. 330.—Diagram for a Hot-cathode Type of Rectifier. Capacity, 2 amperes.

344. The hot-cathode rectifier operates upon the same principle as the mercury-arc rectifier. The commercial type of hot-cathode rectifier is known by various names such as "Tungar" "and Rectigon."[1] The device consists of a glass bulb (Fig. 330) containing a tungsten filament and graphite anode. A transformer is used to reduce the supply voltage to a value suitable for the load to be carried. The filament is heated by current from the same transformer. The bulb is filled with a gas, such as argon, which is ionized by the electrons given off from the hot filament. The current enters at the anode and leaves at the filament or cathode (Fig. 330) as long as the left hand terminal of the transformer is positive, but a high resistance is introduced and the current ceases as soon as it becomes negative. This rectifier, therefore, allows only the positive wave of the alternating current to flow, but this is not a serious disadvantage for small size units, although it reduces the efficiency. A two-bulb type of rectifier is also made, in which both positive and negative waves are used. Rectifiers of this type are at present limited to small ampere capacities, the sizes available ranging from 2 to 12 amperes at voltages from 7.5 to 75 volts. They are employed for charging small storage batteries.

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[1] See "The Tungar Rectifier" by R. E. Russell, *General Electric Review*, 1917, p. 209.

CHAPTER XXXV

SINGLE-PHASE MOTORS

345. The single-phase induction motor resembles the polyphase induction motor in its structural features. The stator or primary has a single winding instead of the two or three windings used for a polyphase motor. The stator winding of the single-phase motor is, however, not distributed over the entire circumference of the stator core as is the case with the polyphase machines. A rotor of the squirrel-cage type is used in small single-phase induction motors. The rotors of the larger machines usually have a set of insulated coils connected to a commutator. The single-phase motor is not inherently self-starting; hence, special starting devices are used in commercial motors. In Fig. 331 is shown a single-phase induction motor with a squirrel-cage rotor. If a single-phase e.m.f. is applied to the stator winding of this motor, an alternating flux *NS* will be produced along the horizontal axis *XOX*. The rotor winding acts like the short-circuited secondary of a transformer, and the current in this rotor winding will be in a direction opposite to that in the stator winding. The force due to the rotor current and the flux *NS* is however, in opposite directions in the two halves of the rotor; hence, the resultant torque is zero and the motor will not start.

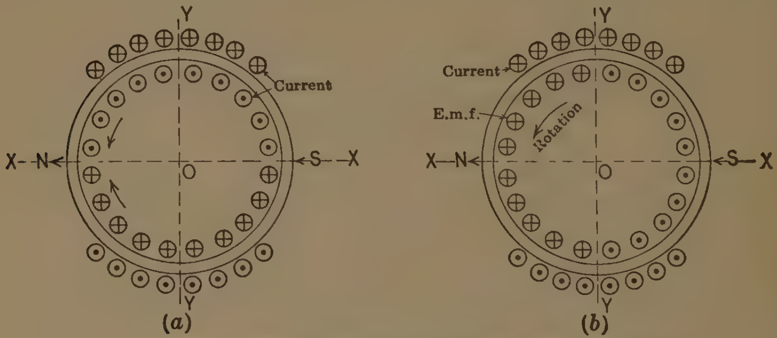


FIG. 331.—Diagram for a Single-phase Squirrel-cage Induction Motor.

The flux produced by the stator winding is alternating in polarity along the line XOX and does not rotate as in the polyphase induction motor. If the rotor were turned at synchronous speed, i.e., 3600 r.p.m. for a two-pole machine supplied at 60 cycles, the rotor conductors would cut this alternating flux NS. Let Fig. 331 *b* represent conditions when this stator flux is at its maximum value. There will be an e.m.f. induced in the rotor conductors by this flux NS, and the direction of this rotor e.m.f. will be as shown in Fig. 331 *b*. Due to this e.m.f., a current will flow in the rotor, the direction of this current being the same as that of the e.m.f. This rotor current would, therefore, tend to set up a flux along the vertical axis YOY. The rotor winding is, however, highly inductive so that this current would lag nearly 90 degrees behind the rotor e.m.f. Therefore, the maximum value of the flux set up by the rotor current along axis YOY would differ by 90 time-degrees from the flux along the horizontal axis XOX which is set up directly by the stator winding. Since these two fields are spaced 90 degrees apart, they act like the fields produced by the two stator windings of a two-phase induction motor. Hence, they combine to produce a rotating field which is of nearly constant strength as long as the rotor turns at synchronous speed. This rotating field turns at synchronous speed, exactly like the field of a polyphase motor. If the motor is standing still, there is no cutting of the stator field and no flux is produced along the YOY axis. The rotating field then no longer exists, and there remains only an alternating field along the axis XOX. At any speed below synchronism, the e.m.f. induced in the rotor by cutting the stator field is less than at synchronism; therefore, the flux along the YOY axis is less. This flux, combining with the stator flux, still produces a rotating field, but the strength of this field is no longer constant and varies at different points during each revolution. The variation in strength of the rotating field becomes greater as the rotor slip increases. The rotating field produced by the combination of the flux due to the rotor and stator magnetizing currents, in the manner described, is sufficient to drive the rotor in the same manner as in a polyphase motor, so that the performance of the single-phase induction motor, after it has once been brought up to speed, resembles that of a polyphase motor. A simple single-phase motor as described will run in either direction, depending upon which way it is started. In practice, single-phase induction motors are usually started either by phase-splitting, or by using the repulsion motor principle. These methods are described in the articles which follow. The efficiency and power factor of a single-phase motor is lower than for a polyphase motor, and the size and weight are greater.

346. The split-phase motor is a single-phase induction motor provided with an auxiliary winding which is used for starting. The method of applying the phase-splitting principle varies with different manufacturers. One arrangement is shown in Fig. 332. The stator has two windings, a main winding and a starting winding. The latter is displaced 90 degrees with respect to the main winding, so that the arrangement is similar to that used for a two-phase motor. At starting, the switch *S* is closed, and both windings are energized. The starting winding has a high resistance and relatively small reactance while the main winding has a low resistance and large reactance. Hence, the currents in the two windings are out of phase with each other. The combined effect of these currents produces a rotating field which starts the motor. When it has reached a pre-determined speed, a centrifugal device on the rotor shaft opens switch *S* and cuts off the starting winding. The motor then operates as a single-phase induction motor in the manner described in the previous article and accelerates until it reaches normal speed.

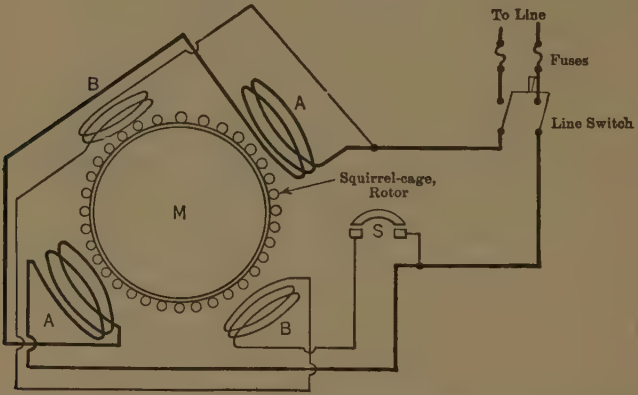
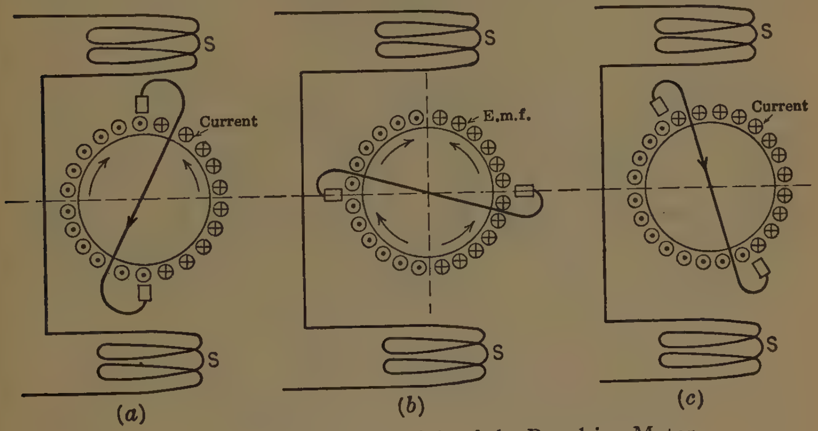


FIG. 332.—Diagram of Connections for a Split-phase Motor. *AA*—Operating winding. *BB*—Starting winding.

As the starting torque which is obtained by phase-splitting is not very large, this type of motor is suited only for applications requiring a small starting torque, such as washing machines, small printing presses, blowers and similar devices. Split-phase motors are made only in small sizes.

347. The repulsion type of single-phase motor is used where a comparatively large starting torque is required. The principle involved in this method of starting a single-phase motor is shown in Fig. 333. Referring to diagram (*a*), the stator winding *SS* would produce a flux with an axis in line with the brushes and a large current would flow, because the brushes are short-circuited. The net torque is zero, however, because half the conductors under a pole carry current in a direction opposite to the direction of current in the remainder of the conductors under this pole. If the brushes were shifted 90 degrees (Fig. 333 *b*), the e.m.f.'s induced in each path in the rotor would neutralize each other and there would be no voltage at the brushes. Thus no current would flow in the armature. If the brushes were shifted to a position between (*a*) and (*b*), (Fig. 333 *c*), there would be a resultant voltage in the armature, and, since the brushes are short-circuited, a current would flow in the armature conductors. The direction of this current is such as to produce a torque which would rotate the armature. A number of manufacturers employ this principle for starting single-phase motors. One example of this is shown in Fig. 334. The rotor or armature has a commutator with radial brushes, which are short-circuited through the brush-holder yoke. When voltage is applied to the stator winding, a large starting torque is produced, and the motor accelerates rapidly. When it reaches approximately full-load speed, a centrifugal device short-circuits all the commutator bars, thus making the rotor winding practically equivalent to a squirrel-cage rotor. The machine then operates as a single-phase



(*a*) (*b*) (*c*)

FIG. 333.—Illustrating Principle of the Repulsion Motor.

motor. In some designs of this type of motor, the centrifugal device lifts the brushes from the commutator at the same time that the rotor winding is short-circuited in order to reduce the wear on the commutator and brushes. The starting torque of this type of single-phase motor varies from 2.25 to 1.50 times full-load torque, the lower value being for a motor with a large number of poles. The maximum running torque is about 1.75 times full-load torque. The

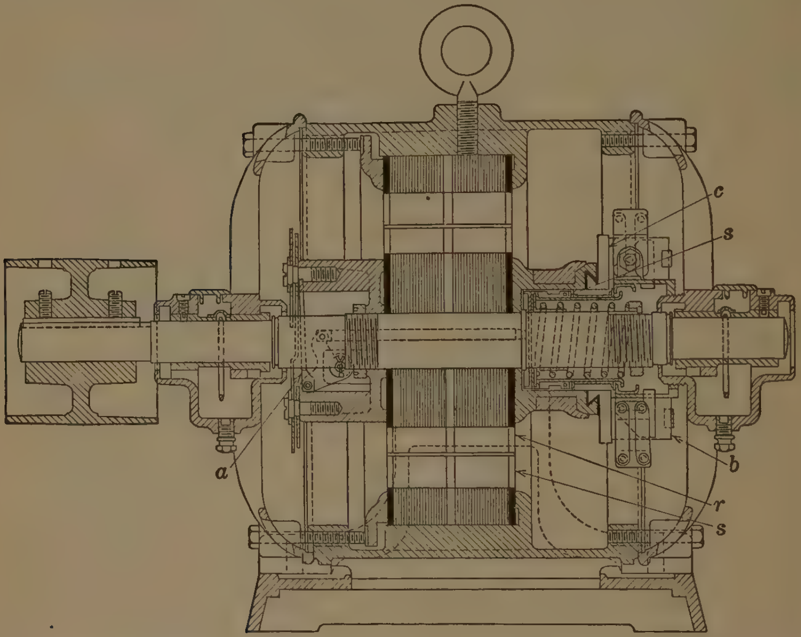


FIG. 334.—Single-phase Induction Motor Started as a Repulsion Motor. Primary and secondary windings are placed in slots in stator core (*s*) and rotor core (*r*). Brushes (*b*) bear on radial commutator (*c*). A centrifugal mechanism (*a*) operates a switch (*s*) which short-circuits the commutator segments and at the same time separates the brushes from the commutator. *The Century Electric Co.*

type of motor described in the preceding paragraph starts as a repulsion motor and runs as a single-phase induction motor after the centrifugal device has short-circuited the rotor coils. Another type of single-phase machine, known as the "repulsion-induction" motor, has a rotor with two windings; one a squirrel-cage, and the other a winding connected to a commutator. As no centrifugal short-circuiting device is used, both windings are in service when starting as well as when running. The motor is, therefore, a combination of single-phase induction motor and a repulsion motor. Motors of the type first described, which operate as single-phase induction motors when at normal speed, have a speed characteristic similar to that of the d.c. shunt motor. The speed characteristic of the repulsion-induction motor resembles that of a d.c. compound motor. Machines of this kind usually have a higher power-factor than the repulsion-starting, induction-running type of motor. This improvement of power factor is secured by means of a compensating field produced by auxiliary brushes, bearing on the commutator.

348. The Single-phase Series Motor.—The direction of rotation of either a series or a shunt-wound motor is the same regardless of the polarity of the voltage applied to the motor terminals, provided there is no change in the relative connection of field and armature windings. Therefore, if a d.c. motor were connected to an a.c. supply, there would be a torque tending to produce rotation of the armature. For a shunt-wound machine, however, the high self-induction of the field winding would permit only a small field current to flow and, hence, would produce only a weak field. Furthermore, the flux produced by the field current would be out of phase with the current in the armature owing to the high self-induction of the field winding and, therefore, the torque would be small. In the series motor, the same current flows in both armature and field, and, if such a motor were connected to an a.c. supply, considerable torque would be developed because the field flux would be nearly in phase with the armature current. An ordinary d.c. series motor would not, however, operate satisfactorily from an a.c. supply for the following reasons:

(1) There would be a large eddy current and hysteresis loss in the field magnet, due to the alternating flux.

(2) The high self-induction of field and armature windings would cause a large reactive drop, which would produce a low power factor for the machine.

(3) There would be excessive sparking at the commutator because the coils short-circuited by the brushes would be exposed to the alternating flux of the main field and these coils would act like short-circuited secondary coils of a transformer so that large currents would flow in the coils undergoing commutation.

The difficulty due to eddy currents is overcome by making the main magnetic circuit entirely of laminated sheet steel. The punchings forming the stationary field poles and yoke are supported in a cast-steel housing. This construction is more expensive than that used for d.c. motors.

The effect of the self-induction of the field and armature windings is illustrated in Fig. 335. The total fall of potential in the field E_f is due to a resistance drop IR_f and a reactance drop IX_f where R_f and X_f are the resistance and reactance of the field winding. This is true also for the armature, where R_a includes the brush resistance. A portion of the impressed voltage E must be used to overcome the fall of potential in the armature and field and the remainder E_c is the counter e.m.f. in the armature. Since the product E_cI is the total power converted into the mechanical form, it is essential that E_c be made as large as possible in order that the efficiency shall be high. The power factor of the motor when running is cos θ (Fig. 335) hence, X_a and X_f should be made as small as possible in order to give as high a power factor and as large a value of E_c as possible. The field reactance X_f is reduced to a minimum by employing the following expedients:

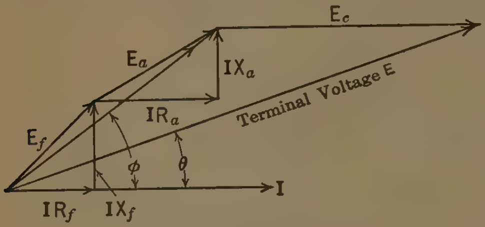


FIG. 335.—Vector Diagram for a Single-phase Series Motor.

(1) The number of turns on the field is reduced, since the self-induction varies as the square of the number of turns.

(2) With a small number of field turns the magnetomotive force of the field winding is small; hence, the field flux is relatively smaller than for a d.c. series motor. To make the flux as great as possible with the small number of turns used, the reluctance of the magnetic circuit is made low by the use of high permeablity sheet steel and a short air gap.

(3) The frequency of the supply must be as low as possible. It is customary to use 25 cycles, since this is the lowest standard frequency. Except for very small machines, the use of 60 cycles is not practicable.[1]

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[1] The so-called "universal motors," which are extensively used for vacuum cleaners, fans, and other devices requiring very small amounts of power are series commutator motors. These motors operate on either 60-cycles alternating current or on direct current.

The required torque for a given motor may be secured by using a strong field and small number of armature turns or by a weak field and a large number of armature turns. For the single-phase series motor it was shown that it is necessary to have a weak field; hence, the armature must have a large number of ampere-turns. But a large number of armature ampere-turns would result in a high value of X_a, which is undesirable. The value of X_a is reduced by neutralizing the field produced by the armature current. This is accomplished by a compensating winding which is placed in slots in the main poles. This winding acts like the compensating winding used on d.c. machines (see Article 141) and neutralizes the field produced by the armature current. The arrangement is shown in Fig. 125. The compensating winding may be connected in series with the armature as in a d.c. machine or it may be short-circuited on itself without any electrical connection with the armature circuit. In this arrangement, the current in the compensating winding is secured by transformer action. If the motor is designed for operation on either direct or on alternating current, the compensating winding must, of course, be connected in the armature circuit.

The excessive sparking caused by the transformer action in the armature coils which are short-circuited by the brushes is eliminated by the use of high resistance leads, sometimes called preventive leads, which join the coils to the commutator bars (Fig. 336). These resistance leads introduce additional resistance in the armature circuit. For the coil undergoing commutation, this resistance is relatively high, since two of these leads are in series; but for the main armature current the loss due to these leads is small, since there are two in parallel at each brush. High resistance brushes, which are somewhat narrower than is usual on d.c. machines, also assist in reducing the sparking. The torque and speed curves of a.c. series motors are much the same shape as those of a d.c. series motor. At light loads, when the speed is high, the power factor is close to unity, but at full load it is lower, reaching about 90 per cent in some cases. When starting or when carrying an overload, the power factor is lower. The motor must be more liberally designed than a d.c. motor and, therefore, is heavier and more expensive. Because of commutating difficulties, series a.c. motors are suitable only for

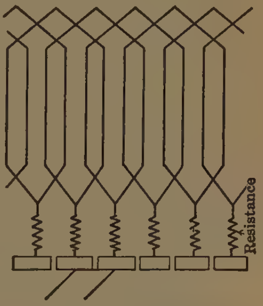

Fig. 336.—Preventive Leads.

comparatively low voltages and are usually designed for not more than about 300 volts. These motors are used for a.c. railway systems. Control of the speed is generally accomplished by varying the voltage applied to the motor by connecting it to different taps on a transformer.

CHAPTER XXXVI

ALTERNATING-CURRENT TRANSMISSION AND DISTRIBUTION

349. The essential parts of an electric transmission and distribution system are shown in Fig. 337. The electric energy produced by the generators is usually transmitted at a high voltage; therefore, "step-up" transformers must be used at the generating station. The electrical energy is carried by the transmission line to one or more substations located at distributing centers near the points where the energy is to be utilized. The transmission voltage is reduced by "step-down" transformers at the substations to suit the requirements of the loads to be supplied. The energy is carried from these substations by distribution circuits, to the different customers.

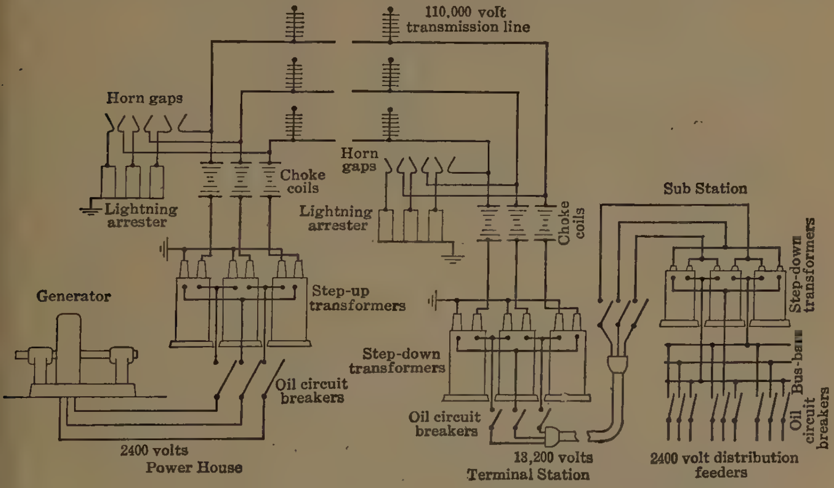

FIG. 337.—Diagram of a Transmission and Distribution System.

350. Transmission Systems.—It was shown, in Article 188, that the amount of copper required to transmit a given amount of power varies inversely as the square of the voltage. Hence, it is desirable to use as high a transmission voltage as other limitations will permit. As the voltage is increased, however, the cost of insulators, switches, transformers, and other parts of the transmission system increase; therefore, the determination of the most economical voltage for given conditions must be based upon proper consideration of all these factors. In general, it is found that even for short distances, such as 5 to 10 miles, a voltage of about 11,000 volts is desirable. For longer distances, a value of about 1000 volts per mile is required. Power may be transmitted by either direct or alternating current, but only the a.c. system is used in the United States. In Europe, a d.c. system, called the Thury System, is used. This system has specially designed, high-voltage, constant-current generators which are connected in series and supply direct current to the transmission line. At the substation, similar high-voltage machines acting as motors are used to drive generators which supply the energy at a low voltage, suitable for distribution.

With the a.c. system, it is possible to produce the energy at a voltage best suited for the generators, to raise this voltage by static transformers to the best value for the transmission system, and to reduce this voltage by similar static transformers to a voltage suitable for distribution. About 95 per cent of all the electrical energy produced in the United States is in the form of alternating current, and most of it is used without conversion to direct current. The three-phase system is generally used for power transmission because this system requires only 75 per cent of the copper which would be needed for either a two-phase or a single-phase system, assuming the same voltage between conductors and equal line losses. Although frequencies of 25 and 60 cycles are both used on transmission systems, by far the greater proportion of these systems use 60 cycles.[1] For underground power transmission systems, voltages of 6600, 11,000 and 13,200 are used extensively and there are a few instances where higher voltages are employed. At present (1926) 132,000 volts is the highest value used on underground systems. Overhead systems are operated at voltages ranging from 11,000 to 220,000 volts.

It is not economical to design alternators for a voltage much above 15,000 volts because of the large amount of insulation required. In general, the generator voltage must be raised to a suitable line voltage by means of static transformers. Hence, the generator voltage is independent of the line voltage and is fixed at a value which will give a moderate size of conductors and bus-bars in the station. The voltages most commonly used are 2400, 6600, 11,000 and 13,200 volts, the higher voltages being used for the larger installations.

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[1] See Article 205, regarding the relative merits of 25 and 60 cycles.

351. Distribution Systems.—Direct-current distribution systems were described in Chapter XXIII. When alternating current is used, there may be a primary and a secondary distribution system. Primary systems usually operate at about 2300 volts. If the customers were scattered over a large area, they would be supplied in groups from transformers connected to the primary system. Where there are a large number of customers in a small area, for example, in the business district of a city, a secondary distributing system is generally used. Secondary voltages are customarily 110 or 220 volts for lighting and 110, 220, 440 or 550 volts for motors. Large motor installations are usually supplied at 2200, 6600 or 11,000 volts. Voltages of 2200 and 6600 are in common use in steel mills. Primary systems are either two- or three-phase. Secondary systems are usually two- or three-phase, although single-phase, three-wire systems are sometimes used. The three-phase, four-wire system (See Article 255) is extensively used for both primary and secondary distribution. A primary system of this type would have a potential of about 2300 volts between line wires and neutral giving 4000 volts between line wires. The secondary system would also be three-phase, four-wire, with about 115 volts to neutral and 200 volts between phases. Lighting loads would be connected between a line wire and neutral; motors, which would be of the three-phase type, would be connected to the three line wires. Individual customers are sometimes supplied directly from the primary system. An individual lighting load would be supplied from a single transformer connected between a line wire and neutral; for a motor load, two transformers, connected to the three line wires in *V*, would be used.

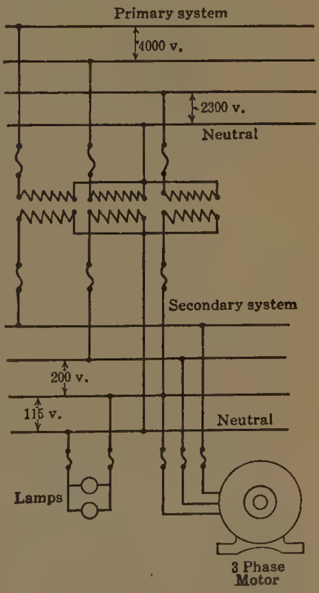

FIG. 338.—Diagram of a Distribution System. Four-wire, three-phase primary and secondary.

A two-phase distribution system has also been used extensively, but it requires more copper than the three-phase system and, in the case of the three-wire two-phase system, there is more difficulty in regulating the voltages on the two phases. Calculation and experience have shown that the four-wire, three-phase system is the most desirable for both primary and secondary distribution, on the basis of first cost, operating and maintenance cost, and reliability.[2] A typical distribution system is shown in Fig. 338.

For industrial installations requiring a considerable amount of power which may be supplied from individual plants or a central station, the three-phase, three-wire system would, in general, be most suitable and is commonly used. Where direct current is necessary, for adjustable speed tools or for cranes, it would be produced by motor-generator sets or synchronous converters located in the plant substation.

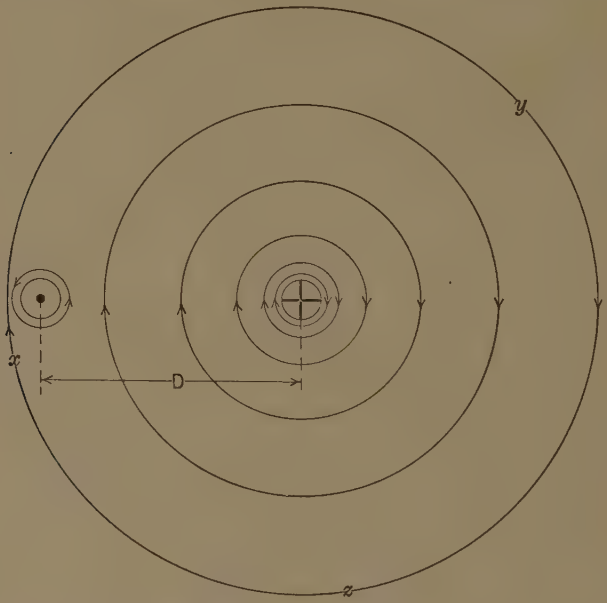

FIG. 339.—Magnetic Field around Two Conductors.

352. Line Reactance.—It has been shown that a magnetic field always surrounds a conductor carrying a current. If this current is alternating, the field is alternating also, and therefore an e.m.f. of self-induction will be produced in the conductor. In the case of an overhead transmission line, the magnetic field at any point on the circuit is comparatively weak, but the total flux surrounding a long line is large and, therefore, the reactive voltage due to this flux cannot, in general, be neglected in determining the voltage drop

[2] *Electrical World*, Feb. 17, 1923.

in the circuit. In Fig. 339 are shown the two conductors of a single-phase circuit which are separated by the distance D. The inductive voltage produced in each of these conductors depends upon the total flux surrounding the conductor. For magnetic paths which pass through the space between the centers of the two conductors, the magnetomotive forces due to the currents in the conductors are in the same direction. For paths outside the conductors, such as x, y and z, the magnetomotive forces are opposed. Hence, the flux surrounding each conductor extends only to a distance D from the conductor. At greater distances, the field is neutralized by the current in the other conductor. If the conductors are close together, the total flux surrounding each will be small since most of it will be neutralized by the current in the return conductor. Hence, the self-induced voltage is a minimum when the two conductors are placed close together. The total flux surrounding a conductor therefore depends upon its length l and upon the distance D to the return conductor. This is given by the following equation:

$$L=\left(0.0007412 \log_{10}\frac{D}{r}+0.0000805\right)l \quad . \quad . \quad . \quad . \quad (91)$$

where L = coefficient of self-induction in henries for one wire of the line;
D = distance between wires in inches;
r = radius of the wire in inches;
l = length of the line in miles.

For stranded wires, the radius r may be taken as that for a solid wire having the same copper area. The value of l is the length of the line, or the distance between the supply and the load. For a single-phase line, the total length of conductor would be $2l$, and the total self induction of the line would be $2L$.

Example 1.—A single-phase overhead line has No. 0000 stranded conductors spaced 24 inches apart. The length of the line is 10 miles. Calculate the self-induction for each wire.

$$L=\left(0.0007412 \log_{10}\frac{24}{0.230}+0.0000805\right)10=0.01574 \text{ henry.}$$

For a three-phase line, having the wires separated by a distance D, Fig. 340 *a*, the total field surrounding one wire is determined by the distance D, since the other two wires can be considered as a return for the current in the other conductor. Hence, Equation (91) can be used to determine the self-induction for each of the three wires.

Example 2.—Calculate the inductance per mile for each wire of a three-phase line with No. 000 stranded conductors spaced 3 feet apart as in Fig. 340(*a*).

$$L=\left(0.0007412\log_{10}\frac{36}{0.205}+0.0000805\right)=0.001746 \text{ henry.}$$

For a three-phase line with irregular spacing (Fig. 340 *b*), the value of D is

$$D=\sqrt[3]{ABC} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

If the wires are all in the same plane (Fig. 340 *c*) and $A=B$, the effective spacing is

$$D=1.26A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

Equations (92) and (93) are correct only if the wires are transposed at regular intervals so that each conductor is between the other two for one-third the distance. If the conductors are not transposed, the self-induction of the outside wires, 1 and 3 (Fig. 340 *c* , is greater than the center wire.[3]

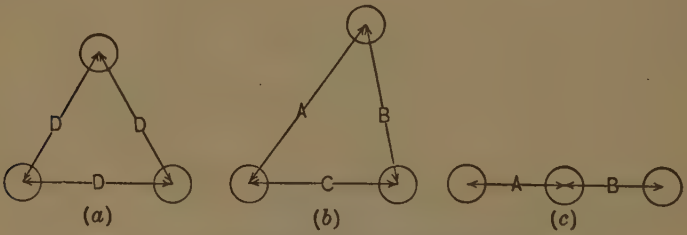

FIG. 340.—Arrangement of Three-phase Line Wires.

353. Effect of Electrostatic Capacity.—The conductors of an electric circuit possess electrostatic capacity and, therefore, a capacity or "charging" current must be supplied to any a.c. line. This charging current is large for underground cables or long overhead lines (see Article 235), but, for short lines at comparatively low voltages, this current is small and is usually neglected in making line calculations. An error of not more than 0.5 per cent is made if the effect of electrostatic capacity is neglected, for lines not exceeding about 125 miles for 25 cycles and 50 miles for 60 cycles.[3]

354. Single-phase Line Calculations.—If electrostatic capacity is neglected, the single-phase line can be treated as a series circuit having resistance and self-induction. Examples of the calculation of voltage drop for a single-phase line at different power factors are given in Article 232, Chapter XXVI. The percentage line drop is based upon voltage at the load end. Percentage energy loss is based upon the power delivered by the line. The *voltage drop* for a line is the numerical difference between the voltages at the power house and the load end of the line. This is not always equal to the impedance volts in the line.

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[3] See "Electrical Characteristics of Transmission Lines," by Wm. Nesbit, *Electric Journal*, August, 1919.

Example 1.—Referring to Example 1, Article 232, the line drop with unity power factor load is:

$$2452-2200=252 \text{ volts.}$$

The drop at a lagging power factor of 0.8 is

$$2463-2200=263 \text{ volts.}$$

The impedance voltage for the line is

$$ZI=\sqrt{250^2+104^2}=271 \text{ volts.}$$

The per cent voltage drop is:

For unity power factor, $\frac{252\times100}{2200}=11.5$ per cent.

For 0.8 power factor lagging, $\frac{263\times100}{2200}=12$ per cent.

The current is 45.5 amperes (see Article 232) in both cases since the load is 100 kv-a. Hence the energy loss for unity power factor and for 0.8 power factor is the same and is

$$P=I^2R=45.5^2\times5.28=10{,}900 \text{ watts.}$$

The per cent energy loss is:

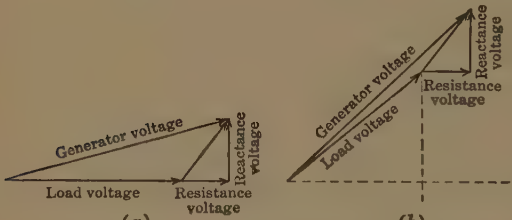
For unity power factor, $\frac{10.9\times100}{100}=10.9$ per cent.

For 0.8 power factor, $\frac{10.9\times100}{100\times0.8}=13.6$ per cent.

In a d.c. line, the per cent voltage drop is the same as the per cent energy loss, but this is not always true of an a.c. line as is shown by the above example. The voltage drop on an a.c. line will not be greater than the impedance volts, but it may be less as is shown in this example. In general, the drop for a lagging power factor will be greater, and for a leading power factor less, than for unity power factor. Sometimes the drop with a leading power factor becomes negative, indicating that the voltage at the load is higher than at the power house. The drop on an a.c. line is affected by the power factor and also by the amount of reactance in the line. For interior wiring, where the conductors of a circuit are either in conduit or are supported on insulators with a close spacing, the reactive effect can be neglected except for large conductors. The total drop will be practically the same as the resistance drop unless the size is greater than is specified below:

Kind of Load	Size of Circuit	
	In Conduit	2.5 In. Spacing
Incandescent lamps—60 cycles.........	No. 0000	No. 00
" " —25 "	600,000 c.m.	400,000 c.m.
Induction motors —60 "	No. 1	No. 3
" " —25 "	No. 0000	No. 00

Where the power factor is near unity the line drop depends principally upon the loss of voltage due to resistance and the reactance volts have a much smaller effect. For low power factors the opposite is the case, the resistance loss not being so important as the reactive loss. This is shown in Fig. 341, where (*a*) represents the conditions for unity power factor and (*b*) for a low power factor. In the first case it will be seen that the line drop is nearly equal to the resistance voltage and a large variation in the reactance voltage would have only a small effect upon the total line drop. On the other hand for a low power factor (Fig. 341 *b*) the line drop depends principally upon the reactive voltage and a large variation of resistance voltage may occur without much change in the total drop.



(*a*) (*b*)

FIG. 341.—Effect of Power Factor on Drop in a Transmission Line.

355. Three-phase line calculations can be made best by determining the resistance and reactance volts for one wire, using the voltage to neutral in determining the drop. This method can be employed for either a delta- or Y-connected load since we are dealing only with the line current.

Example 1.—A three-phase line 20 miles long is composed of No. 0000 stranded conductors spaced three feet apart in a delta arrangement. The load carried by the line is 1300 kv-a. at 0.80 power factor with 10,000 volts at the load. The frequency is 60 cycles. Calculate the voltage drop and energy loss in per cent. The resistance of the conductor is 0.27 ohm per mile. The reactance at 60 cycles is 0.644 ohm per mile. For one wire of the line the values are

$$R=20\times0.27=5.4 \text{ ohms.} \quad X=20\times0.644=12.9 \text{ ohms.}$$

The line current is

$$I=\frac{1{,}300{,}000}{10{,}000\times\sqrt{3}}=75 \text{ amperes.}$$

The fall of potential in one wire is

$$RI= 5.4\times75=405 \text{ volts}$$

$$XI=12.9\times75=966 \text{ volts}$$

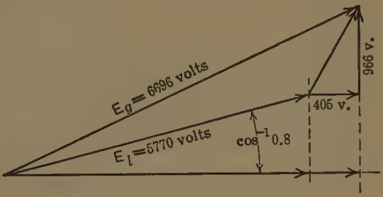


FIG. 342.—Vector Diagram for a Three-phase Line.

The voltage to neutral at the load end of the line is $10{,}000\div\sqrt{3}=5770$ volts. The voltage to neutral at the generator (Fig. 342) is

$$E=\sqrt{(\,(5770\times0.8)+405)^2+(\,(5770\times0.6)+966)^2}=6696 \text{ volts}$$

The phase voltage is $\sqrt{3}\times6696=11{,}600$ volts.

The voltage drop is 1600 volts or 16 per cent.

The power lost is

$$P=3\times I^2R=3\times75^2\times5.4=91{,}000 \text{ watts,}$$

or

$$\frac{91\times100}{1300\times0.8}=8.9 \text{ per cent.}$$

In all cases, the size of conductor must be sufficient to carry the current without overheating. This was explained in Article 193. In general, the size of conductor for short lines would be determined by carrying capacity rather than voltage drop. For longer lines, a conductor large enough to carry the current may give excessive voltage drop; hence, the size would have to be increased. For the ordinary industrial applications, motor circuits may have a total drop of about 10 per cent from the switchboard to the motor. For lighting circuits it is best to limit the total drop to about 5 per cent. For transmission lines a drop of more than 10 per cent may sometimes be allowed. For very long lines it would not be economical to attempt to provide sufficiently large conductors to give satisfactory voltage regulation. The substation voltage is, therefore, controlled by means of synchronous condensers located at the receiving end of the line. (See Article 323.)

356. Arrangement of Circuits.—The voltage drop of a d.c. circuit is not affected by the arrangement of the conductors, but, with a.c. circuits, the spacing of the wires affects the reactance of the circuit and therefore, influences the line drop. It has been shown (Article 352), that the reactance is increased as the spacing between conductors is increased. Therefore, in order to keep the reactive drop as low as possible, the spacing is made as small as other requirements will permit. The spacing of overhead circuits depends upon the voltage and the length of the spans. For interior wiring, the minimum spacing is 2.5 inches if the wires are supported on insulators. If the conductors are installed in iron conduit, all the wires of a circuit must be placed in the same conduit. If separated, the magnetic fields around the wires are not neutralized, and are, in fact, greatly increased by the presence of the iron. As a result, the reactive drop would be high, and the conduit would heat, if the wires were carrying a large current. If the current is so great as to require more than one wire for each lead of the circuit, and it is not feasible to put them all in one conduit, because of the large size of conduit required, the leads should be divided into two or more groups, each containing all the poles of the circuit. The resistance drop on a circuit is decreased proportionally to the increase in cross-section, but the reactive drop does not decrease as rapidly; therefore, large conductors used for a.c. circuits have an excessive reactive drop. For this reason, it is generally not economical to use sizes larger than about 300,000 circular mils for distribution circuits unless the wires are in conduit, when somewhat larger sizes may be used. Circuits of large capacity can be made up best of two or more wires in parallel.

Example 1.—A 500,000 cir. mil feeder (spacing 6 ins.) carrying a load of 300 amperes at 60 cycles a distance of 500 feet has a drop of 18.5 volts at a lagging power factor of 0.8. It is required to find the size of wire, to give the same drop, with the load divided between two feeders. By calculation, it is found that a No. 00 (133,000 cir. mils) circuit carrying 150 amperes gives a drop of 17.2 volts. Hence, two of these could carry as much load as a single 500,000 cir. mil feeder, and the drop would be somewhat less. Therefore, the load can be transmitted by two circuits totaling 266,000 cir. mils with practically the same voltage drop as a single 500,000 cir. mil circuit. The saving in copper thus obtained is partly offset by the additional cost of running two circuits.

If two feeders are used, they should be of the same size. Thus in Example 1, if a No. 000 (168,000 cir. mils) cable and a No. 0 (106,000 cir. mils) cable were used, the smaller wire would be overloaded, although the total section of copper would be 274,000 cir. mils.

357. Skin effect is the term applied to the apparent increase in the resistance of a conductor when carrying alternating current. The central portion of a wire is surrounded by a magnetic field which is stronger than the field around the outside portion. Hence the inductive voltage produced in the central portion is higher, giving a greater opposition to the flow of the current through the center. As a result, the current is crowded towards the surface of the conductor, to such an extent that the center, in the case of large conductors, carries practically no current. The wire acts as if the cross-sectional area had been reduced by taking out the central part; in other words, the apparent resistance is increased. Skin effect increases with the frequency and the size of wire. It can be neglected for copper wires, smaller than 300,000 cir. mils on 60 cycles and 750,000 cir. mils on 25 cycles. The skin effect for iron conductors, such as steel rails, is very much greater. Skin effect not only increases the drop due to resistance but it also increases the heating of the conductor. Therefore, due to skin effect, a large conductor, carrying alternating current, would run somewhat warmer than when carrying the same amount of direct current.

PROBLEMS ON CHAPTER XXXVI

1–36. Calculate the reactance (each wire) at 60 cycles of a 50-mile, three-phase line consisting of 0000 stranded conductors spaced 36 inches apart on an equilateral triangle.

2–36. What would be the reactance of the line of Prob. 1–36 if the three conductors were spaced 36 inches apart in the same plane assuming that the conductors were transposed.

3–36. A three-phase, 60-cycle line consisting of 000 stranded conductors is 25 miles long. The conductors are spaced 24 inches, 30 inches and 38 inches apart respectively and are transposed. Calculate the reactance per wire.

4–36. A single-phase lighting feeder consists of two No. 0 stranded wires spaced 6 inches apart. The reactance per 1000 feet for each wire at 60 cycles is 0.0885. The feeder is 500 feet long and supplies a load of 25 kw. at 250 volts and unity power factor.

(*a*) Calculate the fall of potential in the line.

(*b*) Calculate the required voltage at the power-house end of the feeder.

5–36. Assuming the same line as in Prob. 4–36, with a load of 25 kv-a. at a power factor of 0.80 lagging:

Calculate the voltage at the power-house end of the feeder.

6–36. Determine the size of wire for a single-phase feeder to transmit 200 amperes at 60 cycles and a power factor of 0.90 lagging, a distance of 750 feet. The spacing between wires is 6 inches and the drop is not to exceed 16 volts. Assume 220 volts at load.

Note: If it is found that the size must be larger for alternating current than for direct current, assume a larger size and calculate the drop. Continue this until nearest size is found.

7–36. A single-phase load of 250 amperes at 240 volts, 25 cycles, and 0.85 power factor lagging, is to be transmitted a distance of 1400 feet with a drop not to exceed 20 volts. Assume a 12-inch spacing.

(*a*) Calculate the size of wire if a single feeder is used.

(*b*) Calculate the size of wire if two feeders are used.

8–36. A branch circuit for a 15-hp., 110-volt, 60-cycle, single-phase motor is 65 feet long. The size is to be as per Table 6. Calculate the drop when motor is carrying full load. Spacing 6 inches.

9–36. A single-phase transmission line ten miles long, consists of two No. 2 stranded copper wires spaced 24 inches apart. The load is 500 kv-a. at a voltage of 11,000 and 25 cycles. Calculate:

(*a*) The total resistance of the line.

(*b*) The total reactance of the line.

(*c*) The total fall of potential in the line.

(*d*) The required voltage at the power house if the power factor is unity.

(*e*) The percentage energy loss and percentage drop.

10–36. (*a*) Calculate the required power-house voltage for the line in Prob. 9–36, if the load is 500 kv-a. and the power factor 0.70 lagging.

(*b*) Calculate the percentage energy loss and percentage drop.

11–36. (*a*) Calculate the required power-house voltage for the line in Prob. 9–36, if the load is 500 kv-a. and the power factor is 0.70 leading:

(*b*) Calculate the percentage energy loss and percentage drop.

12–36. Calculate the total reactance of line in Prob. 9–36, if the spacing is made 12 inches.

13–36. A three-phase line 10 miles long consisting of three No. 2 wires spaced on a triangle 24 inches on a side carries a balanced load of 500 kv-a. at 11,000 volts, 25 cycles, and unity power factor.

(*a*) Calculate the fall of potential on one wire.

(*b*) Calculate the fall of potential on the line.

(*c*) Calculate the required power-house voltage.

CHAPTER XXXVII

A.C. SWITCHING AND PROTECTIVE DEVICES

358. Oil Switches and Circuit-breakers are used to interrupt a.c. circuits. They are so designed that the interruption of the circuit is accomplished by contacts which are immersed in oil, contained in a steel tank provided with a suitable vent. By this means, the arcing is reduced, and it becomes possible to interrupt circuits of large capacity by devices which are comparatively compact. Circuit-breakers are built in a large range of sizes as regards current and voltage rating. The type shown in Fig. 343, is typical of circuit-breakers intended for systems of moderate capacity and voltages not exceeding 600 volts. Circuit-breakers for large capacity systems have the tanks containing the individual poles separated by brick or concrete partitions. Figure 344 shows a type of circuit-breaker used for high voltage transmission systems. This style is designed for outdoor use and is electrically operated. The smaller circuit-breakers are usually manually operated. Automatic opening of the circuit-breaker is usually effected by a relay which is operated by current from a current transformer in the line wire. The plunger of the relay may be arranged to trip a latch which

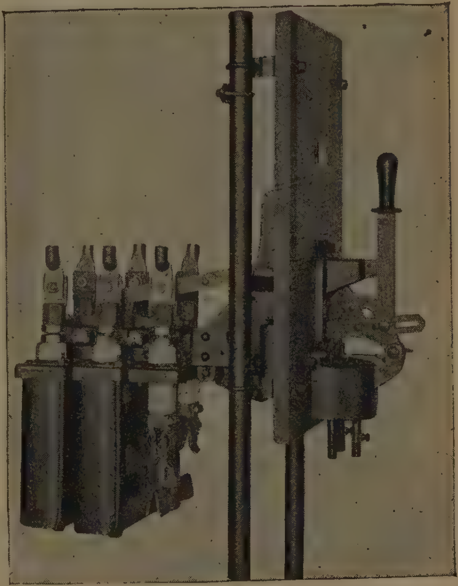


Fig. 343.—Oil Circuit Breaker and Operating Mechanism. Hand-operated Type. Triple-pole, 600 volts, 600 amperes. This type of circuit breaker is made for voltages as high as 7500 volts. *The General Electric Co.*

releases the circuit-breaker contacts or it may close an electrical circuit which operates a trip-coil on the circuit-breaker. The contacts are opened by gravity assisted by springs which are under tension when the contacts are closed.

The size of a circuit-breaker required for a particular installation is determined not only by the ampere carrying capacity and the voltage rating, but also by the capacity of the circuit-breaker to interrupt the circuit under short-circuit conditions. Manufacturers therefore include in the rating the **interrupting capacity** of the circuit-breaker which is the highest current (effective) which the device will interrupt under normal conditions. The amount of current which a circuit-breaker may be called upon to interrupt when a short-circuit occurs must be determined from an estimate of the short-circuit current produced by all the generating machinery connected in the circuit. With large systems, this current is so great that it is necessary to use current-limiting reactors (see Article 226) in the feeder and generator circuits in order that the circuit-breakers may be able to handle the short-circuit current.

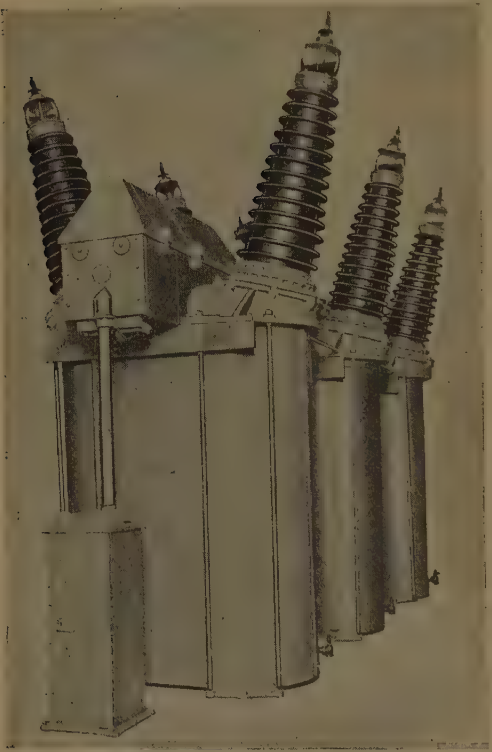
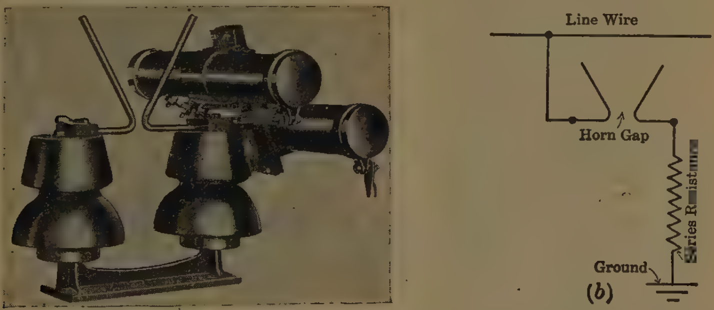


FIG. 344.—Oil Circuit Breaker, Electrically Operated, Outdoor Type. Triple-pole, 165,000 volts, 400 amperes. *The General Electric Co.*

359. Lightning Arresters.—Power transmission systems are exposed to abnormal voltages caused by lightning, arcs over insulators, switching, and similar disturbances. These abnormal voltages would cause damage to the insulation of the system unless means were provided for their immediate discharge whenever they occur. Lightning arresters are therefore designed to provide a discharge path of low resistance and at the same time to prevent a short-circuit on the system. One type is shown diagrammatically in Fig. 345. A horn gap is connected between each line wire and ground through a resistor. The separation of the gap is sufficient to prevent the normal voltage from jumping the gap and, hence, there is no flow of power current. If the voltage between a line and ground rises above normal, however, the gap breaks down and the excessive voltage is discharged to ground. As soon as the gap is broken down the generator voltage sends power current across the gaps and there is a partial short-circuit of the system. The resistor is inserted to limit this short-circuit current to a value which will allow the horn gap to operate and break the arc after the high voltage disturbance has been discharged. Another method, somewhat similar in principle, employs a number of gaps in series. These gaps are formed between brass cylinders.



(a)

FIG. 345.—Horn Gap Type of Lightning Arrester. (a) Outdoor type, 4600-volt arrester for constant current systems; (b) Diagram of connections. *The General Electric Co.*

Lightning arresters, of the type described in the preceding paragraph, are not suited for use on large capacity or very high voltage systems because of the difficulty in extinguishing the power arc which forms when the discharge takes place, and also because these arcs in themselves set up high-voltage strains on the system. One type used extensively on large capacity systems is the **electrolytic or aluminum arrester** (Fig. 346). This consists of a stack of nested, cone-shaped aluminum trays separated by porcelain spacers and containing an electrolyte which forms an insulating film on the surface of the trays when current is passed through the nest of trays. This film is a good insulator up to a voltage of about 300 volts (effective), but, if the voltage applied across this insulating film exceeds this value, the film is punctured and a low-resistance path is provided for discharge of the excessive voltage. As soon as the voltage is reduced by discharge, the film reforms and introduces a high resistance in the circuit. This reduces the power current to a low value, which can be interrupted easily by a horn gap placed between the arrester and the line. The number of trays used in series is determined by the normal line voltage. The trays are mounted in a steel tank which is filled with oil to provide insulation and to assist in dissipating the heat produced during a discharge. If the arrester were connected directly to the line at all times, the small current which flows through the electrolyte would cause a gradual deterioration of the arrester. It is, therefore, necessary to connect a horn gap between it and the line so that, at normal voltage, no current can flow. This gap is adjusted to break down when the potential between line and ground is about 25 per cent above the normal value. The film on the aluminum trays must be reformed

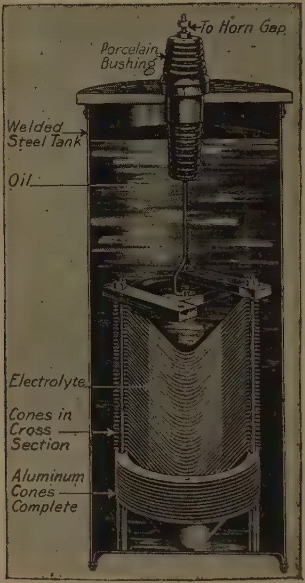


FIG. 346.

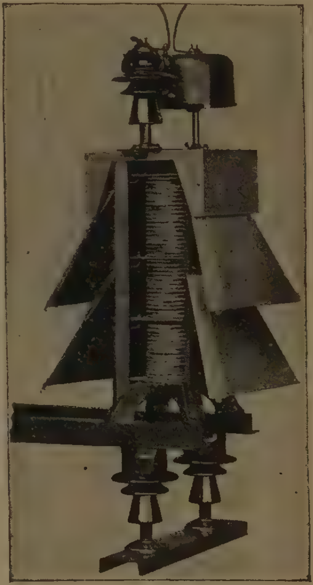


FIG. 347.

FIG. 346.—Sectional View of an Element of an Electrolytic Lightning Arrester. Capacity, 20,000 volts. *The General Electric Co.*

FIG. 347.—Oxide-film Lightning Arrester. Sectional view of one phase, showing arrangement of disks. Capacity, 15,000 to 25,000 volts, outdoor type. *The General Electric Co.*

at frequent intervals. This is done by momentarily shortening the gap until it discharges at normal voltage.

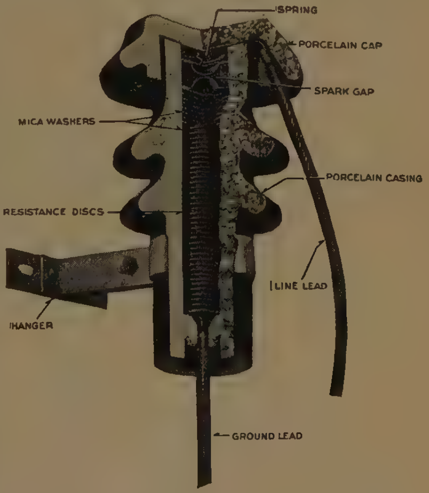


FIG. 348.—Autovalve Lightning Arrester. Outdoor type for 7500 volts. *The Westinghouse Electric & Mfg. Co.*

There are two other types of arresters which are used extensively and which have characteristics similar to the electrolytic arrester. One of these, the **oxide-film arrester**[1] (Fig. 347), uses a stack of metal disks which are separated by porcelain ring insulators. The space between these disks is filled with lead peroxide (PbO_2), and the disks are coated with varnish. When an excessive voltage occurs, the film of varnish is broken down, and a low resistance path is provided for discharge of the abnormal voltage. The heat produced by this discharge immediately raises the temperature of the lead peroxide at the point of discharge and converts it to litharge (PbO), which has an extremely high resistance. This seals the puncture in the varnish film and prevents a flow of power current following the discharge of the high voltage. If a high voltage occurs again, either the varnish or the film of litharge may be broken down and the process repeated. The action is very rapid and the arrester reseals itself immediately after the high voltage discharge has ceased. An air gap is provided between the arrester and the line. The **auto-valve arrester**[2] is a spark-gap type of arrester in which the abnormal voltage is relieved by a glow discharge between the faces of disks which are separated by mica spacers (Fig. 348). By the use of high resistance disks and a close spacing of about 0.005 inch, it has been found possible to produce a valve action resembling that of the electrolytic arrester. If a certain critical voltage of about 350 volts (instantaneous) per gap is exceeded, a glow discharge occurs, and the high voltage is relieved without the formation of an arc. As soon as the voltage drops below the critical value the discharge ceases. The number of disks is so chosen that the discharge occurs when the voltage is about 25 per cent above normal. The high

[1] Transactions of the American Institute of Electrical Engineers, June, 1918.

[2] Transactions of the American Institute of Electrical Engineers, February, 1923.

resistance material of which the disks are made prevents the formation of an arc during the discharge. A short air gap is connected between the line and the pile of disks. This type of arrester is used for protecting both station apparatus and overhead lines. The arrester shown in Fig. 348 is an out-door type used on distribution systems to protect the transformers.

Choke coils (Fig. 201) are used with all types of arresters. These offer a low impedance to the generator current but have a very high impedance for the abnormal voltages which are usually of extremely high frequency. The choke coils, therefore, prevent the high voltage from reaching the apparatus to be protected. (See Article 226.)

CHAPTER XXXVIII

A. C. MOTOR STARTING AND CONTROLLING DEVICES

METHODS OF STARTING SQUIRREL-CAGE MOTORS

360. Switch Starters.—Small-size induction motors, up to about 5 hp. capacity, are usually started by throwing them directly on the line at full voltage. A two-throw switch (Fig. 349) is used. The switch is thrown first to the starting position, which places the motor on the line without any fuse protection except that provided by the branch fuses. Since the starting current is large, these fuses must have a capacity of from 3 to 3.5 times the rated full-load current of the motor.

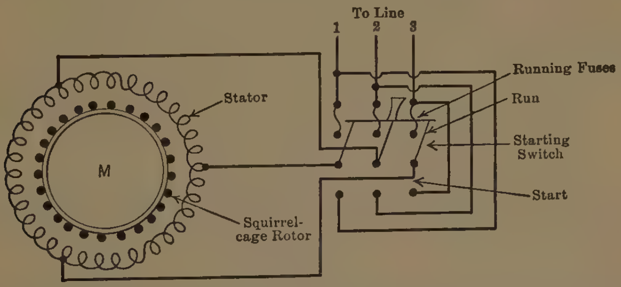


FIG. 349.—Connections for Starting a Squirrel-cage Induction Motor. Switch starter. This arrangement is used for small motors. For large motors, an autostarter is used, see Fig. 350.

After the motor has reached normal speed the switch is thrown to the running position, which cuts in the "running fuses." These are rated at 1.10 to 1.25 times full-load current of the motor and serve to protect the machine from excessive overloads during operation. If the running fuses were in circuit during the starting period, the heavy starting current would blow the fuses. The starting switch is so designed that it cannot be left in the starting position but must be thrown to the running position before the handle is released.

361. Auto-starters or compensators are used to start squirrel-cage induction motors where the starting current must be kept to a minimum. This type of starter is generally used for motors larger than 5 hp. The starter has auto-transformers which are connected to a two-throw switch by means of which the motor is supplied with a reduced voltage for starting and is then thrown on to the full voltage. The switch which produces these changes has oil-immersed contacts to reduce the arcing. The principle of operation is shown in Fig. 350, and an illustration of one make of starter is shown in Fig. 353. These starters can be adjusted to give 50, 65, or 80 per cent of normal voltage at starting. This is done by changing the connections at the auto-transformers. In order to keep the starting current as low as possible, it is important to select the lowest voltage tap that will start the load.

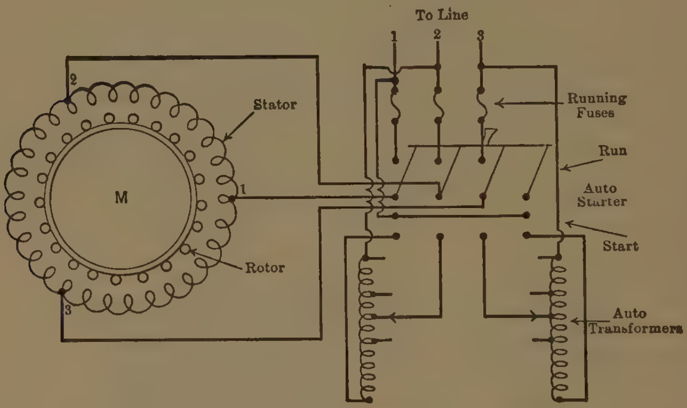


FIG. 350.—Connections for Starting a Squirrel-cage Induction Motor. Auto-starter. Line switch not shown. Overload relays are frequently used instead of the fuses shown.

To obtain full load torque at starting requires about 80 per cent of normal voltage. (See Article 290.)

Auto-starters are made for both manual and automatic operation. The automatic starter is now extensively used, although it is considerably more expensive than the hand-operated type. Overload protection is provided by two inverse-time-limit relays (Fig. 353). Low-voltage protection is also provided so that the switch will be released and open the circuit if the supply voltage fails.

The principal advantage of the auto-starter as compared with the switch starter arises from the fact that the line current taken by the motor is greatly reduced.

Example 1.—A 220-volt, three-phase, squirrel-cage induction motor develops a starting torque of 1.5 times full-load running torque with full voltage applied to the motor terminals. The starting current is four times rated full-load current. The torque with 80 per cent of normal voltage (176 volts) is

$$\frac{176^2}{220^2}=\frac{x}{1.5},$$

$x=0.96$ times full-load torque.

The starting current in the motor winding with 176 volts applied is $I_m=\frac{176}{220}\times 4=3.2$ times full-load current. Due to the use of auto-transformers, the starting current taken from the line is inversely as the primary and secondary voltages, and is

$$\frac{176}{220}\times 3.2=2.56 \text{ times full-load current.}$$

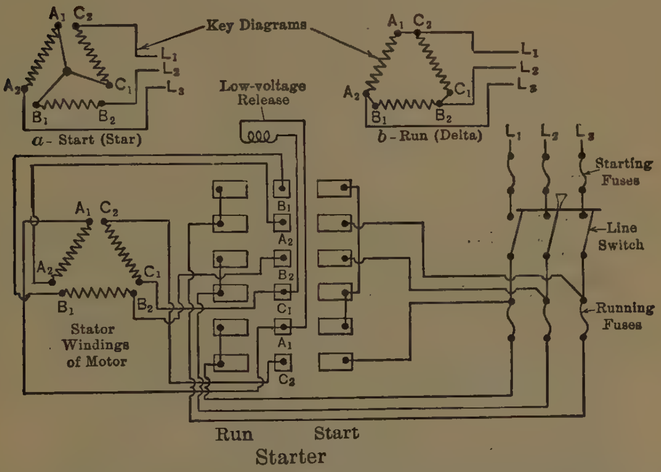


FIG. 351.—Diagram of Star-delta Method of Starting Squirrel-cage Induction Motors. When switch is in "start" position, center contacts connect with corresponding contacts at right, marked start. After the motor has speeded up, center contacts are thrown to left to "run" position, where they are held by the low-voltage release coil.

362. Star-delta Starting.—The reduced voltage necessary for starting a three-phase, squirrel-cage induction motor may be secured by changing the connections of the stator winding. When starting, the three groups of coils are connected in star (or Y) as shown in Fig. 351 *a*. Each winding then receives 58 per cent of the line voltage or 127 volts in the example shown. After the motor has started, the windings are thrown directly across the line by means of a switch (Fig. 351 *b*). The voltage across each winding is thereby raised to the normal value of 220 volts. If the load is not too great, this method is satisfactory, but the starting voltage cannot be adjusted, so that for heavy loads the motor would not receive enough voltage to start. This arrangement is limited in its application to small sizes of motors, where the starting loads are not heavy. The switch used with this method of starting is usually provided with a low-voltage release attachment, similar to that used with auto-starters.

363. Starters and Controllers for Slip-ring Motors.—This type of motor is started by inserting resistance in the rotor circuit. The stator windings are connected directly to the line, and the rotor resistance is cut out in steps as the motor speed increases. When the motor has reached normal speed with all the starting resistance cut out, the slip-rings are short-circuited either by switches connected directly to the brushes, or by switches mounted on the rotor spider in order to reduce the rotor resistance to a minimum. One type of resistance starter is shown in Fig. 352. The resistance is in three sections, which are Y-connected, since the

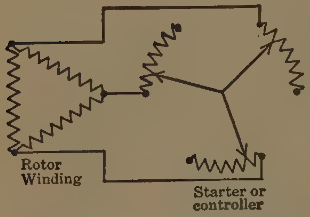


FIG. 352.—Connections for Starting a Slip-ring Induction Motor.

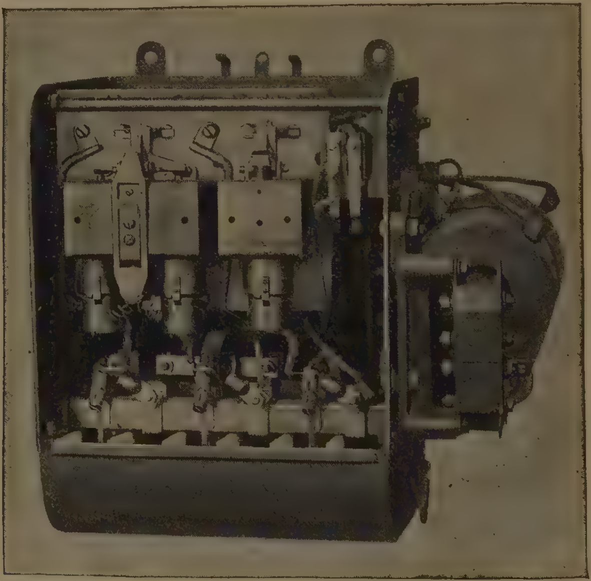
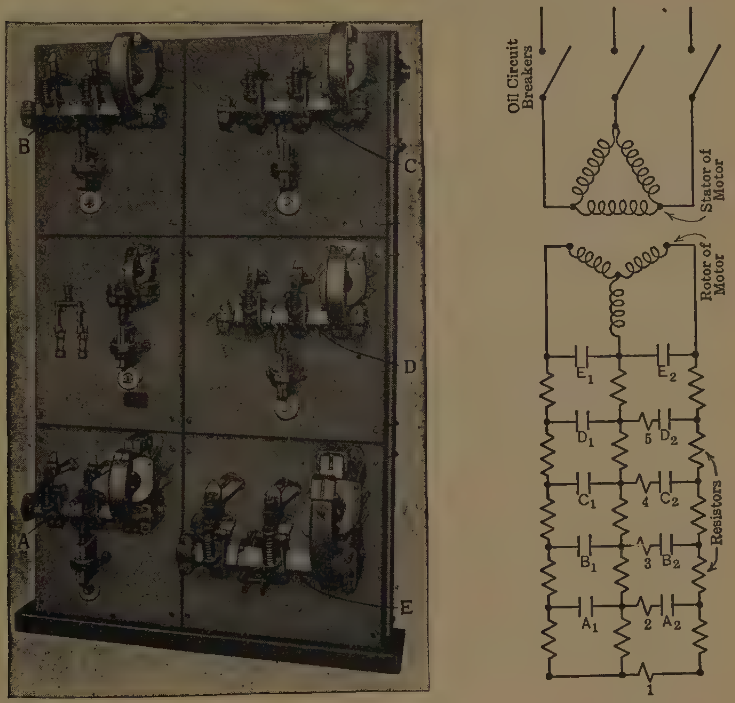


FIG. 353.—Auto-starter for Squirrel-cage Induction Motor. Automatic type. This starter is controlled by a push button or a float-switch. Two overload relays (left hand) and a low-voltage relay are provided. *The Westinghouse Electric & Mfg. Co.*

rotor winding is three-phase. Movement of the controller handle cuts out the resistance in steps. For small motors, this controller is of the face-plate type similar to that used for d.c. motors (Fig. 137). Large slip-ring motors are provided with drum-type con-



(*a*) (*b*)

FIG. 354.—Automatic Starter for a Slip-ring Induction Motor. (*a*) Front view of rotor control panel. (*b*) Diagram of connections. Acceleration is accomplished by cutting out rotor resistances by means of the electrically operated switches *A*, *B*, *C*, *D*, *E*. The operating coils of these switches are connected across the stator side of the motor. When the oil circuit breaker in the stator circuit is closed, the motor starts with all resistance in the rotor circuit. The electrically operated switches are held open, however, until the rotor current has decreased to a predetermined value, then relay 1 energizes the operating coil of switch *A*, which closes contacts A_1 and A_2, thus cutting out a portion of the rotor resistance. As soon as the current has decreased sufficiently, relay 2 closes and energizes the coil of switch *B*, thus closing contacts B_1 and B_2. This process is repeated until all resistance is cut out, when switch *E* is closed. The stator circuit is provided with overload and low-voltage release relays and measuring instruments, connections for which are not shown. *The Westinghouse Electric & Mfg. Co.*

trollers similar to Fig. 140, if they are to be manually controlled or with electrically operated switches or contactors if automatic control is desired. In general, contactors would be used for large motors.

These are sometimes operated from a small drum-type controller, and in other cases are automatically controlled. (See Article 364.)

If the speed of a slip-ring motor is to be adjusted by varying the rotor resistance, the same type of drum controller with resistance is used. In this case, however, the carrying capacity of the rotor resistances must be greater, in order that they may be left continuously in the circuit.

364. Automatic Starters for induction motors are used where the motor must be controlled from a distance or where the motor operates a pump or air compressor, and it is necessary to start the machine whenever the pressure drops. Automatic starters are also preferable for large motors, as they give automatic acceleration and limit the starting current to a minimum. An automatic starter for a squirrel-cage motor is shown in Fig. 353. Control may be secured by means of a push button or a contact-making pressure gage or float. In Fig. 354 is shown an automatic starter for a large-size slip-ring motor. In starting, full voltage is applied to the stator or primary and the rotor resistance is cut out in steps by means of the electrically operated contactors shown on the panel (Fig. 354 *a*). This type of control is used for main roll drives in steel mills. The rotor resistance is not used for controlling the speed and is all cut out during the regular operation of the motor.

365. Starting Synchronous Motors.—It was shown in Chapter XXXII, that in practice, synchronous motors are usually made self-starting by means of a squirrel-cage winding in the field poles. This type of motor is, therefore, started like a squirrel-cage induction motor, using an auto-starter to secure a reduced voltage for starting. If the synchronous motor is started by a separate motor, it must be synchronized like an alternator.

PROBLEMS ON CHAPTER XXXVIII

1–38. A 3-hp. three-phase, 220-volt, squirrel-cage induction motor is started by applying full voltage by means of a switch starter (Fig. 349).

(*a*) What size of running fuse should be used?

(*b*) What size of starting fuse is required?

2–38. Determine sizes of fuses as in Prob. 1–38 for a two-phase motor.

3–38. Referring to the motor of Prob. 1–38, determine the size of starting fuse if the motor were started by an auto-starter, set on the 80-per cent tap.

4–38. A 25-hp., 220-volt, three-phase, four-pole, squirrel-cage motor has a starting torque of 1.5 times full-load running torque with full voltage applied to the motor. The starting current at full voltage is 5 times full-load current.

(*a*) Determine the voltage and starting current to give full-load torque.

(*b*) What is the current taken from the line if the voltage across the motor terminals is secured by resistances in series with the motor?

(*c*) What would be the line current if an auto-starter were used to produce the required voltage?

(*d*) Assuming that the motor has a power factor of 0.40 when starting, calculate the power taken from the line when starting by the methods specified in (*b*) and (*c*).

5–38. A 50-hp., 440-volt, eight-pole, three-phase, squirrel-cage motor gives 1.25 times full-load torque when starting with full voltage applied and requires a starting current 6 times the full-load current. An auto-starter is used which has 65- and 85-per cent taps.

(*a*) Calculate the starting torques available when using these taps.

(*b*) What are the corresponding starting currents?

6–38. A 15-hp., 10-pole, 220-volt, squirrel-cage induction motor is to be started by the star-delta method. The starting torque at full voltage is 1.20 times full-load running torque and the starting current 7 times full-load current.

(*a*) What would be the starting torque?

(*b*) What would be the starting current?

7–38. A 75-hp., 440-volt, three-phase, 60-cycle slip-ring induction motor has a full load speed of 1160 r.p.m. Sufficient rotor resistance is inserted so that the full-load slip is made 5 times as great.

(*a*) What is the full-load speed?

(*b*) Assuming the full-load torque in both cases, what would be the efficiency with and without the additional rotor resistance?

(*c*) What is the speed regulation in each case?

CHAPTER XXXIX

MOTOR APPLICATIONS, A.C. AND D.C.

366. Comparison of Motors.—The relative merits of direct and alternating-current power supply have been discussed in Article 190. For power and lighting service for industrial installations, the supply, whether obtained from a power company or generated by an isolated plant, is usually alternating-current. For that reason a.c. motors would be used exclusively unless special requirements for some machines justified the conversion of a portion of the alternating supply to direct-current. A comparison of the relative merits of d.c. and a.c. motors will help explain this.

Classification of Motors with Reference to Performance

Load Requirements	Suitable Type of Motor	
	A.C.	D.C.
(1) Approximately constant speed, no load to full load.	Induction or synchronous motor.	Shunt motor.
(2) Semi-constant speed, no load to full load.	Induction motor with high rotor resistance.	Compound motor.
(3) Speed adjustable, but remaining constant, no load to full load, for a given adjustment.	Induction motor, Sherbius, brush-shifting or pole-changing types.	Shunt motor with field control.
(4) Speed adjustable, and semi-constant, no load to full load for a given adjustment.	Induction motor with automatically variable rotor resistance.	Compound motor with field control.
(5) Varying speed, decreasing greatly with increase of load.	Induction motor with adjustable rotor resistance.	Series motor.

It will be seen from this comparison that there is a type of direct-current motor suitable for all the various industrial requirements; whereas, with a.c., it is difficult to obtain the same speed characteristics in all cases. In spite of this disadvantage, however, an a.c. system is usually employed because of the advantages in transmission. (See Article 190). Direct current is more commonly used for electric traction purposes, although there are a number of notable examples of the use of alternating current; for example, the New York, New Haven and Hartford R.R. electrification between New York and New Haven, and the Norfolk and Western R.R. in West Virginia. In selecting a motor to drive a particular machine, both the starting and running requirements of the machine must be known and a motor selected to meet these requirements as nearly as possible.

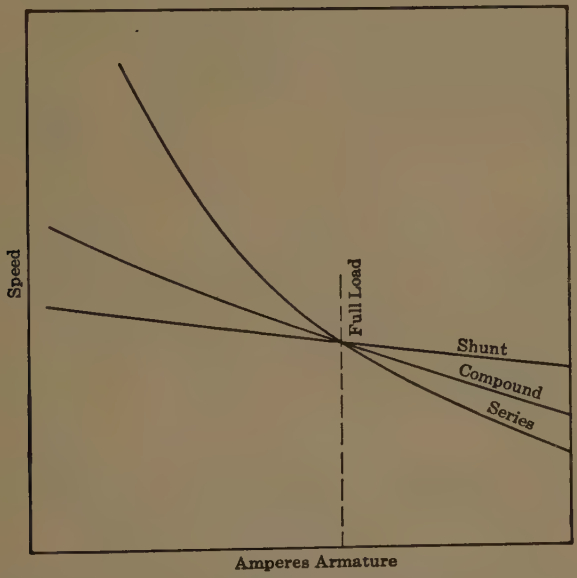


FIG. 355.—Comparative Running Performance of D.C. Motors.

367. Performance of D.C. Motors.—A comparison of the performance of d.c. motors is given by the curves of Figs. 107 and 355 and the tabulation on page 516. The curves are for three motors having the same rated output at the same speed; hence, the torque with full-load current is the same for all.

Considering the starting performance of these motors, it will be seen that the series motor is the best and the shunt motor the poorest, with the compound motor intermediate. Therefore, if a heavy start-

PERFORMANCE OF D.C. MOTORS

Type	Starting Torque	Running Performance	Applications
Series.	Very large for heavy currents. Small for currents less than half full load.	Speed varies widely with load. Motor will reach a dangerous speed if load is all thrown off.	Electric railways, cranes and small hoists, small air compressors, small elevators, propeller fans.
Shunt.	Less than compound motor for large current. More than compound for small values of current.	Speed practically constant. About 3 to 5 per cent drop from no load to full load.	Wood-working machinery, screw machines, lathes, shapers, drills, blowers, centrifugal pumps, line shafts, pressure blowers, centrifugal fans, printing presses, conveyors.
Compound (cumulative).	Less than series for large currents. Less than shunt, but more than series for small currents.	Speed falls off rapidly with increase in load, the amount depending on strength of series winding.	Punch presses, large shears, drop hammers, planers, large printing presses, passenger elevators, bending rolls.
Compound (differential).	Very small unless series winding is cut out when starting.	Speed may be held more nearly constant than with shunt motor. Is likely to be unstable at heavy overloads.	Special, constant-speed applications for small power.

ing duty is imposed, either a series or a compound motor would be required. It should not be thought from this, however, that the shunt motor will not start a fairly heavy load. In fact, a shunt motor would be satisfactory if the starting torque does not exceed 1.5 times the full load torque. Although heavier loads might be started by a shunt motor, there would probably be trouble from sparking. As far as running requirements are concerned, it will be seen from Fig. 355 that the shunt motor has the closest speed regulation with the compound motor next.

368. Performance of A.C. Motors.—Loads requiring a moderate starting torque, not exceeding full-load torque, can be handled satisfactorily by squirrel-cage induction motors, particularly if the motor is started infrequently. Where a greater starting torque is required, the slip-ring type of motor would in general be used, although small squirrel-cage motors may be obtained with high-resistance rotors, which give higher starting torque than the usual low-resistance rotor, but at the expense of lower efficiency and poorer speed regulation. This type of motor is used for cranes and elevators. Loads requiring a large starting torque, particularly if the motor is of large size, can be handled best by slip-ring induction motors which can be designed to produce the maximum torque at starting, that is 1.5 to 2.5 times full-load torque. The smaller figure applies to motors with a large number of poles, and the higher to those with a small number of poles. Self-starting synchronous motors will not give so large a starting torque as slip-ring induction motors, but they are designed to give a starting torque comparable with the torque of a squirrel-cage induction motor. Both synchronous and induction motors are essentially constant speed machines; therefore, it is not so easy to meet adjustable- or varying-speed requirements with a.c. motors as with d.c. motors. The high-torque induction motor gives a speed characteristic similar to the compound motor, but at poorer efficiency because of the extra rotor resistance. Adjustable speed is secured with induction motors using the special regulating set (Sheribus method, Article 295), or a special type of induction motor with a commutator and adjustable brushes (Article 296), but these machines are expensive and the former is best adapted for large-size units.

Induction motors will withstand more severe operating conditions than d.c. motors because of the absence of a commutator and they rarely need to be entirely enclosed to protect them from dust. For the same reason, induction motors do not need so much protection against moisture, and there is not the same danger of setting fire to inflammable dust as there is where a commutator and brushes are used. Comparing induction and synchronous motors, the former is simpler and does not require as much skill in starting. The induction motor gives a fixed, lagging power-factor; whereas, the power-factor of the synchronous motor is variable and can be made nearly unity in practice. This is a decided advantage in many cases where power is purchased. Synchronous motors will not recover speed again if they fall out of step due to low voltage or excessive load, while the induction motor will recover when the voltage is restored or the overload is removed. Synchronous motors are specially adapted for air compressors where a low speed is required as this is not favorable to efficient design of an induction motor because of the large number of poles required. A comparison of the performance of a.c. motors is given in the tabulation below:

PERFORMANCE OF A.C. MOTORS

Type	Starting Torque	Running Performance	Applications
(1) Two or three-phase. Squirrel cage (low slip).	Relatively low. Starting current high.	Speed nearly constant. Speed regulation 1.5 to 5 per cent depending on size of motor.	Small blowers, cement and steel mills, screw machines, lathes, drills, pumps, conveyors, wood-working machinery.
(2) Squirrel cage (high slip).	Higher than for (1). Starting current less.	Speed decreases rapidly with load, somewhat like compound, d.c. motor.	Small cranes and elevators, punches and shears, large band saws.
(3) Slip-ring.	Higher than (1) and (2). Starting current small.	With starting resistance out, acts like (1). With resistance in circuit, speed can be adjusted to any desired value at a sacrifice in efficiency.	Elevators, cranes, air compressors, ventilating fans, steel mills, hoists, woodworking machinery.
(4) Single-phase (split-phase).	Low starting torque. Large starting current.	Similar to (1).	Used only in small sizes. Employed for constant speed applications only, such as small printing presses, sewing machines, etc.
(5) Single-phase (repulsion).	Fairly high starting torque, but not so good as polyphase motors.	Similar to (1).	For constant speed applications. Used principally in sizes below 15 hp. where only single-phase service is available.
(6) Synchronous.	Low starting torque, large current.	Speed constant and cannot be adjusted.	Large air compressors, line shafts, pumps.

369. Desirable Speeds for Motors.—In general, the motor speed should be as high as conditions will permit. A high-speed motor costs and weighs less than a slow-speed motor and occupies less space. This is shown by the following tabulation:

EFFECT OF SPEED UPON COST AND WEIGHT OF A MOTOR

Three-phase, 60-cycle, 220-volt, Squirrel-cage Motor

5 Horsepower			10 Horsepower			50 Horsepower		
Speed r.p.m.	Cost Dollars	Weight Pounds	Speed r.p.m.	Cost Dollars	Weight Pounds	Speed r.p.m.	Cost Dollars	Weight Pounds
1750	92	125	1750	130	225	1750	347	820
1160	113	190	1160	161	315	1160	410	1150
870	146	220	870	196	345	870	479	1230
680	183	340	690	255	405	690	587	1910
575	200	415	575	280	660	575	650	1950

Speed is for full load. Cost is for bare motor without starter.

On the other hand, a high-speed motor may be more noisy. With a.c. motors only a limited number of speeds are available, the highest being approximately 1800 r.p.m. for 60 cycles and 1500 r.p.m. for 25 cycles. In any case, a standard speed should be selected if possible. Unless the motor is directly connected to the machine, there is some freedom of choice. Where belts are used, a pulley ratio of more than 6 to 1 is undesirable. A ratio of machine and motor speeds greater than this would require counter shafts or idler pulleys to increase the arc of contact on the motor pulley. These should be avoided where possible.

370. Relation between Load and Motor Rating.—Both the average and maximum conditions of load must be considered when selecting a motor. Sometimes the maximum load requirements occur at starting; in other cases, they may represent an occasional overload of short duration. It is obvious that the motor should be as small as will properly meet the requirements, in order to reduce the first cost to a minimum. This should not, however, lead one to underestimate the load requirements. A motor which is too small would be subjected to frequent overloads, would operate at an excessive temperature and in the case of a d.c. machine there would probably be difficulty in keeping the commutator in good condition. As a result, the cost of maintenance and repairs would be excessive. If the motor is larger than necessary, besides costing more, the operating efficiency would be lower and in the case of induction motors the power-factor would be poor. Motors are so designed that they have high efficiency between one-half and full load. Below half load the efficiency falls off rapidly. As a general rule, the size should be so chosen that the motor will operate between three-quarters and full load most of the time. Heavy loads of comparatively short duration can be taken care of by the overload capacity of the motor. Manufacturers' standard sizes and speeds should be chosen, because they cost less and repair parts can be more easily secured.

371. Open and Enclosed Motors.—Since the output of a motor is usually limited by the heating, it is apparent that the various parts should be as freely ventilated as possible. In many cases, however, it is necessary to enclose the windings to protect them against mechanical injury, excessive dust or, in some cases, to prevent communicating fire to inflammable materials. Motors for industrial purposes are therefore classified according to the amount of such protection that is provided. While there are a number of different classes, they may all be grouped as either open, semi-enclosed or enclosed motors. In the **open motor** the windings and rotating parts are freely exposed to a circulation of air, which serves to cool the windings. This type gives the best ventilation possible, and therefore for a given load will be the cheapest type of motor to use. The open type would therefore always be selected where the conditions permit, that is, where dust or moisture are not excessive. Where there is considerable dust or dirt, motor bearings are sometimes made dust proof by the addition of felt rings at each end. **The semi-enclosed motor** is similar to the open type except that the openings at each end of the frame are covered by gratings or screens, which allow a fairly free ventilation, but protect the motor against damage from pieces of wood, metal, or other substances which might be dropped into it. This arrangement does not protect the motor against fine dust which could pass through the openings. A semi-enclosed motor will run slightly hotter than an open motor, since the covers shut off some of the air circulation. **The enclosed motor** has solid covers closing all openings to the inside of the machine. Such a motor is practically dust and moisture proof. These motors are used for very severe operating conditions, where there is a large amount of dust or where considerable moisture is present. When d.c. motors are installed where inflammable dust exists, the enclosed type is necessary. When a motor is enclosed in this way there is no free circulation of air over the various parts of the windings and the entire cooling of the motor must be accomplished by cooling the outside frame of the machine. A given size of motor would therefore run much hotter if enclosed than if open.

Stated another way, to carry a given load the motor must be much larger if enclosed. To partly offset this difference, enclosed motors are allowed to run somewhat hotter than open motors. An enclosed motor is from 25 to 40 per cent heavier than an open motor and is therefore correspondingly more expensive. In some cases, particularly with large enclosed motors, air is blown into the motor by means of a blower to assist in the cooling.

372. Effect of Low Efficiency.—If the motors operate at low efficiency, the cost of power may be considerably increased. The low efficiency may result either because the motors are not fully loaded or because they are poorly designed. It is not always true, however, that the most efficient motor is the best to use. A low-efficiency motor would cost less and take up less space and the saving in cost may sometimes be invested in the business where it will give a greater return. The saving in power by the use of a more efficient motor depends upon the length of time the motor is used each day and upon the cost of power. The high-efficiency motor shows the greatest saving when the cost of power is high and the period of operation long.

373. Determining Size of Motor Required.—Information regarding the amount of power required to drive a machine can be obtained from the manufacturer, or, by testing with a temporary motor, or in some cases by calculation. Wherever possible, data based on tests should be used. The current required for various motors at full load is given in Tables 4, 5 and 6 in the Appendix. Methods of determining the load required to drive a machine are given in Article 150.

374. Individual and Group Drives.—When machines are driven electrically, either group or individual drive may be used. **Group drive** is a modified form of mechanical drive. The main shafting and belts are eliminated and machines of a similar kind, arranged in groups, are belted to short lengths of comparatively light shafting. Each group is then driven by a motor. By this means the friction losses of the shafting are greatly reduced. With **individual drive** each machine is fitted with one or more motors, thus making it an independent unit. The **advantages** of the individual drive are: (*a*) Increased production, because slipping of belts is eliminated, and speed may always be adjusted to the best value for the work. There is also a greater overload capacity. Since the labor cost of production is a very large item (often 50 per cent or more of the total), this advantage is very important. Experience with the electric drive has shown increases of from 10 to 20 per cent in the output of a plant. (*b*) The location of the tool is independent of the line shaft and an arrangement can therefore be used which will facilitate rapid production with a proper routing of the work through the shop. (*c*) New tools can be added and tools easily rearranged. (*d*) Line shafts, with their troublesome belts, are eliminated. This means greater safety to workmen, a cleaner and better lighted workroom, headroom for overhead cranes, and in some cases a considerably cheaper building, because less headroom and a lighter construction can be used. (*e*) The power losses in transmission are less than with shafting. The total loss, including generator, feeders and motors, is from 20 to 30 per cent. The first cost of individual drive is, however, greater (about 20 per cent) than group drive. The group drive is particularly adapted for light machine work, where the individual machines are small and little or no speed adjustment is required.

REQUIREMENTS OF MACHINES

375. Machine Tools.—Group drive is generally preferable for bench and speed lathes and moderate size engine lathes. Heavy lathes are usually driven from individual motors. Other machines which are generally group-driven include automatic screw machines, sensitive drills, vertical and radial drilling machines, boring machines, grinders, shapers, slotters, and milling machines. Individual drive would be used only for very large sizes of these machines or where they are isolated from the other tools. Large planers are best driven by individual motors. Where individual drive is used for lathes, drills, etc., the shunt-wound motor is best, because wide speed adjustment can be secured by a field rheostat. If induction motors are used they would be of the squirrel-cage type and the speed adjustment would be made by cone pulleys or by a set of gears contained in a "gear box." For driving large planers, a motor which reverses with each stroke of the table is now being used extensively. The motor is compound wound and specially designed for this service. The controller is arranged to permit independent adjustment of the speed of the cutting and return strokes over a wide range. Motors for machine tools are usually rated on a two-hour basis, since there are frequent periods of shut-down or light-load operation. The motor would in general be selected to take care of the average load, since the overload capacity of the motor is sufficient for the maximum load unless it is of long duration. In every case, care must be taken that the motor selected is large enough properly to start the machine under the most severe conditions. The sizes of motors on machines used by piece workers would in general have to be larger than on those where day labor is used.

376. Wood-working Machinery.—For most wood-working machinery, the starting load is light and the machine must operate at a constant speed. The ordinary types of induction motors are therefore well adapted to this service and are generally used. They are more satisfactory than d.c. motors because of the absence of a commutator and their large overload capacity. For most machines, standard squirrel-cage motors are satisfactory. Large circular saws and band saws for sawing logs, large planers and matchers, require a large starting torque and are subject to heavy fluctuations in load. For this service slip-ring motors with resistance placed permanently in the circuit, or squirrel-cage motors with a large slip, are used.

377. Shears and Punch Presses.—The starting torque for these machines is light and the load small except during the cutting stroke. If the motor alone were depended upon to supply the peak load, an excessively large motor would be required. Accordingly, a heavy flywheel is used in order that it may assist the motor during the working part of the stroke. To do this, it is necessary that the motor shall slow down so that the flywheel may give up part of its energy. A shunt-wound or squirrel-cage motor with low resistance rotor would not be suitable, as they would not decrease in speed enough and would, therefore, be overloaded. A series motor would tend to overspeed during the non-cutting periods. Therefore, a compound motor or an induction motor, with high-resistance rotor would be used, since these have considerable speed variation, which allows the flywheel to act. The motor will accelerate the flywheel back to normal speed again during periods between the cutting strokes.

378. Pumps.—The service which a motor has to perform depends first upon the kind of pump used. **Reciprocating or plunger pumps** require a heavy starting torque unless the pressure on the pump is relieved with a by-pass. Only small pumps are, accordingly, started under pressure. Usually a squirrel-cage induction motor or a shunt motor is employed. Where it is desired to change the quantity of liquid discharged by the pump, the speed is varied, by means of a field rheostat, in the case of d.c. motors, and by the use of a slip-ring motor where alternating current is used. **Centrifugal pumps** have to be treated somewhat differently because of their peculiar action under different heads. With a reciprocating pump operating at constant speed, an increase in the resistance against which the pump is working increases the pressure, since the volume discharged keeps nearly the same. Hence the load on the motor increases. With a centrifugal pump, on the other hand, an increase in the resistance reduces the load. If the head is reduced on a reciprocating pump, the power required is less; with a centrifugal pump, however, a reduction of head increases the volume of water delivered and the load increases. Some centrifugal pumps are so designed that the increase in load at the minimum head is only about 25 per cent, and this can be taken care of by the overload capacity of the motor. In others, however, the load is considerably greater than this, so this point must be carefully investigated when selecting a motor. If the speed of a centrifugal pump is reduced below normal, the quantity of water discharged is reduced. Even the slightly lower speed of a shunt motor when first started (with windings cold) is sufficient to affect the output considerably. For this reason a field rheostat frequently is provided so that the speed can be maintained normal at all times. The starting characteristics of centrifugal pumps should also be considered. In starting, the discharge valve is usually entirely closed to make the starting load as light as possible. Under these conditions, the torque is from 15 to 25 per cent of full-load torque and drops slightly as soon as the pump turns over. The pump casing is filled with water, which is churned round when the motor is up to speed, and this produces about half load on the pump even with the valve closed. Shunt motors, or either squirrel-cage or slip-ring induction motors, are suitable for driving pumps. Reciprocating pumps are generally geared or belted to the motor, while centrifugal pumps, because of their high speeds, can generally be direct-connected to advantage.

379. Blowers and Fans.—Centrifugal fans have the inlet at the center and the discharge at an opening near the outside circumference. Propeller or disk fans move the air in a direction parallel to the shaft, similar to the propeller of a ship. Blowers include positive-pressure rotary blowers which create a pressure by direct compression. (For example, the Root blower.) The horsepower of any fan or blower increases greatly with the speed.[1] The horsepower of a centrifugal fan at constant speed, in general, decreases as the area of the discharge opening is decreased, and with the opening entirely closed is about 20 per cent of the power required with full opening. The horsepower for a propeller fan increases as the area of the discharge opening is decreased, being about double when the opening is entirely closed. With a positive-pressure blower, the horsepower increases very rapidly as the area of the discharge opening is decreased. With centrifugal fans, therefore, the discharge is closed to reduce the load, while with the other two types the discharge is left wide open. For centrifugal fans and pressure blowers, shunt motors are used for d.c.

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[1] Proportional to the cube of the speed.

systems. For propeller fans, either shunt or series motors can be used, the latter being the better. With a.c. systems, standard squirrel-cage motors are generally used. Fans and blowers frequently must be capable of speed adjustment. With d.c. motors, this is accomplished by field or armature rheostats or both, and with induction motors, by means of resistance in the rotor circuit. Motors may be direct-connected to centrifugal fans and to small propeller fans. For blowers and large-sized fans, however, the speeds are so low that a belt or chain drive is preferable.

380. Compressors.—The load on air compressors is generally intermittent. In some cases the machine is started and stopped as the demand for air changes, but generally the compressor is run continuously and an unloading valve is used to reduce the output of the compressor as required. When the compressor is started and stopped, the unloading valve is used to reduce the starting load. The starting conditions are therefore not severe in either case. Flywheels are used to equalize the load. Small compressors which are started and stopped frequently are driven by d.c. series motors or induction motors with a high resistance squirrel-cage rotor. Large reciprocating compressors which run at a low speed are best driven by synchronous motors rather than induction motors because of the large number of poles required.

381. Elevators require a nearly constant running speed. The starting torque is large, because they must be accelerated rapidly. A series motor would give the required starting torque, but it would run at excessive speed when the car was lightly loaded. A compound motor is therefore used in order to secure a high starting torque. After the car has started and reached normal speed, which is 500 to 600 feet per minute for express passenger elevators, the series winding is cut out, and the motor runs as a shunt machine, thus giving a practically constant speed. One of the great problems in high-speed elevator work is to make accurate landings. This is very satisfactorily accomplished with d.c. motors by making them act as generators feeding into a resistance. With a.c. motors the control is not as satisfactory, because a friction brake must be depended upon to bring the car to rest. Slip-ring induction motors are generally used with a.c. systems. For high-speed service a two-speed slip-ring motor is used. Two separate windings are provided, one high-speed for starting and running and the other low-speed for stopping. By this means, the motor can be made to act as a generator until the elevator has reached one-third or one-fifth normal speed, after which a friction brake must be used to bring the car to rest.

382. Hoists require a large starting torque. The running speed must be varied to suit the amount of load. For small hoists, d.c. series motors are generally used, although sometimes compound motors are employed. For a.c. service, a high torque, squirrel-cage motor would be used. Larger hoists would have slip-ring induction motors. For very large hoists, such as would be used in mines, slip-ring induction motors with variable rotor resistance are extensively used. In some installations, the Ward-Leonard system is used (see Article 163) with a synchronous or induction motor driving the motor-generator set. If it is necessary to reduce the load fluctuations caused by the hoist, an Ilgner-Ward Leonard system (see Article 163) is used. A slip-ring induction motor is used to drive the motor-generator set used in this installation. An adjustable water rheostat like that shown in Fig. 15, is connected in the rotor circuit of this motor. When a heavy load is thrown on the motor-generator set, it tends to take more current from the line. This causes more resistance to be inserted in the rotor circuit, the motor slows down and allows the flywheel to deliver power to assist in driving the motor-generator set. In mine hoists driven by induction motors, it is common practice to lower the load by connecting the motor to the line and allowing it to act as an induction generator (see Article 299), which it will do if allowed to run slightly above synchronous speed. The lowering speed can be increased by increasing the rotor resistance. Another method is to excite the stator of the hoist motor with direct current and consume the power generated in the rotor by a rotor resistance.

383. Cranes.—The starting torque of cranes is high and the running speed may be allowed to vary with the load; therefore, d.c. series motors are well adapted for this service and are extensively used. With a.c. systems, the slip-ring type of motor would be used except for small cranes where squirrel-cage motors with a high resistance rotor would be used. Direct current is so much superior to alternating current for crane service that frequently motor-generator sets or converters are installed where only an alternating current supply is available.

384. Steel Mills.—Motors used in steel mills must have high overload capacity, great mechanical strength, and frequently must operate under very severe conditions as regards dust and dirt. They are, therefore, usually of the totally enclosed type. A.c. motors are used almost exclusively in steel mills except for the cranes, which are d.c. driven and for reversing mills which are driven by large d.c. motors supplied by a flywheel motor-generator set operated on the Ward-Leonard system (Art. 163). For driving rolling mills that do not have to be reversed, very large induction motors are used. Where adjustable speed is required, the Sherbius (see Article 295) or Kraemer systems are used for large installations and the brush shifting motor (Article 296) where smaller amounts of power are required.

385. Cement Mills.—Only induction motors are suitable for cement mills because of the dust. The bearings should be made dust proof. Slip-ring motors are used on ball and tube mills and on crushers which require a large starting torque. They are also used on kilns and dryers when speed adjustment is required. For the other machines, standard squirrel-cage motors are satisfactory. The dust causes rapid deterioration of belts, and therefore direct connection is preferable wherever possible. A flexible coupling should be used to reduce the shocks on the motor.

386. Tanneries.—Either the shunt-wound, d.c. motor or the squirrel-cage induction motor has suitable operating characteristics for driving machinery in tanneries. In some parts of the process, however, the operating conditions are very severe owing to the presence of moisture and acid fumes. For this reason induction motors are best adapted for the service and are generally used. The windings are specially treated to withstand the acid fumes. Switches and fuses are enclosed in moisture-proof boxes. Group driving is commonly employed, although individual drive is favored for some of the machines. Group drive is best adapted for staking, rolling and glazing machines, for the tanning and washing drums and for shaving and shanking machines. Individual drive is best for belt-knife splitters, fleshing and unhairing machines, color drums, exhaust fans, pumps, etc.

387. Textile Mills.—As a rule, the loads are steady and extremely close speed regulation is required. The starting loads are not severe. There is always more or less lint in the air and therefore the motors have to be specially protected. Squirrel-cage induction motors are generally used. These motors are especially designed to give very close speed regulation and a high efficiency and power-factor under working conditions. Because of the lint, all air ducts in the motors are eliminated and the bearings are made dust tight. Enclosed motors are not used generally, but the rotor is made without any projections to catch the lint. There is a tendency at present to use individual motor drive, particularly for looms and spinning frames. Overhead shafting and belts are objectionable because they are likely to cause damage to the goods from dirt, oil, etc. It is also difficult to maintain constant speed due to slipping of belts.

388. Electric Traction.—For this service, the power required to start and accelerate the car is much greater than is required to keep the car moving at a constant speed. Furthermore, it is desirable that the car shall run at a lower speed up grade than on a level in order to avoid excessive demands on the power house. For these reasons, a series motor is best suited to traction service, since it gives high starting torque and slows down as the car goes up grade; whereas, the shunt motor tends to run at constant speed either on a level or upgrade. This would require an excessively large motor and would produce unnecessarily heavy loads on the power house. Both d.c. and a.c. series motors are used although the latter are more expensive and heavier than d.c. motors.

D.c. systems are now in operation at voltages as high as 3000 volts, but the common voltage for city and suburban service is about 600 volts. The single-phase a.c. system, using series motors on the locomotives or cars has an advantage that much higher voltages up to 11,000 volts may be used on the trolley, and this results in a saving in copper where an extensive railroad system is to be electrified. This trolley system is supplied by static transformers, requiring only occasional inspection and no protective building, as they may be installed outdoors beside the tracks. The d.c. system on the other hand requires converting machinery which must be housed and must have more attention than transformers used in the a.c. system. At present, the use of automatically controlled d.c. substations which do not require the continual attendance of an operator has resulted in decreasing the cost of operation of d.c. railway systems. The a.c. system has the disadvantage that the alternating current may cause serious disturbances in overhead telephone or telegraph lines adjoining the trolley system.[2]

389. Costs of Electrical Machinery.—An approximate idea of the cost of electrical machinery is given below. The cost depends to a large extent upon the speed (see Article 369) and various details of design, so that these figures can be used only for a rough approximation of cost. The costs given are based on net retail prices. Jobbers and large users of apparatus would pay lower prices. Costs for speeds different from those given may be determined by interpolation.

DIRECT CURRENT MOTORS

1750 r.p.m. up to 100 hp., $170+ $7.30 per horsepower.
1150 r.p.m. up to 50 hp., $120+$11.60 per horsepower.
1150 r.p.m. 50 to 200 hp., $360+ $7.00 per horsepower.
850 r.p.m. up to 50 hp., $240+$13.20 per horsepower.
850 r.p.m. 50 to 200 hp., $530+ $7.25 per horsepower.
575 r.p.m. up to 50 hp., $280+$14.40 per horsepower.
575 r.p.m. 50 to 200 hp., $470+$10.70 per horsepower.

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[2] For an extended treatment of the subject of motors and their applications see "Principles of Electric Motors and Control," by Gordon Fox.

These prices are for the motor only and are for either 115 or 230 volts. For 550 volts, the prices are about 10 per cent greater for high speeds and 20 per cent for low speeds. Manually operated starters cost about 10 per cent of the cost of the motor; automatic starters cost from 20 to 30 per cent of motor cost, the higher figure applying to the smaller motors. Motors are rated on a 40-degree basis. (See Article 154.)

SQUIRREL-CAGE INDUCTION MOTORS

1750 r.p.m., up to 50 hp., $ 60+$5.00 per horsepower
1750 r.p.m., 50 to 200 hp., $120+$3.90 per horsepower
1160 r.p.m., up to 50 hp., $100+$5.40 per horsepower
1160 r.p.m., 50 to 200 hp., $140+$4.60 per horsepower
870 r.p.m., up to 50 hp., $120+$6.20 per horsepower
870 r.p.m., 50 to 200 hp., $180+$5.00 per horsepower
690 r.p.m., up to 50 hp., $160+$7.60 per horsepower
690 r.p.m., 50 to 200 hp., $280+$5.00 per horsepower

These prices are for motor only and are for either two- or three-phase, 60 cycles, and 220, 440, or 550 volts. Prices for 2200 volts, for 50 to 200 horsepower, are 10 to 30 per cent more for high speeds and 5 to 20 per cent more for slow speeds, the smaller percentage applying to the larger machines. Manually operated starters cost about 20 per cent of the motor price for sizes above 50 horsepower and 30 to 50 per cent for smaller motors; automatic starters for 50 horsepower and above cost from 30 to 50 per cent of motor price, the higher percentage applying to the smaller motors. Motors are rated on a 40-degree basis. (See Article 154.)

SLIP-RING INDUCTION MOTORS

1750 r.p.m., up to 50 hp., $130+$8.00 per horsepower
1750 r.p.m., 50 to 200 hp., $290+$4.75 per horsepower
1160 r.p.m., up to 50 hp., $200+$7.60 per horsepower
1160 r.p.m., 50 to 200 hp., $300+$5.50 per horsepower
870 r.p.m., up to 50 hp., $230+$8.60 per horsepower
870 r.p.m., 50 to 200 hp., $360+$5.85 per horsepower
690 r.p.m., up to 50 hp., $340+$8.80 per horsepower
690 r.p.m., 50 to 200 hp., $500+$5.55 per horsepower

These prices are for the motor only and are for either two- or three-phase, 60 cycles, and 220, 440 or 550 volts. Prices for 2200 volts for 50 to 200 horsepower are 5 to 20 per cent more for both high and low speeds, the smaller percentage applying to the larger machines. Manually operated, drum-type starters cost 25 to 45 per cent of cost of motor. Motors are rated on a 40-degree basis. (See Article 154.)

SYNCHRONOUS MOTORS—HIGH-SPEED

1200 r.p.m., 100 to 500 hp., $ 600+$3.30 per horsepower
900 r.p.m., 100 to 1000 hp., $ 600+$3.30 per horsepower
900 r.p.m., 1000 to 2000 hp., $2300+$1.65 per horsepower
600 r.p.m., 100 to 1000 hp., $1000+$3.65 per horsepower
600 r.p.m., 1000 to 2000 hp., $2600+$2.00 per horsepower
360 r.p.m., 100 to 1000 hp., $1200+$5.60 per horsepower

These prices are for the motor only and are for three-phase, 60 cycles, 220, 440, or 2200 volts, based on operation at unity power factor. The exciter costs about 10 per cent additional. Manually operated starters cost 30 to 40 per cent of motor cost. Motors are rated on a 50-degree basis. (See Article 154.)

SYNCHRONOUS MOTORS—SLOW-SPEED

300 r.p.m., 100 to 1000 hp., $1100+$2.70 per horsepower
180 r.p.m., 100 to 1000 hp., $1850+$2.70 per horsepower
138 r.p.m., 100 to 1000 hp., $2500+$2.70 per horsepower
120 r.p.m., 100 to 1000 hp., $3250+$2.70 per horsepower

These prices are for an engine-type motor, without bearings, designed for direct connection to load. The machines are three-phase, 60 cycles, 220, 440, or 2200 volts, and for operation at unity power factor. The exciter costs about 15 per cent additional. Bearings and bedplate cost from 25 to 35 per cent additional. Manually operated starters cost 15 to 20 per cent of motor cost for the low speeds and 25 to 35 per cent of motor cost for the high speeds. Motors are rated on a 50-degree basis. (See Article 154.)

PROBLEMS ON CHAPTER XXXIX

1–39. What type of a.c. motor is most commonly used for industrial applications?

2–39. What type of a.c. motor would be used where a nearly constant speed is required and the starting conditions are not severe?

3–39. What are some of the advantages and disadvantages of a.c. motors as compared with d.c. motors?

4–39. A shunt motor was purchased to drive a fan but gave trouble by sparking and overheating. An investigation showed that the fan was running at 1030 r.p.m. although its normal speed was 900 r.p.m. Why did the motor give trouble?

5–39. Compare the cost of small (up to 25 hp.) d.c. shunt motors, with squirrel-cage induction motors, including manually operated starter.

6–39. How does the cost of squirrel-cage motors compare with slip-ring motors, including manually operated starters:

(*a*) For small sizes (up to 25 hp.).
(*b*) For moderate sizes (25 to 100 hp.).

7–39. Compare the cost of slip-ring induction motors of about 200-hp. capacity with corresponding synchronous motors.

8–39. A certain railroad repair shop has loads in the various buildings as follows:

	Connected Horsepower
(1) Blacksmith Shop:	
Group drives (3 motors)	80
Individual drives, constant speed (14 motors)	43
Blowers (4 motors)	130
Heater fan	50
Tin shop, group drive (4 motors)	20
(2) Car Repair Shops:	
Car painting, group drive (2 motors)	30
Passenger-car repair group drive (1 motor)	7
Wheel shop, individual drive (5 motors)	60
Steel-truck shop, group drive (2 motors)	22.5
Freight-repair shop, group drive (3 motors)	50
Cranes	95
(3) Machine and Erecting Shops:	
Group drives (above 10 motors)	225
Individual drives, constant speed (above 10 motors)	158
Individual drives, adjustable speed (above 10 motors)	317
Cranes	336
(4) Boiler Shop:	
Group drives (5 motors)	75
Individual drives (5 motors)	40
Blowers and fans	75
Rolling mill, group drives (3 motors)	30
Flue rattler, group drive	50
Cranes	245

The power supply available is 11,000 volts, three-phase, 60 cycles. The lengths of feeders from site of sub-station to the different buildings are as follows:

Blacksmith shop	525 feet
Car-repair shop	1120 feet
Machine and erecting shop	850 feet
Boiler shop	750 feet

Assume an average efficiency of 85 per cent and a power factor of 0.85 for a.c. motors. Assume 85-per cent efficiency for d.c. motors. Allow a demand factor of 0.50 for cranes. Allow for 30-per cent increase in load as far as feeders are concerned.

(*a*) Should an alternating or a direct distribution system be used? Give reasons.

(*b*) If alternating current is selected, how will you take care of the adjustable speed tools?

(*c*) What motor voltage would you use?

(*d*) Determine size and number of transformers required to step down to motor voltage.

(*e*) If direct current is used, determine size and number of d.c. units.

(*f*) Determine the ampere load on each feeder.

(*g*) Determine size of each feeder.

(*h*) Determine drop on the feeders.

CHAPTER XL

LIGHT AND ILLUMINATION

390. Light Sources.—The principal sources of artificial light are the incandescent filament of carbon or tungsten, the electric arc between carbon or metallic electrodes and the mercury-vapor arc. Incandescent lamps are used generally for interior lighting and some street lighting. The use of arc lamps is at the present time confined generally to exterior lighting of streets, yards, and similar places. Mercury-vapor lamps are used only for factory lighting where their peculiar color is not objectionable and for photographic purposes.

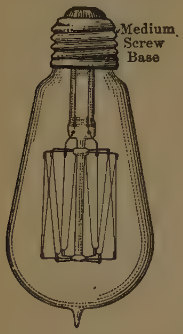

Fig. 356.—Vacuum-type Tungsten Lamp. 40-watt, Mazda B, ¼ scale.

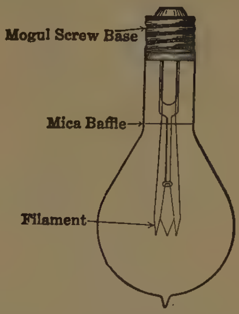

Fig. 357.—Gas-filled Type of Tungsten Lamp. 750-watt, Mazda C, ⅑ scale.

391. Incandescent Lamps are at the present time almost exclusively of the tungsten type. The carbon-filament lamp is used only when there is excessive vibration which the tungsten lamp could not withstand. Tungsten lamps are of two types. The vacuum-type, known generally as the Mazda B lamp, has a filament of small diameter tungsten wire arranged in several hairpin-shaped loops which are supported from a glass stem (Fig. 356). This lamp has a high vacuum. The gas-filled type, known as the Mazda C lamp, has a filament consisting of tungsten wire closely wound in the form of a very small helix or single-layer coil (Fig. 357), so that the filament is concentrated in a small space. The bulbs of Mazda C lamps are filled with an inert gas, such as nitrogen, which is at about atmospheric pressure when the filament is lighted. By the use of gas in the bulb, instead of a vacuum, the tendency of the filament to evaporate is reduced so that the filaments of gas-filled lamps may be operated at a higher temperature than those of the vacuum-type lamps without danger of excessive blackening of the glass. The higher temperature, thus made possible, produces a whiter light at increased efficiency. The filament of a vacuum-type lamp operates at a temperature of about 2100° C. and produces about 9 lumens[1] per watt, the gas-filled filament has a temperature of about 2400° C. and gives from 12 to 20 lumens per watt, depending on the size. For the same size, the gas-filled lamp is about 30 per cent more efficient. Vacuum-type lamps are used only in sizes up to 60 watts; gas-filled lamps are employed in sizes from 75 to 1000 watts.

Incandescent lamps are frequently used with frosted bulbs to diffuse the light. This is particularly necessary with the gas-filled lamps because of the brightness and concentrated arrangement of the filament. The so-called **daylight lamps** are gas-filled lamps with blue-glass bulbs, designed to produce a light approximating very close to that of daylight. They are used for color matching and for similar purposes where correct color values are essential.

Over 95 per cent of the energy supplied to a tungsten lamp is given off in the form of heat. With the gas-filled lamp, the heat is carried to the upper part of the bulb by convection currents in the gas so that when burning tip down as is usual, the base and socket become very hot, particularly in the case of the larger sizes. Special precautions, therefore, must be employed with these lamps to prevent overheating of the fixture, and non-inflammable fixture wire must be used.

392. Arc Lamps.—Modern arc lamps are of two types, the flame-arc and the metallic-electrode arc. The ordinary carbon arc is now seldom used for interior illumination but is employed for stereopticons and moving-picture machines. The flame-arc employs carbon electrodes impregnated with chemicals which make the arc luminous. This lamp is sometimes used for illuminating large rooms in industrial plants but the gas-filled tungsten is tending to displace this type of arc lamp. It is also used for street lighting. The metallic-electrode arc uses, as the positive electrode, a composite block of copper and iron and, as the negative electrode, a mixture of powdered iron oxide or magnetite and several light-producing oxides. This lamp is used extensively on series or constant-current systems for street lighting.

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[1] See Article 401 for explanation of this term.

A multiple type is used for illumination of parks, railroad yards, and similar applications. They can be operated on d.c. systems only.

393. Mercury-vapor Lamps employ, as a light-producing element, an arc of mercury at low pressure confined in a glass tube about 50 inches long and 1 inch in diameter. The tube is inclined at a small angle and has an enlarged space at the lower end which contains a pool of mercury. This is the negative electrode of the lamp. The positive electrode which is made of iron or graphite is at the opposite end of the tube. In operation, the entire length of the tube is filled with mercury vapor which is rendered luminous by the passage of the current. A diagram of connections for the a.c. lamp is shown in Fig. 358. The mercury vapor will conduct current in one direction only. (See Article 343.) Hence, where alternating current is used, two positive terminals are provided, as shown in Fig. 358. In the d.c. lamp, only one positive terminal is required with a ballast resistance in series instead of the auto-transformer.

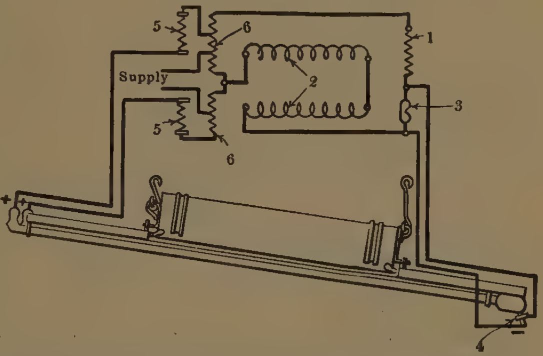


Fig. 358.—Diagram of Alternating Current Mercury-vapor Lamp. 1. Starting resistance. 2. Inductance coil. 3. Shifter. 4. Starting band. 5. Series resistance (ballast) to steady arc. 6. Auto-transformer. *The Cooper Hewitt Elec. Co.*

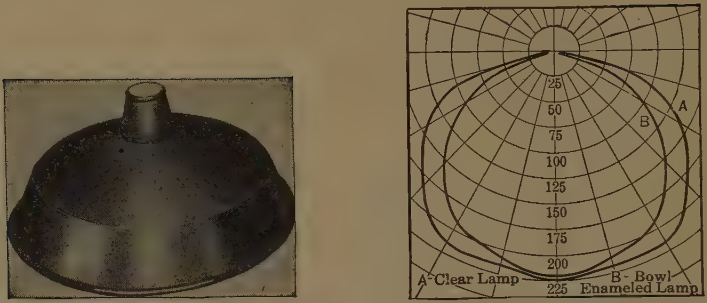
The light produced has a greenish-blue color owing to a deficiency of red rays; therefore, objects illuminated by this lamp do not appear in their true colors. For this reason, the lamps are not suitable for general illumination but they do find extensive use in industrial lighting, particularly where the work requires close attention to details, such as inspecting, automobile body finishing, and drafting. The light produced by the mercury-vapor arc is more active photographically than other artificial illuminants and is, therefore, particularly well suited to photographic work.

394. Reflection and Color.—A portion of the light which strikes an opaque object is always absorbed, the amount depending upon the character of the object. Substances of different kinds do not reflect light in the same way, and from this results differences in appearance and in color. A **black** body reflects no light, and hence it can be seen only by contrast with nearby objects which reflect light. A body illuminated by white light will appear white provided it reflects the primary colors of which white light is composed, in such an amount as to maintain the proper proportion of each. On the other hand, if it reflects more light of one color than another, the object will not appear white even when illuminated by white light. A body which appears red reflects the red light and absorbs the other colors. It should be noted, however, that if an object appears of a certain color, the light which is used to illuminate the object must also contain that color. For this reason, the mercury-vapor lamp, which produces no red rays, gives very peculiar effects. Objects which would appear red, when illuminated by white light, appear black under the mercury-vapor lamp since no light rays are reflected. Any color which is composed partly of red would not appear in its true color, and objects normally white have a greenish-blue color when illuminated by the mercury-vapor lamp. The color of the light in a room is not always the same as the color of the source. If white light is produced in a room with green walls, most of the light which strikes the walls is absorbed, while the green rays are reflected, thus giving a green tint to the illumination. Dark walls and ceilings result in a low efficiency of the lighting system, since much of the light produced is absorbed and lost.

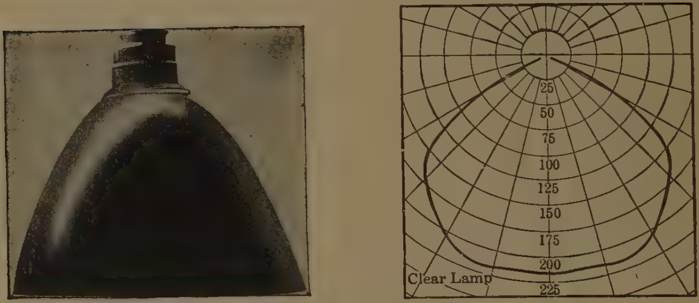
395. Intensity of Illumination.—Since the purpose of all illumination is to enable objects to be seen, the first consideration is to provide sufficient illuminating intensity. Fortunately the eye can adapt itself to a wide range of intensities by changes in the size of the pupil of the eye, and adjustments of its internal parts. Changes in the size of the pupil occur rapidly and tend to adjust automatically the amount of light which enters the eye. A further adaptation is made internally to suit a variation in the amount of light admitted. In brief, the eye attempts to adjust itself to secure comfortable vision. If the intensity is too low, the object cannot be seen distinctly because not enough light enters the eye even when the pupil is enlarged as much as possible; on the other hand, if the intensity is too high, more light is admitted to the eye, even when the pupil is contracted, than can be accommodated, and this results in discomfort and sometimes permanent injury to the eye. We can see well and without discomfort with daylight intensities ranging from 1 to 500 foot-candles, while under artificial lighting conditions the eye will adjust itself to intensities from less than 1 foot-candle to several hundred foot-candles. A smaller range of intensities for artificial illumination will, however, meet the usual requirements. The best intensity depends upon the use which is made of the illumination. For reading, 5 to 10 foot-candles intensity gives satisfactory results, while for work requiring close application to detail, such as drafting or tool-making, high intensities of from 10 to 20 foot-candles are necessary. The values of illumination intensity required for proper vision are influenced greatly by the surroundings and the contrasts in intensity which exist.

396. Glare is caused by excessive brightness of surfaces or objects in the field of vision. The worst examples of glare occur when the light source itself is in view or an image of the light source is reflected from some polished surface. When a bright surface is in the field of vision, the eye tries to adjust itself to the intensity of the bright surface; consequently an insufficient amount of light enters the eye from other objects and they are therefore seen only indistinctly, if at all. It is impossible, for example, to distinguish distant objects with a bright light nearby in the line of vision. Glare, besides reducing a person's visual power, may result also in eye-strain or, if long continued, may even permanently injure the eyes. All forms of modern illuminants, except the vapor-tube lamps, are so bright that the bare lamp should never be placed in the usual line of vision. If it is necessary to place the units where they are visible when looking in the usual directions, enclosing globes of opaque glass or frosted lamps should be used. With incandescent lamps, the reflectors usually enclose the lamps sufficiently to protect the eyes. Glare due to light reflected from a glazed paper, polished table top or even polished metal parts in process of manufacture may often be nearly as objectionable as the glare from an exposed lamp. Such surfaces therefore are to be avoided where possible, and in other cases diffusion of the light and a suitable direction for the light will eliminate the glare.

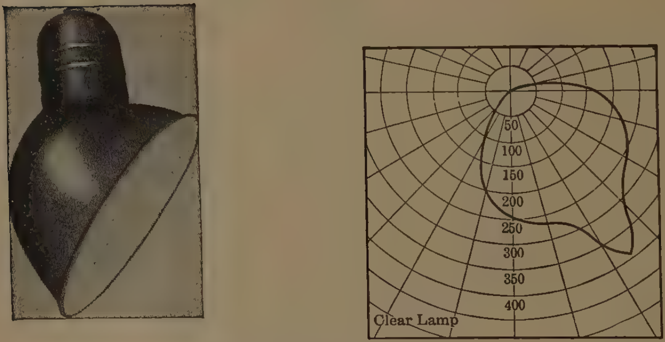
397. Diffusion of Illumination.—This should be distinguished from diffusion of the light from the lamp, which is considered in the next paragraph. If the light which illuminates an object comes from a number of different directions the illumination is said to be diffused. If all the light comes from a single source, for example a tungsten lamp, then the diffusion is poor and there are likely to be deep shadows and extreme contrasts in the illumination of different parts of the objects in the room. If the light comes from a number of sources, or if it is reflected first to the ceiling of the room, thus increasing the light-giving area, better diffusion is secured, shadows are eliminated to a greater or less degree, and the glare from polished surfaces is avoided. The highest degree of diffusion is secured with indirect or semi-indirect systems. With a direct system, satisfactory diffusion for most purposes can be secured if the lamps are closely spaced.



(*a*) Dome Type. (RLM.)



(*b*) Deep Bowl Type.



(*c*) Angle Type.

FIG. 359.—Typical Steel Reflectors for Tungsten Lamps. Showing characteristic light distribution curves. *Edison Lamp Works of G. E. Co.*

398. Lighting Accessories. **Diffusing globes** or balls entirely surround the lamp and are intended primarily to reduce its brilliancy. They also modify to some extent the distribution of the light. They are extensively used with large size tungsten lamps which require protection to avoid glare. **Reflectors** are used primarily to redirect the light in a useful direction. For industrial lighting, enameled-steel reflectors are extensively used. Common types of these reflectors are shown in Fig. 359. The light distribution curves given in this figure show the relative candle-power measured at various angles. Glass reflectors of the prismatic or translucent type are used where a more pleasing effect is desired or where it is necessary to allow a portion of the light to be directed towards the ceiling (Fig. 360). Opaque glass reflectors, having a silvered surface, are used extensively for window and show-case lighting.

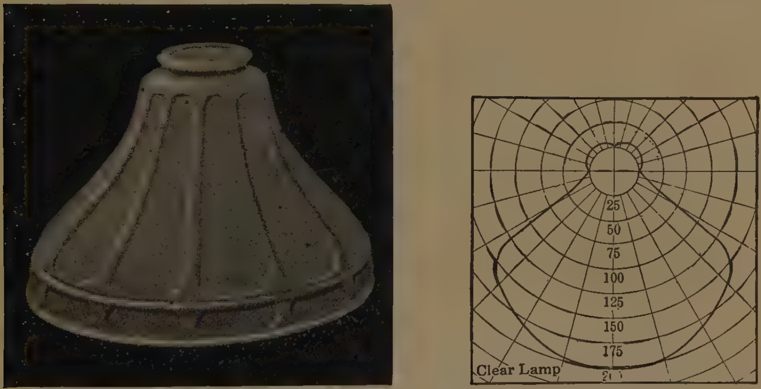
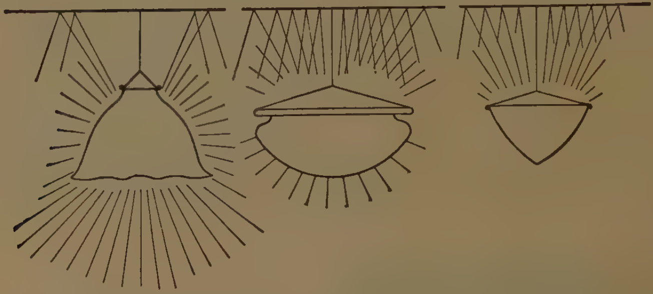


Fig. 360.—Deep Bowl, Translucent Glass Reflector. *Edison Lamp Works of G. E. Co.*

399. Systems of Illumination.—Objects may be illuminated either by the direct or by the indirect method and the systems of illumination are classified according to the method used. With the **direct system,** the largest proportion of the light is directed on the surface to be illuminated without being reflected by walls or ceiling. With the **indirect system,** all of the light is projected to the ceiling or walls and is then reflected on the object to be illuminated. The **semi-indirect system** is a combination of the other two, in which the greater proportion of the light is reflected from the ceiling or walls, the remainder reaching the object directly through diffusing globes which form a part of the lighting unit. In the direct system, a considerable part of the total light may be reflected from the ceiling and walls if they are light in color and if transparent reflectors are used on the lamps. Fig. 361 illustrates these different systems.

400. Lighting Fixtures vary in design, according to the type of illumination, and the importance attached to the appearance of the unit. For industrial lighting, using a direct system with metal reflectors, the arrangement shown in Fig. 362 is typical. An enclosing globe, suitable for high candle-power units is shown in Fig. 363. A semi-indirect unit consisting of a translucent-bowl type of reflector is shown in Fig. 364.



a. Direct. *b.* Semi-indirect. *c.* Indirect.
Fig. 361.—Systems of Illumination.

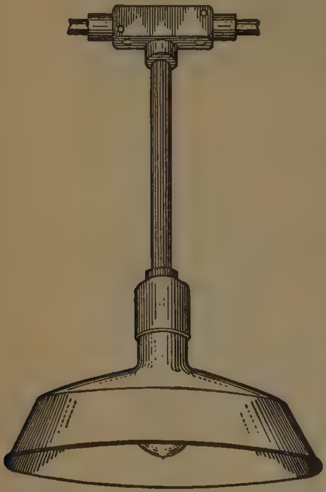


Fig. 362.—Industrial Type Fixture.

401. Illumination Units.—The candle-power or intensity of light emitted by a lamp is expressed in terms of a standard source of light, known as the international candle. A lamp which gives, in a specified direction, a light intensity twenty times as great as the standard candle would be said to give 20 candle-power in that direction. The light intensity produced by modern illumination sources varies in different directions as is shown by the light distribution curves in Fig. 363.

The **output of light** from a lamp is called the **light flux** and is measured in **lumens.** We can compare a lamp with a metal ball supplied with water under pressure. If a large number of small holes, equally spaced, are drilled in the ball, then neglecting the effect of the supply pipe, jets of water would issue from the ball in all directions. These jets may be compared with the light flux given off by a lamp. The total amount of water issuing from the ball would be the aggregate from all the holes and would correspond to the total light which would be given off by the lamp. This would be expressed in lumens. We may speak of the total lumens or total light flux produced by a lamp which includes the light given off in *all* directions, or we may consider only a particular portion of the light given off, as for example, the lumens in the lower hemisphere. We may also use the term **lumens per watt,** which is found by dividing the total lumens output by the power required to produce this output. This quantity is convenient for comparing the efficiency of two lamps. For example, a 500-watt tungsten lamp gives 8750 lumens, or 17.45 lumens per watt. A certain arc lamp requires 495 watts and has an output of 3650 lumens, or 7.37 lumens per watt. We can therefore say that the tungsten lamp is 2.4 times as efficient as the particular arc lamp considered. For purposes of comparison, it is also necessary to determine the amount of illumination produced at a given point by the light source. This is expressed by the

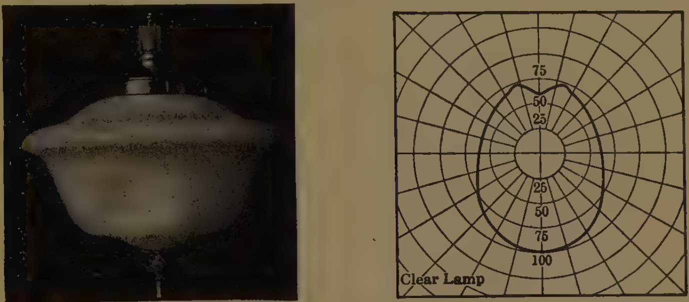


FIG. 363.—Opal Glass, Enclosing Globe. *Edison Lamp Works of G. E. Co.*

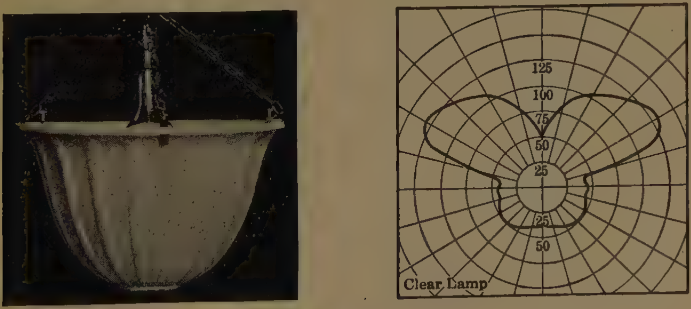


FIG. 364.—Opal Glass, Semi-indirect Reflector. *Edison Lamp Works of G. E. Co.*

term **foot-candles** [2] or lumens per square foot. Thus, if the flux of light on a given surface is such that there are 4 lumens per square foot, then we would say that the illumination produced was 4 foot-candles. It is necessary to distinguish carefully between the *candle-power* of a lamp and the *quantity of light* which reaches a particular object. The candle-power of a lamp in a certain direction is definite and would be the same if measured close to the lamp or at a distance of hundreds of feet, as has already been explained. The quantity of light which falls on a given area, however, that is, the lumens per square foot, depends upon the distance from the lamp.

402. Methods of Illumination.—In designing illuminating systems for interiors, the general practice is to approximate a uniform distribution of the light flux over the working area. In some cases, this arrangement is modified to the extent of providing a uniform general illumination of moderate intensity and supplementing this with additional light-units at points requiring a higher intensity.

The direct system of lighting is generally employed for industrial lighting and is also frequently used in many commercial installations, for example, store and office lighting. The semi-indirect system is extensively used for commercial installations where a reasonably high efficiency is required, but where it is also necessary that the light-units shall harmonize with the furnishings of a room. This system requires a larger expenditure of power than the direct system for the same intensity of illumination. The indirect system finds a limited use for drafting rooms, hotel bedrooms, etc. This system is less efficient than the semi-indirect, but it reduces the sharpness of shadows.

403. The Determination of Intensity of Illumination involves a knowledge of the character of work carried on in the room to be lighted. The desirable intensity must be determined by experience with installations having similar requirements. A considerable amount of data is available as a guide to selecting the proper intensity. The table on the next page gives a general idea of suitable illumination intensities. When solving specific illumination problems, however, more detailed data should be used. This information may be found in handbooks.[3]

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[2] By definition, a foot-candle is the illumination produced on a surface situated 1 foot distant from a light source of 1 candle-power. The foot-candle illumination of a surface can be obtained in any case by dividing the candle-power of the lamp in the direction considered by the square of the distance in feet from the lamp to the given surface. One foot-candle is about equal to the illumination produced on a surface situated 5 feet horizontally from a 25-watt tungsten lamp without reflector. This unit is now being replaced by the term *lumens per square foot.*

[3] A very comprehensive table of satisfactory illumination intensities is given in Standard Lighting Bulletin No. II, Illumination Design Data, published by H. C. Cushing, Jr.

ILLUMINATION INTENSITIES FOR INDUSTRIAL LIGHTING

Class of Work	Minimum * Foot-candles	Recommended † Foot-candles
Yards and roads	0.02	0.25
Storage	0.25	2.00
Stairways and aisles	0.25	
Rough manufacturing	0.5—1	5.00
Medium manufacturing	2.00	
Fine manufacturing	3.00	10.00
Very fine manufacturing	5.00	20.00
Office work	......	10.00
Toilets and wash rooms	......	5.00

* Minimum values are substantially the same as are required by the factory inspection boards of a number of states.

† Recommended intensities are those accepted as good practice.

The above values are based on recommendations of the American Engineering Standard's Code, compiled by the engineering societies of the United States.

In choosing an intensity, allowance should be made for accumulation of dust on the reflectors. The amount that the intensities, given in the table, should be increased to allow for depreciation caused by dust, varies from 30 to 50 per cent depending upon the frequency with which the reflectors are cleaned and the amount of dust in the air.

404. Determination of Power Required.—The power required to produce a given illumination intensity depends principally upon the reflector equipment used, the size of the room, and the color of walls and ceiling. This is shown in the tabulation, on the next page which gives the approximate power required to produce unit illumination with different kinds of reflector equipment. The values given may be used to approximate the amount of power required for a given installation, although for more accurate work, the dimensions of room, height of unit and reflecting character of the walls and ceiling must be taken into consideration.[4]

Example 1.—A machine shop is 128 feet long by 44 feet wide, and is divided into 16 bays, 16 feet by 22 feet. The ceiling is 14 feet high. Rather fine manufacturing is carried on in this shop, so an intensity of 8 foot-candles will be allowed. (See Article 403.) The ceiling is fairly light and the walls fairly dark; hence, according to the tabulation on the next page, it would require 0.14 watt per square foot to produce one foot-candle illumination. To produce 8 foot-candles, allowing for 30 per cent depreciation due to dust, would require $8 \times 0.14 = 1.12$ watt per square foot. Therefore, the total power required to light the room is $128 \times 44 \times 1.12 = 6330$ watts.

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[4] See "Interior Wiring," by Arthur L. Cook.

Power Required to Produce One Foot-candle Illumination

TUNGSTEN LAMPS,* UNIFORM LIGHTING

Watts per Square Foot †

Reflector Equipment	Color of Ceiling Very light			Color of Ceiling Fairly light			Color of Ceiling Fairly dark	
	Color of Walls Fairly Light	Color of Walls Fairly Dark	Color of Walls Very Dark	Color of Walls Fairly Light	Color of Walls Fairly Dark	Color of Walls Very Dark	Color of Walls Fairly Dark	Color of Walls Very Dark
(1) Enameled-steel reflectors, clear lamp	0.13	0.14	0.16	0.14	0.14	0.16	0.14	0.16
(2) Light density, opal-glass reflectors, bowl enameled lamp......	0.22	0.23	0.28	0.25	0.28	0.33	0.31	0.35
(3) Prismatic-glass reflectors, clear lamp	0.16	0.17	0.20	0.17	0.18	0.20	0.20	0.21
(4) Diffusing globe.....	0.25	0.29	0.36	0.27	0.33	0.38	0.37	0.42
(5) Semi-indirect, light opal glass, bowl reflectors............	0.26	0.30	0.35	0.31	0.36	0.42	0.47	0.52
(6) Indirect, mirror reflectors............	0.31	0.35	0.40	0.42	0.50	0.56	0.83	0.93

* For gas-filled lamps, with an allowance of 30 per cent for depreciation due to dust.

† The values given apply to mounting heights and floor areas as follows:

(1) Mounting height, 10 feet above working plane, 2500 to 7500 square feet area.

(2), (3) and (4) Mounting height 10 feet above working plane, 1500 to 2500 square feet area.

(5) and (6) Ceiling 12 feet above working plane, 700 to 1500 square feet area.

The required power is greater for smaller rooms or higher mounting height.

405. Arrangement of Outlets.—The spacing of outlets depends upon the mounting height above the plane of the work[5] and upon the type of reflector used. For the ordinary designs of reflectors, such as items (1), (2), or (3) or for enclosing globes, item (4), in the tabulation of Article 404, the distance between outlets should be about 50 per cent greater than the height of the *reflector* above the working plane. For semi-indirect or totally indirect units items (5) or (6), the spacing is about 50 per cent greater than the height of the *ceiling*, measured from the working plane. In general, a symmetrical arrangement of the outlets should be used, and only one size of outlet should be

[5] Usually this is assumed to be 2.5 feet above the floor.

employed in a particular area. Each outlet should be over the center of an imaginary square (Fig. 365 *a*) and not over the corners (Fig. 365 *b*).

Example 1.—Referring to the machine shop of Example 1, Article 404, the mounting height above the plane of the work can be about 10 feet. Using two units per bay, the longitudinal spacing would be 16 feet. The spacing between units in a bay could be 11 feet, 4 inches, with the row nearest the wall 5 feet away, in order to take care of benches. If there were no benches, the distance to the wall could be made greater. This arrangement would require 16×2=32 units, and if 200-watt units were used, the total power would be 200×32=6400 watts, which checks with the value determined in Example 1, Article 404. The reflectors would be mounted about 12 inches below the ceiling.

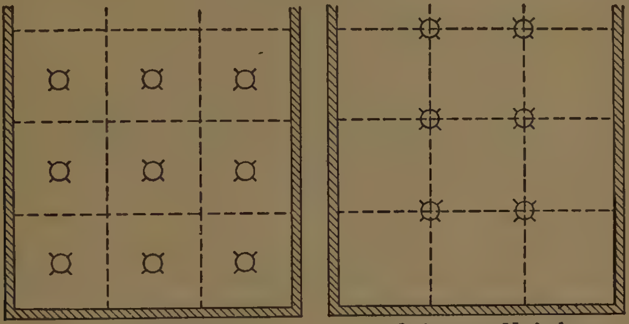
a—Correct Method *b*—Incorrect Method

FIG. 365.—Method of Locating Light Units for Uniform Illumination.

PROBLEMS ON CHAPTER XL

1–40. A tungsten lamp is designed to operate at 115 volts and to give a life of 1000 hours when operated at that voltage.

(*a*) If the lamp were operated at 125 volts what would be the effect upon, (1) the color of the light produced? (2) the life of the lamp?

(*b*) If the lamp were operated at 100 volts, what would be the effect on (1) and (2)?

2–40. Which gives the more light for a given expenditure of energy, a lamp (*A*) which has a rating of 10.2 lumens per watt, or a lamp (*B*) which has a rating of 15.2 lumens per watt?

3–40. An incandescent lamp (*C*) consumes 25 watts and has a total light output of 240 lumens; another lamp (*D*) consumes 500 watts and has a light output of 9050 lumens. Which lamp is the more efficient?

4–40. What are the principal points of difference in construction of a tungsten lamp of the vacuum type (Mazda B) and the gas-filled type (Mazda C)?

5–40. Why is a concentrated filament used in a gas-filled (Mazda C) lamp?

6–40. Why is nitrogen or a similar gas used in Mazda C lamps?

7–40. Why is it that the base and socket containing a gas-filled lamp will run hotter than a similar socket containing a vacuum-type lamp which consumes the same number of watts?

8–40. Why is rubber-covered wire to be avoided in the wiring of sockets for large gas-filled lamps?

9–40. How is the light produced in a mercury-vapor lamp?

10–40. State the principal differences between the a.c. and the d.c. Cooper Hewitt lamps.

11–40. What are the principal applications of mercury-vapor lamps?

12–40. What are the advantages and disadvantages of the mercury-vapor lamps?

13–40. What is the disadvantage of using dark-colored walls or ceilings in a room?

14–40. A certain tungsten lamp consumes 100 watts and gives a total light output of 1280 lumens. An arc lamp consuming 715 watts gives an output of 10,000 lumens. Which lamp is the more efficient?

15–40. A certain room requires for its proper illumination a total of 175,000 lumens of light flux. If tungsten lamps, which produce 12.6 lumens per watt are used, how many watts are required to light the room?

16–40. A room is illuminated by 100-watt tungsten lamps, each giving a total output of 1000 lumens. Assume that for proper illumination there must be a total of 35,000 lumens directed below the horizontal center line of the lamps, and that the light above horizontal can be neglected because the ceiling is dark. Calculate the total power in watts required to light the room:

(*a*) When bare lamps are used.
(*b*) When dome-type steel reflectors are used.
(*c*) When prismatic-glass reflectors are used.

Note: Assume 50 per cent of light below horizontal for bare lamp, 80 per cent for steel reflector and 65 per cent for prismatic-glass reflector.

17–40. Referring to Prob. 16–40, assume that the ceiling is made light in color so that 20 per cent of the light delivered above the horizontal is reflected to the floor and becomes useful in lighting the work. Assuming as before, 35,000 lumens required below the horizontal, calculate the total power for:

(*a*) With bare lamps.
(*b*) Dome-type steel reflectors.
(*c*) Prismatic reflectors.

Compare results with those in Prob. 16–40.

Note: Assume 20 per cent of light absorbed by steel reflector, and 15 per cent by glass reflector.

18–40. In Prob. 16–40, a total light flux of 35,000 lumens is assumed to strike the working surface, which has an area of 8750 square feet. If the light is distributed practically uniformly, what is the illumination of the working surface, expressed in foot-candles?

19–40. A room, 30 feet by 125 feet, requires a uniform illumination of 4.5 foot-candles over the entire floor area. Calculate the number of lumens of light flux which must strike the floor.

20–40. Assume in Prob. 19–40 that tungsten lamps are used with reflectors which deliver 45 per cent of the total light output to the floor level. Calculate the total light flux which must be produced by the lamps.

21–40. A room, 35 feet by 95 feet, is lighted by twenty-five 100-watt, 120-volt Mazda C lamps, uniformly spaced and equipped with prismatic-glass reflectors. Ceiling very light, walls fairly light.

(*a*) Calculate the total power required to light the room.

(*b*) Calculate the average illumination intensity on the working plane.

22–40. For the lamps specified in Prob. 21–40,

(*a*) Calculate the illumination intensity if semi-indirect lighting were used.

(*b*) Calculate the illumination intensity if indirect, mirror reflectors were used.

23–40. Calculate results as required in Probs. 21–40 and 22–40 under the conditions specified, except ceiling and walls are fairly light.

24–40. A room, 25 feet by 60 feet, has ten 100-watt, Mazda C tungsten lamps equipped with steel reflectors, using a direct system. If the ceilings and walls are fairly light determine the illumination intensity.

25–40. If in Prob. 24–40 the same lamps were used with translucent glass enclosing globes, calculate the intensity.

26–40. The room specified in Prob. 21–40 is to be lighted by the semi-indirect system, using bowl-type reflectors. (*a*) If the illumination is kept the same as in Prob. 21–40,

(*b*) Calculate the power required to operate the tungsten lamps.

(*c*) What would be the power required if an indirect system, with mirror reflectors, were used and the illumination were kept the same?

27–40. A factory doing fine work uses a direct system with gas-filled tungsten lamps. The total floor area is 100,000 square feet. Calculate the power required for lighting.

28–40. For the machine shop specified in Example 1, Article 404, assume that the walls and ceiling become so dirty by neglect that they may be classed as fairly dark and very dark, respectively; also that the reflectors have not been cleaned for six months. What would be the approximate illumination intensity on the working plane?

APPENDIX

TABLE 1

Copper Wire Table, Solid Conductors

Size A. W. Gage (B.&S.)	Diameter Mils	Area Circular Mils	Area Square Mils	* Resistance Ohms per 1000 ft.	Weight Pounds per 1000 ft.	Ampere Carrying Capacity †		
						Rubber Insulation	Varnished Cloth Insulation	Other Insulations
0000	460	211,600	166,000	0.0500	641	225	270	325
000	410	167,800	132,000	.0630	508	175	210	275
00	365	133,100	105,000	.0795	403	150	180	225
0	325	105,500	82,900	.100	320	125	150	200
1	289	83,690	65,700	.126	253	100	120	150
2	258	66,370	52,100	.159	201	90	110	125
3	229	52,640	41,300	.201	159	80	95	100
4	204	41,740	32,800	.253	126	70	85	90
5	182	33,100	26,000	.320	100	55	65	80
6	162	26,250	20,600	.403	79.5	50	60	70
7	144	20,820	16,400	.508	63	40	50	55
8	129	16,510	13,000	.641	50	35	40	50
9	114	13,090	10,300	.808	39.6	30	35	40
10	102	10,380	8,160	1.02	31.4	25	30	30
11	91	8,234	6,470	1.28	24.9	20	25	30
12	81	6,530	5,130	1.62	19.8	20	25	25
13	72	5,178	4.070	2.04	15.7	18	20	25
14	64	4,107	3,230	2.58	12.4	15	18	20
15	57	3,257	2,560	3.25	9.86	9	. . .	10
16	51	2,583	2,030	4.09	7.82	6	. . .	10
17	45	2,048	1,610	5.16	6.20	4	. . .	6
18	40	1,624	1,280	6.51	4.92	3	. . .	5
19	36	1,288	1,010	8.21	3.90			
20	32	1,022	802	10.4	3.09			
21	28.5	810.1	636	13.1	2.45			
22	25.4	642.4	505	16.5	1.95			
23	22.6	509.5	400	20.8	1.54			
24	20.1	404.0	317	26.2	1.22			
25	17.9	320.4	252	33.0	0.97			
26	15.9	254.1	200	41.6	0.769			
27	14.2	201.5	158	52.5	0.610			
28	12.6	159.8	126	66.2	0.484			
29	11.3	126.7	99.5	83.4	0.384			
30	10.0	100.5	78.9	105	0.304			
31	8.9	79.70	62.6	133	0.241			
32	8.0	63.21	49.6	167	0.191			
33	7.1	50.13	39.4	211	0.152			
34	6.3	39.75	31.2	266	0.120			
35	5.6	31.52	24.8	336	0.0954			
36	5.0	25.00	19.6	423	0.0757			
37	4.5	19.83	15.6	533	0.0600			
38	4.0	15.72	12.4	673	0.0476			
39	3.5	12.47	9.8	848	0.0377			
40	3.1	9.89	7.8	1070	0.0299			

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* At 25° C. Temperature coefficient 0.00385 per degree C. K = 10.6 ohms per circular-mil-foot at 45° C. (Bureau of Standards Circular 31.)

† National Electric Code.

TABLE 2

COPPER WIRE TABLE, STRANDED CONDUCTORS

Size A. W. Gage or Circular Mils	Diameter Mils	Area Circular Mils	* Resistance Ohms per 1000 ft.	Weight Pounds per 1000 ft.	Ampere Carrying Capacity †		
					Rubber Insulation	Varnished Cloth Insulation	Other Insulations
4	232	41,700	0.259	129	70	85	90
3	260	52,600	0.205	163	80	95	100
2	292	66,400	0.162	205	90	110	125
1	332	83,700	0.129	258	100	120	150
0	373	106,000	0.102	326	125	150	200
00	418	133,000	0.0811	411	150	180	225
000	470	168,000	0.0642	518	175	210	275
0000	528	212,000	0.0509	653	225	270	325
250,000	575	250,000	0.0431	772	250	300	350
300,000	630	300,000	0.0360	926	275	330	400
350,000	681	350,000	0.0308	1080	300	360	450
400,000	728	400,000	0.0270	1240	325	390	500
450,000	772	450,000	0.0240	1390	350	435	550
500,000	814	500,000	0.0216	1540	400	480	600
550,000	855	550,000	0.0196	1700	425	510	640
600,000	893	600,000	0.0180	1850	450	540	680
650,000	929	650,000	0.0166	2010	475	570	720
700,000	964	700,000	0.0154	2160	500	600	760
750,000	998	750,000	0.0144	2320	525	630	800
800,000	1031	800,000	0.0135	2470	550	660	840
850,000	1062	850,000	0.0127	2620	575	690	880
900,000	1093	900,000	0.0120	2780	600	720	920
950,000	1123	950,000	0.0114	2930	625	750	960
1,000,000	1152	1,000,000	0.0108	3090	650	780	1000
1,100,000	1209	1,100,000	0.00981	3400	690	830	1080
1,200,000	1263	1,200,000	0.00899	3710	730	880	1150
1,300,000	1315	1,300,000	0.00830	4010	770	920	1220
1,400,000	1364	1,400,000	0.00770	4320	810	970	1290
1,500,000	1412	1,500,000	0.00719	4630	850	1020	1360
1,600,000	1459	1,600,000	0.00674	4940	890	1070	1430
1,700,000	1504	1,700,000	0.00634	5250	930	1120	1490
1,800,000	1548	1,800,000	0.00599	5560	970	1160	1550
1,900,000	1590	1,900,000	0.00568	5870	1010	1210	1610
2,000,000	1631	2,000,000	0.00539	6180	1050	1260	1670

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* At 25° C. Temperature coefficient 0.00385 per degree C. K = 10.6 ohms per circular-mil-foot at 25° C. (Bureau of Standards, Circular 31.)

† National Electric Code.

TABLE 3

Demand Factors for Motor Loads*

No. of Motors	Character of Load	Demand Factor
1	Individual drives—tools, etc.	1.25
2–5	Individual drives—tools, etc.	1.00†
6–10	Individual drives—tools, etc.	0.75
Above 10	Individual drives—tools, etc.	0.60 to 0.40
1	Group drives	1.25
2–5	Group drives	1.00†
6–10	Group drives	0.85
Above 10	Group drives	0.70 to 0.45
1	Fans, compressors, pumps, etc.	1.25
2 or more	Fans, compressors, pumps, etc.	1.00 to 0.85

* Ratio of maximum load to connected load.

Above is for 40-degree motors. For 50-degree motors, the factor for one motor would be from 1.10 to 1.15 and correspondingly less for a larger number of motors.

† Note that feeders supplying a small number of motors, up to six, should have a carrying capacity sufficient for the starting current of the largest motor and the sum of the full-load currents of all the others.

The above values make no allowance for future increase in the load.

TABLE 4

Current and Size of Wire for Direct Current Motors

Horse-power	Amperes at Full Load			Size of Wire *								
				Rubber Insulation			Varnished Cloth Insulation †			Other Insulation		
	115 Volts	230 Volts	550 Volts	115 Volts	230 Volts	550 Volts	115 Volts	230 Volts	550 Volts	115 Volts	230 Volts	550 Volts
0.5	5	2.5	1.1	14	14	14	14	14	14	14	14	14
1	8.8	4.4	1.8	14	14	14	14	14	14	14	14	14
2	16	8	3.4	12	14	14	12	14	14	14	14	14
3	24	12	5	8	14	14	10	14	14	10	14	14
5	40	20	8.4	6	10	14	6	12	14	8	12	14
7.5	58	29	12.1	3	6	14	4	8	14	5	8	14
10	76	38	15.9	1	6	12	2	6	12	3	8	14
15	112	56	23.4	00	4	8	0	4	8	1	6	10
20	146	73	30.5	0,000	1	6	000	2	8	0	3	8
25	182	91	38.1	0,000	0	6	0,000	0	6	00	2	8
30	216	108	45.2	300,000	00	4	0,000	0	6	000	1	6
35	252	126	52.6	400,000	000	4	300,000	00	4	0,000	0	6
40	288	144	60.2	500,000	0,000	3	400,000	000	4	300,000	0	5
50	356	178	74.4	600,000	0,000	1	500,000	0,000	2	350,000	00	3
60	428	214	89.5	800,000	350 000	0	600,000	0,000	2	500,000	000	2
75	532	266	111	1,100,000	450,000	00	750,000	300,000	0	600,000	250,000	1
100	710	355	148	1,700,000	600,000	0,000	1,200,000	500,000	00	900,000	350,000	0
125	886	443	185	Two 850,000	850,000	300,000	1,700,000	600,000	0,000	1,200,000	500,000	00
150	1076	538	224	Two 1,100,000	1.100,000	400,000	Two 900,000	900,000	250,000	1,600,000	700,000	0,000

* Based on 40-degree motors and 25 per cent overload. For 50-degree motors the fusing would be for about 15 per cent overload.

† Note that varnished cloth wires smaller than No. 6 may be used only by special permission.

TABLE 5

Current and Size of Wire for Three-phase Induction Motors

Horse-power	Amperes, Full Load				Size of Wire † Rubber or Other Insulation			
	110 V.	220 V.	440 V.	550 V.	110 V.	220 V.	440 V.	550 V.
0.5*	3.6	1.8	0.9	0.7	14	14	14	14
1.0*	6.4	3.2	1.6	1.3	12	12	14	14
2.0*	11.6	5.8	2.9	2.3	8	12	14	14
3.0*	16.4	8.2	4.1	3.3	6	10	14	14
5.0	26.8	13.4	6.7	5.4	6	10	12	14
7.5	39.2	19.6	9.8	7.9	5	8	12	14
10.0	53.2	26.6	13.3	10.7	3	6	8	12
15.0	77.0	38.6	19.3	15.5	1	4	8	10
20.0	109.0	54.6	27.3	21.8	00	3	6	8
25.0	125.0	62.6	31.3	25.1	000	1	5	8
35.0	......	85.6	42.8	34.3	....	0	4	6
50.0	......	122.0	61.0	48.6	....	000	1	3
75.0	......	179.0	89.0	72.0	....	300,000	0	1
100.0	......	237.0	118.0	95.0	....	500,000	000	0
150.0	......	353.0	176.0	141.0	....	Two 300,000	300,000	0,000
200.0	......	451.0	226.0	181.0	....	Two 400,000	400,000	300,000
250.0	......	560.0	280.0	224.0	....	Two 500,000	500,000	400,000
300.0	......	670.0	335.0	268.0	....	Two 600,000	600,000	500,000

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* These motors are thrown directly on the line; all others are provided with auto-starters set to give a starting torque equal to full-load running torque.

† Sizes of wire are for squirrel-cage motors. For slip-ring motors, smaller sizes can be used. See Article 293. The rules of the National Electric Code permit branch circuits which are insulated with rubber or varnished cloth, when used to supply a single induction motor, to be fused at the same values as for weatherproof or other similar insulations.

TABLE 6

Current and Size of Wire for Two-phase Induction Motors

Horse-power	Amperes, Full Load †				Size of Wire ‡ Rubber or Other Insulation			
	110 V.	220 V.	440 V.	550 V.	110 V.	220 V.	440 V.	550 V.
0.5*	3.2	1.6	0.8	0.6	14	14	14	14
1.0*	5.6	2.8	1.4	1.1	12	14	14	14
2.0*	10.0	5.0	2.5	2.0	8	12	14	14
3.0*	14.4	7.2	3.6	2.9	6	10	14	14
5.0	23.2	11.6	5.8	4.7	6	10	12	14
7.5	34.0	17.0	8.5	6.8	5	8	12	12
10.0	46.0	23.0	11.5	9.2	4	6	10	10
15.0	66.8	33.4	16.7	13.4	1	6	8	8
20.0	94.4	47.2	23.6	18.9	0	3	6	8
25.0	108.4	54.2	27.1	21.7	00	2	6	6
35.0	......	74.2	37.1	29.7	....	1	5	6
50.0	......	105.0	52.6	42.1	....	00	2	4
75.0	......	155.0	77.3	61.9	....	0,000	0	2
100.0	......	205.0	103.0	82.0	....	300,000	00	0
150.0	......	306.0	153.0	123.0	....	Two 0,000	0,000	000
200.0	......	390.0	195.0	156.0	....	Two 300,000	300,000	0,000
250.0	......	484.0	242.0	194.0	....	Two 400,000	400,000	300,000
300.0	......	580.0	290.0	232.0	....	Two 500,000	500,000	400,000

* These motors are thrown directly on the line; all others are provided with auto-starters set to give a starting torque equal to full-load running torque.

† Values of current are for a two-phase, four-wire system; if three wires are used, current in common wire would be 1.41 times value given. Values for single-phase motors would be double the values given in the table.

‡ Sizes of wire are for squirrel-cage motors. For slip-ring motors, smaller sizes can be used. See Article 293. The rules of the National Electric Code permit branch circuits which are insulated with rubber or varnished cloth, when used to supply a single induction motor, to be fused at the same values as for weatherproof or other similar insulations.

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